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Cordiality in the path union of vertex switching of wheel graphs in increasing order *

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Abstract The *cordial labeling* of a graph G is a function $f : V(G) \to \{0,1\}$ such that every edge uv in G is consigned the label $|f(u) - f(v)|$ with the property $|v_f(0) - v_f(1)| \le 1$ and $|e_{f*}(0) - e_{f*}(1)| \le 1$, where the number of vertices is denoted by $v_f(i)$ for i = 0,1 and the number of edges is denoted as $e_{f*}(i)$ for i = 0,1. The graph which concedes cordial labeling is called a *cordial graph*. In this paper, we prove that the path union of vertex switching of wheel graphs is cordial.

Key words Cordial labeling, Path union, Vertex switching, Wheel graph.

2010 Mathematics Subject Classification 05C78, 05C99.

1 Introduction

Graph labeling methods started its origin due to the graceful labeling introduced by Rosa [7] in 1967. One among the graph labeling is the cordial labeling introduced by Cahit [2] in 1987. The *cordial labeling* of a graph G is a function f from $V(G)$ to $\{0,1\}$ such that each edge xy in G is assigned the label $|f(x) - f(y)|$ with the property $|v_f(0) - v_f(1)| \le 1$ and $|e_{f*}(0) - e_{f*}(1)| \le 1$, i.e. the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1 and the number of edges labeled 0 and the number of edges labeled 1 also differ by at most 1. So many graphs are demonstrated to be cordial. Andar et al. [1] have proved that the helms, closed helms, flowers, gears and sunflower graphs are cordial. In [3] Cahit showed that every tree is cordial, all fans are cordial, the wheel W_n when $n \not\equiv 3\,(mod 4)$ is cordial, the complete graph K_n is cordial if and only if $n \le 3$, the bipartite graph $K_{m,n}$ is cordial for all m and n, the friendship graph $C_3^{(t)}$ is cordial if and only if $t \not\equiv 2\,(mod 4)$. For a graph G and a vertex v of G Vaidya et al. [8] defined a vertex switching G_v as the graph obtained from G by removing all edges incident to v and adding edges joining v to every vertex not adjacent to v in G. They proved that the graphs obtained by the switching of a vertex in C_n admit cordial labelings.

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Ghodasara and Rokad [5] proved the star of $K_{n,n}$ $(n > 2)$ is cordial, the path union of $K_{n,n}$ $(n > 2)$ is cordial and the graph obtained by joining two copies of $K_{n,n}$ $(n > 2)$ by a path is cordial. In [6] the same authors prove that a vertex switching of any non-apex vertex of a wheel graph, a vertex switching of any internal vertex of a flower graph, a vertex switching of any non-apex vertex of a gear graph, and a vertex switching of any non-apex vertex of a shell graph are cordial graphs. An extensive survey of cordial labeling methods is done in [4] by Gallian.
In this paper, we prove that the path union of vertex switching of wheel graphs in increasing order is cordial.

2 Preliminary definitions and main result

Below we give three preliminary definitions and then state and prove the main result of this paper in the Theorem 2.4.

Definition 2.1. A wheel W_n is defined as $K_1 + C_{n-1}, n \geq 4$. In a wheel W_n, the vertex corresponding to K_1 is known as the apex vertex, the vertices corresponding to the cycle C_{n-1} are known as rim vertices while the edges corresponding to the cycle are known as rim edges and edges joining the apex vertex and the vertices of the cycle are called the spoke edges.

Definition 2.2. [8] A *vertex switching* G_v of a graph G is the graph obtained by taking a vertex v of G, removing all the edges incident to v and adding edges joining v to every other vertex which are not adjacent to v in G.

Definition 2.3. The path union of a graph G is the graph obtained by adding an edge from n copies $G_1, G_2, \ldots, G_n$ $(n \geq 2)$ of G from G_i to G_{i+1} for $i = 1, 2, \ldots, n-1$. We denote this graph by $P(n \cdot G)$.

Theorem 2.4. *The path union of vertex switching of wheel graphs in increasing order is cordial.*

Proof. Let G be a wheel W_n with n vertices that are denoted by $v_1, v_2, \ldots, v_n$ in the anticlockwise direction where, $v_1, v_2, \ldots, v_{n-1}$ denote the rim vertices and v_n denotes the apex vertex. Let H be the vertex switching graph of the graph G with anyone of the rim vertex say $v_1 \in G$ as the switching vertex. The edges incident with the switching vertex are removed and the vertices which are not adjacent to it are joined by an edge. The graphs G and H are described below in the Fig. 1.

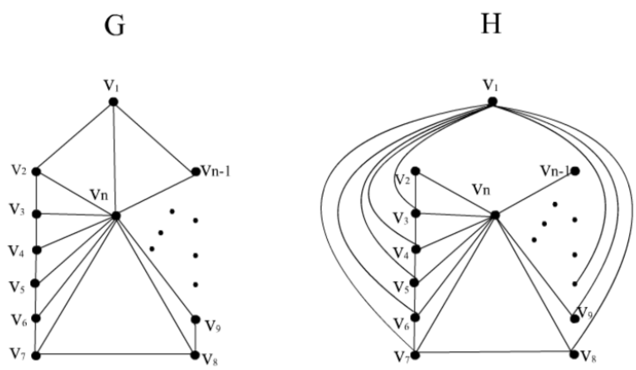

Fig. 1: The wheel graph and its vertex switching.

Case 1: Even wheels ($n_i \equiv 0\,(mod 2)$ and $n_i \geq 6$)
Let $H_1, H_2, \ldots, H_m$ be the copies of H in increasing order as revealed in the Fig.2 below. The first copy H_1 of H is described as follows: denote the switching vertex of H_1 as v_1^1. Denote the remaining vertices in H_1 as $v_2^1, v_3^1, v_4^1, v_5^1$ in the anticlockwise direction and the apex vertex is denoted as v_6^1. The second copy H_2 of H is described as follows: denote the switching vertex of H_2 as v_1^2. Denote the remaining vertices in H_2 as $v_2^2, v_3^2, \ldots, v_7^2, v_8^2$ in the anticlockwise direction. Finally the last copy H_m

of H is described by denoting the switching vertex as v_1^m. The remaining vertices of H_m are denoted as $v_2^m, v_3^m, \ldots, v_k^m$ where $k = n_i$ for $1 \leq i \leq m$ in the anticlockwise direction.
Let H' be the graph obtained by adding an edge e_i between the switching vertices v_1^i and v_1^{i+1} of the copies H_i and $H_{i+1}, 1 \leq i \leq (m-1)$. The graph H' so obtained is called the path union of vertex switching of wheel graphs as presented in the Fig. 2.

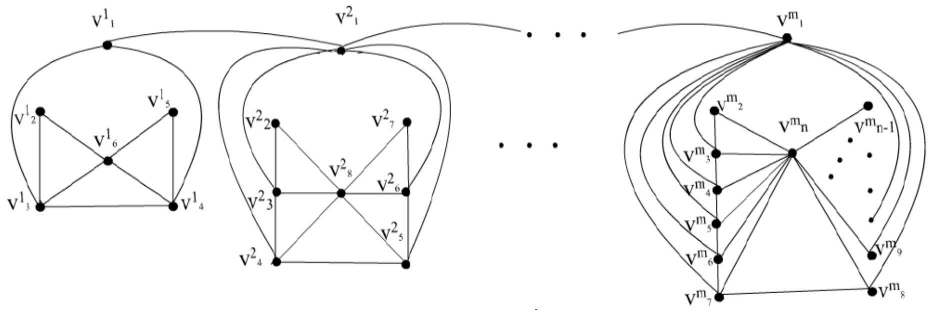

Fig. 2: The graph H'.

Note that in H' the switching vertices are v_1^i for $1 \leq i \leq m$ and the remaining vertices are v_j^i for $(1 \leq i \leq m), (2 \leq j \leq n_1)$. If p denotes number of vertices in H' then $p = \sum_{i=1}^{m} n_i$ and if q denotes the number of edges in H' then $q = 3\sum_{i=1}^{m} n_i - 8m - 1$ where $n_i \geq 6$.
The vertices of H' are labeled as follows:
For $1 \leq i \leq m$ where $i \equiv 1, 2\,(mod4)$

$$f\left(v_j^i\right) = \begin{cases} 1, & 1 \leq j \leq n_i - 1, \quad j \equiv 0, 1\,(mod4) \\ 0, & 1 \leq j \leq n_i - 1, \quad j \equiv 2, 3\,(mod4) \end{cases}$$

For $1 \leq i \leq m$ where $i \equiv 0, 3\,(mod4)$

$$f\left(v_j^i\right) = \begin{cases} 0, & 1 \leq j \leq n_i - 1, \quad j \equiv 0, 1\,(mod4) \\ 1, & 1 \leq j \leq n_i - 1, \quad j \equiv 2, 3\,(mod4) \end{cases}$$

$$f\left(v_{n_i}^i\right) = \begin{cases} 0, & 1 \leq i \leq m, \quad i \equiv 0, 1\,(mod4) \\ 1, & 1 \leq i \leq m, \quad i \equiv 2, 3\,(mod4) \end{cases}$$

$\Rightarrow v_f(0) = \frac{p}{2}, v_f(1) = \frac{p}{2}$ and $e_{f*}(0) = \left\lceil \frac{q}{2} \right\rceil, e_{f*}(1) = \left\lfloor \frac{q}{2} \right\rfloor$

$$\Rightarrow |v_f(0) - v_f(1)| = \left| \frac{p}{2} - \frac{p}{2} \right| = 0$$

$$\Rightarrow |e_{f*}(0) - e_{f*}(1)| = \left| \left\lceil \frac{q}{2} \right\rceil - \left\lfloor \frac{q}{2} \right\rfloor \right| = 1$$

From the above labelings it is clear that $|v_f(0) - v_f(1)| \leq 1$ and $|e_{f*}(0) - e_{f*}(1)| \leq 1$. Therefore the graph H' is cordial. The above case is illustrated below in the Fig. 3.

Illustration 2.5.

$$m = 4, n_1 = 6, n_2 = 8, n_3 = 10, n_4 = 12, p = 36, q = 75$$

$$v_f(0) = 18, v_f(1) = 18 \Rightarrow |v_f(0) - v_f(1)| = |18 - 18| \leq 1$$

$$e_f(0) = 38, e_f(1) = 37 \Rightarrow |e_{f*}(0) - e_{f*}(1)| = |38 - 37| \leq 1$$

Case 2: Odd wheels ($n_i \equiv 1\,(mod2)$ and $n_i \geq 7$)
Let $H_1, H_2, \ldots, H_m$ be the copies of H in an increasing order as shown in the Fig.4. The first copy H_1 of H is described as follows: denote the switching vertex of H_1 as v_1^1. Denote the remaining vertices in H_1 as $v_2^1, v_3^1, \ldots, v_6^1$ in the anticlockwise direction and the apex vertex is denoted as v_7^1. The second copy H_2 of H is described as follows: denote the switching vertex of H_2 as v_1^2. Denote the remaining vertices in H_2 as $v_2^2, v_3^2, \ldots, v_9^2$ in the anticlockwise direction. Finally the last copy H_m of H is described

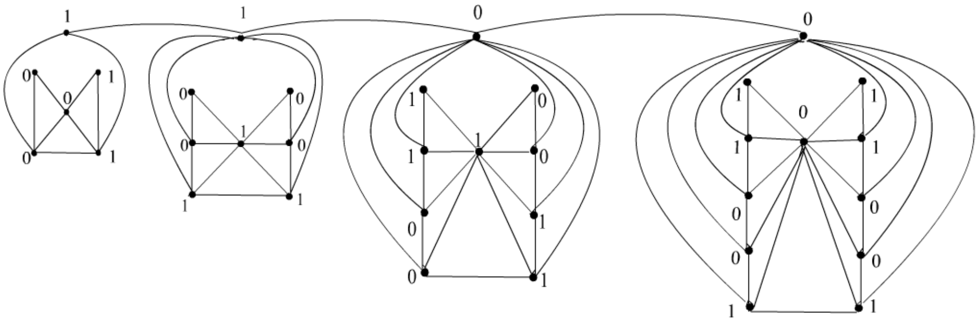

Fig. 3: Path union of four copies in the increasing order.

by denoting the switching vertex as v_1^m. The remaining vertices of H_m are denoted as $v_2^m, v_3^m, \ldots, v_k^m$ where $k = n_i$ for $1 \leq i \leq m$ in the anticlockwise direction.
Let H' be the graph obtained by adding an edge e_i between the switching vertices v_1^i and v_1^{i+1} of the copies H_i and $H_{i+1}, 1 \leq i \leq (m-1)$. The graph H' so obtained is called the path union of vertex switching of wheel graphs as shown in the Fig. 4.

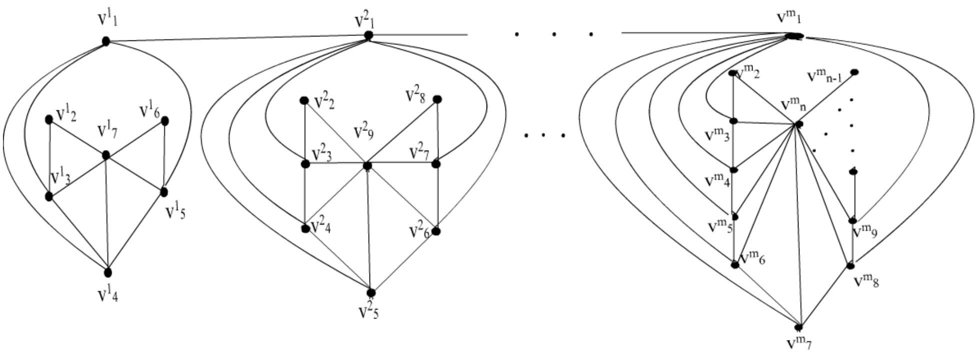

Fig. 4: The graph H'.

Note that in H' the switching vertices are v_1^i for $1 \leq i \leq m$ and the remaining vertices are v_j^i for $(1 \leq i \leq m), (2 \leq j \leq n_i)$. If p denotes number of vertices in H' then $p = \sum_{i=1}^{m} n_i$ and if q denotes the number of edges in H' then $q = 3\sum_{i=1}^{m} n_i - 8m - 1$ where $n_i \geq 7$.
The vertices of H' are labeled as follows:
For $1 \leq i \leq m$ where $i \equiv 1, 2\,(mod4)$

$$f\left(v_j^i\right) = \begin{cases} 1, & 1 \leq j \leq n_i - 1, \quad j \equiv 0, 1\,(mod4) \\ 0, & 1 \leq j \leq n_i - 1, \quad j \equiv 2, 3\,(mod4) \end{cases}$$

For $1 \leq i \leq m$ where $i \equiv 0, 3\,(mod4)$

$$f\left(v_j^i\right) = \begin{cases} 0, & 1 \leq j \leq n_i - 1, \quad j \equiv 0, 1\,(mod4) \\ 1, & 1 \leq j \leq n_i - 1, \quad j \equiv 2, 3\,(mod4) \end{cases}$$

$$f\left(v_{n_i}^i\right) = \begin{cases} 1, & 1 \leq i \leq m, \quad i \equiv 0, 1\,(mod4) \\ 0, & 1 \leq i \leq m, \quad i \equiv 2, 3\,(mod4) \end{cases}$$

Case i: m is even

$\Rightarrow v_f(0) = \frac{p}{2}, v_f(1) = \frac{p}{2}$ and $e_{f*}(0) = \lceil \frac{q}{2} \rceil, e_{f*}(1) = \lfloor \frac{q}{2} \rfloor$

$$\Rightarrow |v_f(0) - v_f(1)| = \left| \frac{p}{2} - \frac{p}{2} \right| = 0$$

$$\Rightarrow |e_{f*}(0) - e_{f*}(1)| = \left| \left\lceil \frac{q}{2} \right\rceil - \left\lfloor \frac{q}{2} \right\rfloor \right| = 1$$

Case ii: m is odd

$\Rightarrow v_f(0) = \lfloor \frac{p}{2} \rfloor, v_f(1) = \lceil \frac{p}{2} \rceil$ and $e_{f*}(0) = \frac{q}{2}, e_{f*}(1) = \frac{q}{2}$

$$\Rightarrow |v_f(0) - v_f(1)| = \left| \left\lfloor \frac{p}{2} \right\rfloor - \left\lceil \frac{p}{2} \right\rceil \right| = 1$$

$$\Rightarrow |e_{f*}(0) - e_{f*}(1)| = \left| \frac{q}{2} - \frac{q}{2} \right| = 0$$

From the above definition it is clear that $|v_f(0) - v_f(1)| \leq 1$ and $|e_{f*}(0) - e_{f*}(1)| \leq 1$. Therefore the graph H' is cordial. The above case is described below in the Fig. 5.

Illustration 2.6.

$$m = 3, n_1 = 7, n_2 = 9, n_3 = 11, p = 27, q = 56$$

$$v_f(0) = 14, v_f(1) = 13 \Rightarrow |v_f(0) - v_f(1)| = |14 - 13| \leq 1$$

$$e_f(0) = 28, e_f(1) = 28 \Rightarrow |e_{f*}(0) - e_{f*}(1)| = |28 - 28| \leq 1$$

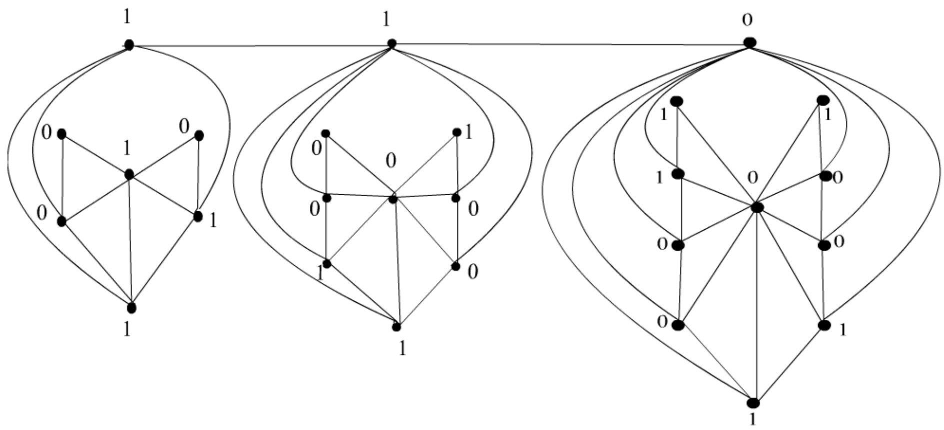

Fig. 5: Path union of three copies.

□

3 Conclusion

We proved that the path union of vertex switching of odd and even wheel graphs in increasing order is cordial. Further we intend to prove that the path union of some other connected graphs is also cordial.

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