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Odd and even edge magic total labeling *

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Abstract An (m, n) graph G is edge magic if there exists a bijective function $f : V(G) \cup E(G) \to \{1, 2, 3, \ldots, m+n\}$ such that $f(u) + f(v) + f(uv) = k$ is a constant, called the valence of f, for any edge uv of G. Moreover, G is said to be super edge magic if $f(V(G)) = \{1, 2, 3, \ldots, n\}$. In this paper, we introduce odd and even edge magic total labeling.

Key words Odd edge magic total labeling, even edge magic total labeling.

2010 Mathematics Subject Classification 05C78.

1 Introduction

This article deals with finite undirected graphs without loops and multiple edges. It could be denoted by $V(G)$ and $E(G)$ the set of vertices and set of edges of a graph G, respectively. The set of vertices adjacent to u in G is denoted by $N_G(u)$, and $deg_G(u) = |N_G(u)|$ is the degree of u in G.
Let G be a graph with n vertices and m edges. A one to one correspondence f from $V(G) \cup E(G)$ to $\{1, 2, 3, \ldots, n+m\}$ is called an edge magic labeling of G if there exists a constant k such that $f(u) + f(v) + f(uv) = k$ for any edge uv of G. An edge magic labeling f may be called super edge magic if $f(V(G)) = \{1, 2, \ldots, n\}$ and $(E(G)) = \{n+1, n+2, \ldots, n+m\}$. A graph G is called edge magic (and respectively super edge magic) if there exists an edge magic (and respectively super edge magic) labeling of G.
Sedlacek [4] introduced the magic labeling of graphs in 1963, and since then onwards they have arrived at many results in magic labeling, especially in edge magic labeling. For new results in graph labeling we refer to [3]. Preliminary results in consecutive edge magic total labeling can be found in [5].

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Let $f : V \cup E \rightarrow \{1, 2, 3, \ldots, n + m\}$ be an edge magic labeling for a graph G. Define the labeling $f' : V \cup E \rightarrow \{1, 2, 3, \ldots, n + m\}\}$ as follows.

$$f^{'}(x) = n + m + 1 - f(x) \ \forall \ x \in V$$

$$f^{'}(xy) = n + m + 1 - f(xy) \ \forall \ xy \in E$$

Then f' is called the dual of f (see [6]), i.e., it could be said that the dual of an edge magic labeling for a graph G is also an edge magic labeling.

In [1], the concept of magic strength of a graph is discussed. It is well known that for any magic labeling f of G there is a constant $c(f)$, such as $f(x) + f(y) + f(xy) = c(f)$ for any edge $xy \in E$. The magic strength of G, $m(G)$ is defined as the minimum of all $c(f)$ here the minimum is taken over by all magic labelings of G. That is $m(G) = \min\{c(f) : f$ is a magic labeling of G$\}$.

In [2], the super edge magic strength is determined for the path $P_n$on n vertices , the star graph $K_{1,n}$, the $n-$ bistar $B_{n,n}$ obtained from two disjoint copies of $K_{1,n}$, by joining the center vertices with an edge, the tree $(K_{1,n} : 2)$ obtained from $B_{n,n}$ by subdividing the middle edge with a new vertex, the odd cycle C_{2n+1}, P_n^2 and the odd copies of P_2, (that is, $2(n + 1)P_2$).

2 Definitions and Main results

The concept of super edge magic total labeling motivates us to define the following new definition of odd edge magic total labeling and even edge magic total labeling.

Definition 2.1. An edge magic total labeling f is called an odd edge magic total labeling if $f(V(G)) = \{1, 3, 5, \ldots, 2n - 1\}$. A graph G is called an odd edge magic if there exists an odd edge magic total labeling of G.

Definition 2.2. An edge magic total labeling f is called an even edge magic total labeling if $f(V(G)) = \{2, 4, 6, \ldots, 2n\}$. A graph G is called an even edge magic if an even edge magic total labeling of G exists there.

Theorem 2.3. *A cycle C_n is an odd edge magic iff n is odd.*

Proof. Suppose there exists an odd edge magic total labeling f of C_n with the magic number k.

$$\begin{aligned} kn &= \sum_{uv\in E(C_n)} \{f(u) + f(v) + f(uv)\} \\ &= 2 \sum_{v\in V(C_n)} f(v) + \sum_{uv\in E(C_n)} f(uv) \\ &= 2\{1 + 3 + 5 + \cdots + 2n - 1\} + \{2 + 4 + \cdots + 2n\} \\ kn &= 2n^2 + n(n + 1) \\ k &= 2n + n + 1 \\ k &= 3n + 1. \end{aligned}$$

Any edge of C_n is adjacent to only the vertices

$$f(u) + f(e) + f(v) = k$$

Here $f(u)$ and $f(v)$ are odd and $f(e)$ is even.
Therefore $k = 3n + 1$ is even.
$k = 3n + 1$ is even iff n is odd.
Let n be odd. Let $V(C_n) = \{v_1, v_2, \ldots, v_n\}$ and

$$E(C_n) = \{e_i = v_i v_{i+1} \mid 1 \le i \le n - 1\} \cup \{e_n = v_n v_1\}$$

Define $f(V \cup E) = \{1, 2, \ldots, 2n\}$ as follows

$$f(v_i) = \begin{cases} n - i + 1, & \text{if } i \text{ is odd} \\ 2n - i + 1, & \text{if } i \text{ is even} \end{cases}$$

and $f(e_i) = 2i,\ 1 \leq i\ \leq n.$
It is clearly vertified that f is an odd edge magic total labeling of C_n with the magic number $k = 3n+1$. $\square$

Example 2.4. An odd edge magic total labeling of C_9 is given in Fig. 1.

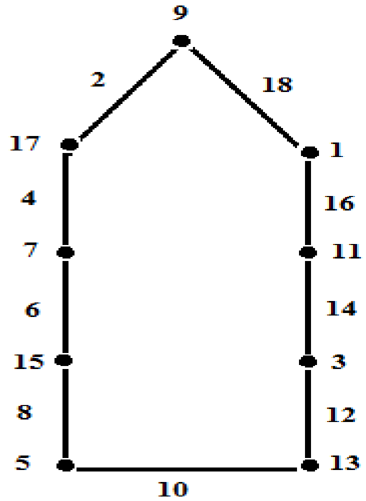

Fig. 1: $n = 9, m = 9, k = 28.$

Theorem 2.5. *A cycle C_n is an even edge magic iff n is odd.*

Proof. Suppose there exists an even edge magic total labeling f of C_n with the magic number k.

$$kn = \sum_{uv\in\ E(C_n)} \{f(u) +\ f(v) +\ f(uv)\}$$

$$= 2 \sum_{v\in\ V(C_n)} f(v)\ \ +\ \sum_{uv\in\ E(C_n)} f(uv)$$

$$= 2\{2+4+6+\cdots+2n\} +\ \{1+3+\cdots+2n-1\}$$

$$kn = 2n(n+1) +\ n^2$$

$$k = 2(n+1) +\ n$$

$$k = 3n+2.$$

Any edge of C_n is adjacent to only the vertices

$$f(u) +\ f(e) +\ f(v) =\ k$$

Here $f(u)$ and $f(v)$ are even and $f(e)$ is odd.
Therefore $k = 3n+2$ is even.
$k = 3n+2$ is odd iff n is odd.
Let n be odd. Let $V(C_n) = \{v_1, v_2, \ldots,\ v_n\}$ and

$$E(C_n) = \{e_i = V_i v_{i+1} \mid 1 \leq i\ \leq n-1\} \cup \{e_n = v_n v_1\}$$

Define $f(V\ \cup E) = \{1, 2,\ \ldots, 2n\}$ as follows

$$f(v_i) = \begin{cases} n+i, & \text{if } i \text{ is odd} \\ i, & \text{if } i \text{ is even} \end{cases}$$

and $f(e_i) = 2i,\ 1 \leq i\ \leq n.$
It is clearly vertified that f is an even edge magic total labeling of C_n with the magic number $k = 3n+2$. $\square$

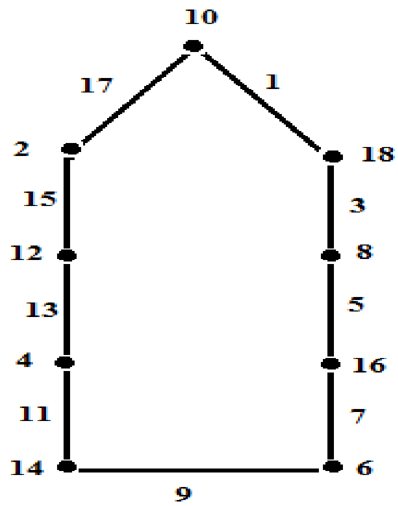

Fig. 2: $n = 9, m = 9, k = 29$.

Example 2.6. An odd edge magic total labeling of C_9 is given in Fig. 2.

Theorem 2.7. *rC_s is an odd edge magic iff r and s are odd.*

Proof. Suppose there exists an odd edge magic total labeling f of rC_s with the magic number k. Then $krs = \sum_{uv\in E(rC_s)} \{f(u) + f(v) + f(uv)\}$

$$= 2 \sum_{v\in V(rC_s)} f(v) + \sum_{uv\in E(rC_s)} f(uv)$$

$$krs = 2(rs)^2 + (rs)(rs+1)$$

$$k = 3rs + 1$$

Any edge of rC_s is adjacent to only two vertices

$$f(u) + f(e) + f(v) = k.$$

Here $f(u)$ and $f(v)$ are odd and $f(e)$ is even.
Therefore $k = 5rs + 1$ is even iff r and s are odd.
Let r and s be odd integers.
Assume that the graph rC_s has the vertex set $= V_1 \cup V_2 \cup V_3 \cup \cdots \cup V_r$, where $V_i = \{v_i^1, v_i^2, v_i^3, \ldots, v_i^s\}$ and the edge set $E = E_1 \cup E_2 \cup E_3 \cup \cdots \cup E_r$, where $E_i = \{e_i^1, e_i^2, e_i^3, \ldots, e_i^s\}$, and $e_i^j = v_i^j v_i^{(j+1)}$ for $1 \le i \le r,\ 1 \le j \le s-1,\ e_i^s = v_i^s v_i^1$.
Define $f(V \cup E) = \{1, 2, 3, \ldots, 2rs\}$ as follows:

$$f(v_i^1) = 2(rs - i) + 1,\ i = 1, 2, \ldots, r$$

$$f(v_i^2) = \begin{cases} 2(rs - 2r + 2i) - 1, & 1 \le i \le \frac{(r-1)}{2} \\ 2(rs - 3r + 2i) - 1, & \frac{(r+1)}{2} \le i \le r\ . \end{cases}$$

For $j = 3, 4, 5, \ldots, s$

$$f\left(v_i^j\right) = \begin{cases} 2rs - (2j-1)r - 2i, & 1 \le i \le \frac{(r-1)}{2} \\ 2rs - (2j-3)r - 2i, & \frac{(r+1)}{2} \le i \le r\ . \end{cases}$$

$$f(e_i^1) = \begin{cases} r - 2i + 1, & 1 \le i \le \frac{(r-1)}{2} \\ 3r - 2i + 1, & \frac{(r+1)}{2} \le i \le r\ . \end{cases}$$

$$j = 2, 4, 6, \ldots, s-1$$
$$f\left(e_i^j\right) = rs - 2i + 2 + (j+1)\,r, \;\; 1 \le i \le r.$$
$$j = 3, 5, 7, \ldots, s$$
$$f\left(e_i^j\right) = \begin{cases} 4i + (j-1)\,r, & 1 \le i \le \frac{(r-1)}{2} \\ 4i + (j-3)\,r, & \frac{(r+1)}{2} \le i \le r\,. \end{cases}$$

It is clearly verified that f is an odd edge magic labeling of rC_s with the magic number $k = 3rs + 1$. □

Example 2.8. An odd vertex magic labeling of $5C_3$ is given in Fig. 3.

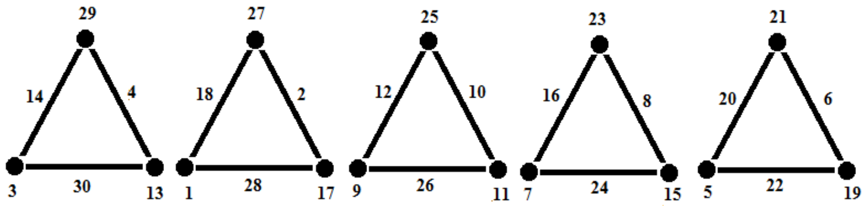

Fig. 3: $n = 15, m = 15, k = 46.$

Theorem 2.9. *rC_s is an Even edge magic iff r and s are odd.*

Proof. Suppose there exists an even edge magic total labeling f of rC_s with the magic number k. Then

$$krs = \sum_{uv \in E(rC_s)} \{f(u) + f(v) + f(uv)\}$$
$$= 2 \sum_{v \in V(rC_s)} f(v) + \sum_{uv \in E(rC_s)} f(uv)$$
$$krs = 2\,(rs)\,(rs+1) + (rs)^2$$
$$k = 3rs + 2$$

Any edge of rC_s is adjacent to only two vertices

$$f(u) + f(e) + f(v) = k.$$

Here $f(u)$ and $f(v)$ are odd and $f(e)$ is odd.
Therefore $k = 3rs + 2$ is odd iff r and s are odd.
Let r and s be odd integers.
Assume that the graph rC_s has the vertex set $= V_1 \cup V_2 \cup V_3 \cup \cdots \cup V_r$, where $V_i = \{v_i^1, v_i^2, v_i^3, \ldots, v_i^s\}$ and the edge set $E = E_1 \cup E_2 \cup E_3 \cup \cdots \cup E_r$, where $E_i = \{e_i^1, e_i^2, e_i^3, \ldots, e_i^s\}$, and $e_i^j = v_i^j v_i^{(j+1)}$ for $1 \le i \le r,\; 1 \le j \le s-1,\; e_i^s = v_i^s v_i^1$.
Define $f\,(V \cup E) = \{1, 2, 3, \ldots, 2rs\}$ as follows:

$$f\left(v_i^1\right) = 2i, \qquad i = 1, 2, \ldots, r$$

$$f\left(v_i^2\right) = \begin{cases} 4r - 4i + 2, & 1 \le i \le \frac{(r-1)}{2} \\ 6r - 4i + 2, & \frac{(r+1)}{2} \le i \le r\,. \end{cases}$$

For $j = 3, 5, 7, \ldots, s$

$$f\left(v_i^j\right) = \begin{cases} (2j-1)\,r + 2i + 1, & 1 \le i \le \frac{(r-1)}{2} \\ (2j-3)\,r + 2i + 1, & \frac{(r+1)}{2} \le i \le r\,. \end{cases}$$

$$f\left(e_i^1\right)=\begin{cases} 2rs+2i-r, & 1\le i\le \frac{(r-1)}{2} \\ 2rs-3r+2i, & \frac{(r+1)}{2}\le i\le r\ . \end{cases}$$

$$j=2,4,6,\ldots,s-1$$

$$f\left(e_i^s\right)=rs+2i-1-(j+1)\,r,\ 1\le i\le r.$$

$$j=3,5,7,\ldots,s$$

$$f\left(e_i^j\right)=\begin{cases} 2rs-4i+1-(j-1)\,r, & 1\le i\le \frac{(r-1)}{2} \\ 2rs-4i+1-(j-3)\,r, & \frac{(r+1)}{2}\le i\le r\ . \end{cases}$$

It is clearly verified that f is an even edge magic labeling of rC_s with the magic number $k=3rs+2$. □

Example 2.10. An Even edge magic labeling of $5C_3$ is given in Fig 4.

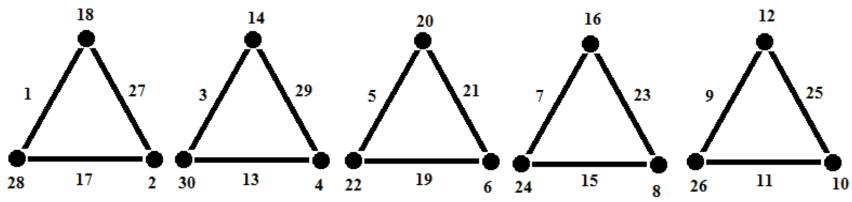

Fig. 4: $n=15, m=15, k=47$.

3 Conclusion and scope

We give two new types of labeling namely odd edge magic total labeling and even edge magic total labeling in this paper and we proved that odd cycles are odd and even edge magic graph. We also proved rC_s is odd and even edge magic graph where r and s are odd. The investigation of the number of non-isomorphic on larger size of the remaining graphs is still an open problem.

References

[1] Avdayappan, S., Jeyanthi, P. and Vasuki, R. (2000). Magic strength of a graph, *Indian J.Pure Appl. Math.*, 31(7), 873–883.

[2] Avdayappan, S., Jeyanthi, P. and Vasuki, R. (2001). Super magic strength of a graph, *Indian J. Pure Appl. Math.* 32(11), 1621–1630.

[3] Gallian, J.A. (2018). A dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, #DS6.

[4] Sedlacek, J. (1964). Problem27, in Theory of Graphs and its Applications, (Proceedings Symposium Smolenice, 1963), Publ. House Czechoslovak Acad. Sci., Prague, 163–164.

[5] Sugeng, K.A. and Miller, M. (2005). Properties of edge consecutive magic graphs, in: Proceeding of the sixteenth Australian workshop on Combinatorics Algorithms 2005, Ballarat, Australia, 2005, 311–320.

[6] Wallis, W.D. (2001). Magic graphs, Birkhauser, Boston.