Bulletin of Pure and Applied Sciences
Section - E - Mathematics & Statistics
Vol. 41E, No.1, January-June 2022.P.81-87

Print version ISSN 0970 6577
Online version ISSN 2320 3226
DOI: 10.5958/2320-3226.2022.00012.1

Original Article

Content Available online at: https://bpasjournals.com/math-and-stat/

Global Strong Domination in α- Cut Diminish Fuzzy Graph $G^{\alpha}(\sigma^{\alpha}, \mu^{\alpha})$ by α- Cut Strong Arc

V. Senthilkumar*

Author's Affiliation:
Assistant Professor, Department of Mathematics, CPA College, Bodinayakanur, Tamil Nadu 625582, India.

***Corresponding Author: V. Senthilkumar,** Assistant Professor, Department of Mathematics, CPA College, Bodinayakanur, Tamil Nadu 625582, India
E-mail: 1984senthilvkumar@gmail.com

How to cite this article: Senthilkumar V. (2022). Global Strong Domination in α- Cut Diminish Fuzzy Graph $G^{\alpha}(\sigma^{\alpha}, \mu^{\alpha})$ by α- Cut Strong Arc. *Bull. Pure Appl. Sci. Sect. E Math. Stat.* 41E(1), 81-87.

ABSTRACT

In this paper, an innovative concept of $\boldsymbol{\alpha}$- Cut diminish fuzzy graph $\boldsymbol{G^{\alpha}(\sigma^{\alpha}, \mu^{\alpha})}$ and $\boldsymbol{\alpha}$- Cut strong arc of $\boldsymbol{\alpha}$- Cut diminish Fuzzy graph are introduced in the new domain. Further complement of $\boldsymbol{\alpha}$- Cut diminish fuzzy graph are discussed with new approach. Definition of $\boldsymbol{\alpha}$- Cut strong domination and Global Strong domination in $\boldsymbol{\alpha}$-Cut diminish fuzzy graph are also introduced by $\boldsymbol{\alpha}$- Cut strong arc. Moreover, some standard theorems and related results in Global domination of $\boldsymbol{\alpha}$- Cut diminish fuzzy graph are presented with suitable example of Standard $\boldsymbol{\alpha}$- Cut diminish fuzzy graph.

KEYWORDS: $\boldsymbol{\alpha}$- Cut diminsh fuzzy graph,$\boldsymbol{\alpha}$ -Cut strong arc, $\boldsymbol{\alpha}$- Cut non strong arc, complement ofα-Cut,$\boldsymbol{\alpha}$-Cut Strong arc domination, $\boldsymbol{\alpha}$- Cut strong global domination.

AMS classification: 05C72

1. INTRODUCTION

Fuzzy graph is the generalization of the ordinary graph. Therefore it is natural that though fuzzy graph inherits many properties similar to those of ordinary graph, it deviates at many places. The earliest idea of dominating sets date back to the origin of game of chess in India over 400 years ago in which placing the minimum number of a chess piece (such as Queen, knight ext..) over chess board so as to dominate all the squares of chess board was investigated. The formal mathematical definition of domination was given by Ore.O in 1962. In 1975 A. Rosenfeld introduced the notion of fuzzy graph and several analogs of theoretic concepts such as path, cycle and connectedness. A. Somasundaram and S. Somasundaram discussed the domination in fuzzy graph using effective arc. A. Nagoorgani and V.T. Chandrasekarn discussed the domination in fuzzy graph [3]. V. Senthilkumar and C. Y. Ponnappan are discussed the strong arc and strong arc domination [6-13]. For $0 \leq \boldsymbol{\alpha} \leq \mathbf{1}$, $\boldsymbol{\alpha}$- Cut Graph(Crips graph) are discussed by J.N. Mondeson and P.S.Nair [2].A dominating set D of a graph G, is a global dominating set in G if D is also a dominating set of $\overline{\boldsymbol{G}}$ of G. The global domination number $\gamma_g(G)$ of G is the minimum cardinality of global dominating set. E. Sampathkumar introduced the concept of Global dominating set in graph theory. Before introducing new the results, we are placed few preliminary definitions and results for new one.

Received on 10.12.2021 / Revised on 15.05.2022/ Accepted on 30.05.2022
Online First Published on June 15, 2022 at https://www.bpasjournals.com

2. PRELIMINARIES

Definition 2.1
Fuzzy graph G(σ,μ) is pair of function $V \to [0,1]$and$\mu: V \times V \to [0,1]$where for all $u, v\ in\ V$, we have $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$.

Definition 2.2
A Pathρ of a fuzzy graph G(σ,μ) is a sequence of distinct nodes v_1, v_2, v_3,…v_n such that $\mu(v_{i-1}, v_i) > 0$ where $1 \leq i \leq n$. A path is called a cycle if $u_0 = u_n$and$n \geq 3$

Definition 2.3
The complement of a fuzzy graph G(σ,μ) is a subgraph $\bar{G} = (\bar{\sigma}, \bar{\mu})$where$\bar{\sigma} = \sigma$ and
$\bar{\mu}(u,v) = \sigma(u) \wedge \sigma(v) - \mu(u,v)$for all$u, v in V$. A fuzzy graph is self complementary if $G = \bar{G}$

Definition 2.4
The order p and size q of a fuzzy graph G(σ,μ) is defined as
$p = \sum_{u \in V} \sigma(u)$ and$q = \sum_{(u,v)\in E} \mu(u,v)$.

Definition 2.5
The degree of the vertex u is defined as the sum of weight of arc incident at u, and is denoted by d(u)**.**

Definition 2.6
Let u,v be two nodes in G(σ,μ). If they are connected by means of a path ρ then strength of that path is $\wedge_{i=i}^{n} \mu(u_{i-1}, v_i)$.

Definition 2.7
Two nodes that are joined by a path are said to be connected. The relation connected is reflexive, symmetric and transitive. If u and v are connected by means of length k, then $\mu^k(u,v) = \sup\{\mu(u,v_1) \wedge \mu(v_1,v_2) \ldots \wedge \mu(v_{k-1},v_k) \mid u, v_1, v_2, \ldots v$in such path$\rho$ }

Definition 2.8
A Strongest path joining any two nodes u,v is a path corresponding to maximum strength between u and v. The strength of the strongest path is denoted by $\mu^{\infty}(u,v)$.
$\mu^{\infty}(u,v) = \sup\{\mu^k(u,v) \mid$ k = 1,2,3…}

Example 2.9

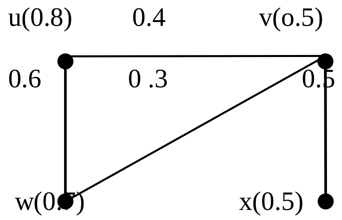


Figure: 2. 1:

In this fuzzy graph, Fig(i)(a), u , w, v, x is a u-x path of length 3 and strength is 0.3. Another path of u-x is u, v, x of length 2 and strength is 0.4. But strength of the strongest path joining u and x is $\mu^{\infty}(u,x) = \sup\{0.3, 0.4\} = 0.4$

Definition 2.10
A node is a fuzzy cut node of G(σ,μ) if removal of it reduces the strength of the connectedness between some other pair of nodes. That is, w is a fuzzy cut node of G(σ,μ) iff there exist u,v such that w is on every strongest path from u to v .

Definition 2.11
For $\leq \alpha \leq 1$, α-cut graph of a fuzzy graph G(σ,μ) is a crisp graph with $G_\alpha(V_\alpha, E_\alpha)$ such that V_α ={$u \in V \,/\, \sigma(u) \geq \alpha$ } and $E^\alpha = \{(u,v) \,/\, \mu(u,v) \geq \alpha\ \}$.

3. α- CUT DIMINISH FUZZY GRAPH $G^{\alpha}(\sigma^{\alpha},\mu^{\alpha})$

Definition 3.1

Theα- Cut diminish Fuzzy graph $G^{\alpha}(\sigma^{\alpha},\mu^{\alpha})$is pair of function $\sigma^{\alpha}: V \to [\alpha, 1]$ and $\mu^{\alpha}: V \times V \to [\alpha, 1]$, for any threshold $\alpha \in [0,1]$ and for all u, v in V we have $\mu^{\alpha}(u,v) \leq \sigma^{\alpha}(u) \wedge \sigma^{\alpha}(v)$.
Further $V^{\alpha} = \{u \in V \,/\, \sigma(u) \geq \alpha\}$ and $E^{\alpha} = \{(u,v) \in V \times V \,/\, \mu(u,v) \geq \alpha\}$

Definition 3.2

A strong α- Cut diminish Fuzzy graph $G^{\alpha}(\sigma^{\alpha},\mu^{\alpha})$is pair of function $\sigma^{\alpha}: V \to (\alpha, 1]$ and $\mu^{\alpha}: V \times V \to (\alpha, 1]$ where $\alpha \in [0,1]$, for all u, v in V we have $\mu^{\alpha}(u,v) \leq \sigma^{\alpha}(u) \wedge \sigma^{\alpha}(v)$.
with vertex set $V^{\alpha} = \{u \in V \,/\, \sigma(u) > \alpha\}$ and edge set $E^{\alpha} = \{(u,v) \in V \times V \,/\, \mu(u,v) > \alpha\}$

Example 3.3

Consider a fuzzy graphG(σ, μ)

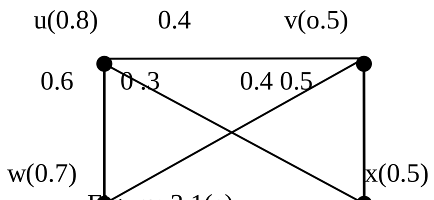


Figure: 3.1(a)

α- Cut diminish Fuzzy graph $G^{\alpha}(\sigma^{\alpha},\mu^{\alpha})$ of a fuzzy graph G(σ, μ) are drawn below.
For α = 0.5 $\in [0,1]$

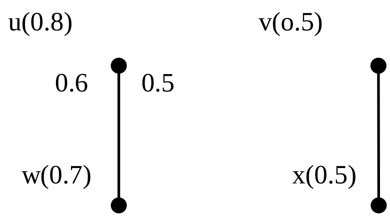


Figure:3.1(b)

Strong α- Cut diminish Fuzzy graph $G^{\alpha}(\sigma^{\alpha},\mu^{\alpha})$ of a fuzzy graph G(σ, μ) are drawn below

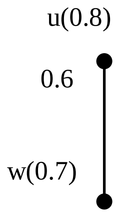


Figure:3.1(c)

Note: In the above example

$(i)\alpha$- Cut diminish Fuzzy graph$G^{\alpha}(\sigma^{\alpha},\mu^{\alpha})$ of a connected fuzzy graph G(σ, μ) is a sub graph when$\alpha \neq 0$.
(ii) α- Cut diminish Fuzzy graph$G^{\alpha}(\sigma^{\alpha},\mu^{\alpha})$ of a connected fuzzy graph G(σ, μ) may be connected or disconnected.
(iii) Ifα- Cut diminish Fuzzy graph $G^{\alpha}(\sigma^{\alpha},\mu^{\alpha})$ is same as fuzzy graph G(σ, μ) then $G^{\alpha}(\sigma^{\alpha},\mu^{\alpha})$ is Isoα- Cut diminish Fuzzy graph.(this case only possible if α = 0).
Therefore, Isoα- Cut diminish Fuzzy graph is called 0-Cut diminish Fuzzy graph

Definition 3.4

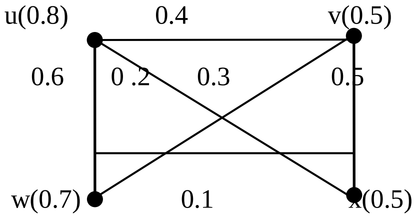
An arc (u,v) of α- Cut diminish Fuzzy graph $G^{\alpha}(\sigma^{\alpha},\mu^{\alpha})$ is called a α- Cut strong arc if $\mu^{\alpha}(u,v) = \mu^{\alpha^{\infty}}(u,v)$ else arc(u,v) is called α-Cut non strong arc.
α- Cut Strong neighbourhood of u$\in V^{\alpha}$ is $N_s^{\alpha}(u) = \{v \in V^{\alpha}: arc(u,v)$isα- Cutstrong} .
$N_s^{\alpha}[u] = N_s^{\alpha}(u) \cup \{u\}$ is the closed neighborhood of u.

The minimum cardinality of α- Cut strong neighbourhood $\delta_s^{\alpha}(G^{\alpha}) = \min\{|N_s^{\alpha}(u)|: u \in V^{\alpha}(G^{\alpha})\}$.
Maximum cardinality of α- Cut strong neighbourhood $\Delta_s^{\alpha}(G^{\alpha}) = \max\{|N_s^{\alpha}(u)|: u \in V^{\alpha}(G^{\alpha})\}$.

Definition 3.5
Let$G^{\alpha}(\sigma^{\alpha},\mu^{\alpha})$ be a fuzzy graph. Let u, v be two nodes of $G^{\alpha}(\sigma^{\alpha},\mu^{\alpha})$. We say thatu α- Cut strong arcdominates v if arc (u,v) is a α- Cut strong arc. A subset D^{α} of V^{α} is called a α- Cutstrong arc dominating set of $G^{\alpha}(\sigma^{\alpha},\mu^{\alpha})$ if for every $v \in V^{\alpha} - D^{\alpha}$, there exists $u \in D^{\alpha}$ such that u ***α- Cut strong arc dominates*** v. A dominating set D^{α} is called a minimal α- Cutstrong arc dominating set if no proper subset of D^{α} is a α- Cut strong arc dominating set. The minimum fuzzy cardinality taken over all α- Cut strong arc dominating sets of a graph $G^{\alpha}(\sigma^{\alpha},\mu^{\alpha})$ is called the α- Cut strong arc domination number and is denoted by$\gamma_s^{\alpha}(G^{\alpha})$and the corresponding dominating setD^{α} is called minimumα- Cut strong arc dominating set. The number of elements in the minimum α- Cut strong arc dominating setD^{α} is denoted by n[$\gamma_s^{\alpha}(G^{\alpha})$]

Moreover, $\gamma_s^{\alpha}(G^{\alpha}) = \sum \sigma^{\alpha}(u)$; u∈ D^{α} is the minimum α- Cut strong arc dominating set.

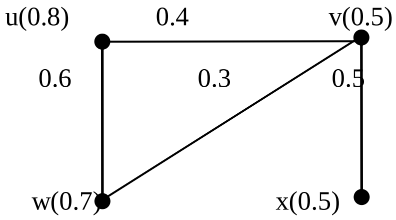
Example 3.6



Fuzzy graph G($\boldsymbol{\sigma},\boldsymbol{\mu}$)

Figure: 3.2:

Fix $\boldsymbol{\alpha}$ = 0.3



$\boldsymbol{\alpha}$- Cut diminish Fuzzy graph $\boldsymbol{G^{\alpha}(\sigma^{\alpha},\mu^{\alpha})}$

Figure: 3.3:

In Figure 3.3,
Arc (u, v), (u, w), (v, x) are 0.3- *Cut strong arcs* and (v, w) is 0.3 - *Cut non strong arc*.

Illustration
u , w, v is a u-v path of length 2 and strength is 0.3. Another path of u-v is u, v of length 1 and strength is 0.4. But strength of the strongest path joining u and v is $\mu^{\alpha\infty}(u,v) = \sup\{0.3,0.4\} = 0.4$
$\mu^{\alpha}(u,v)$ = 0.4 and $\mu^{\alpha\infty}(u,v)$ = 0.4 .Therefore $\mu^{\alpha}(u,v) = \mu^{\alpha\infty}(u,v)$ = 0.4
Arc (u, v) is 0.3- *Cut strong arcs*

Therefore, u α- Cut strong arc dominates v (or)
v α- Cut strong arc dominates u
Similarly, $\mu^{\alpha}(v,w)$ = 0.3 and $\mu^{\alpha\infty}(v,w)$ = 0.4 .Therefore $\mu^{\alpha}(v,w) \neq \mu^{\alpha\infty}(v,w)$
Hence, Arc (v, w) is 0.3 - *Cut non strong arc*

From this, v cannot α- Cut strong arc dominates w (or)
w cannot α- Cut strong arc dominates v
D_1^{α} = {u, x} ,D_2^{α} = {w, x}, D_3^{α} = {w, v} ,D_4^{α}= {u,v} areα- Cut strong arc dominating sets. Also D_1^{α}, D_2^{α}, D_3^{α}, D_4^{α} are minimal α- Cut strong arc dominating sets.

Therefore $|D_1^{\alpha}| = 0.8 + 0.5 = 1.3, |D_2^{\alpha}| = 0.7 + 0.5 = 1.2,$
$|D_3^{\alpha}| = 0.5 + 0.7 = 1.2 \; and \; |D_4^{\alpha}| = 0.8 + 0.5 = 1.3.$

Therefore, min{1.3, 1.2, 1.2, 1.3 } = 1.2. Hence $D_2^{\alpha}$and D_3^{α} areα- Cut strong arc minimum dominating sets.
$\gamma_s^{\alpha}(G^{\alpha}) = 1.2$ andn[$\gamma_s^{\alpha}(G^{\alpha})$] = 2

Observation 3.7
n[$\gamma_s^{\alpha}(G^{\alpha})$]. $\boldsymbol{\alpha} \leq \gamma_s^{\alpha}(G^{\alpha}) \leq$ n[$\gamma_s^{\alpha}(G^{\alpha})$]

Proof: It can be illustrated by example
From the above example
we have α = 0.3,$\gamma_s^{\alpha}(G^{\alpha}) = 1.2$ and n[$\gamma_s^{\alpha}(G^{\alpha})$] = 2
n[$\gamma_s^{\alpha}(G^{\alpha})$]$\boldsymbol{\alpha} \leq \gamma_s^{\alpha}(G^{\alpha}) \leq$ n[$\gamma_s^{\alpha}(G^{\alpha})$]
⇒ 2(0.3) ≤ 1.2 ≤2
⇒ 0.6 ≤ 1.2 ≤ 2

Theorem 3.8
Let G(σ,μ) be a fuzzy graph and let $G^{\alpha}(\sigma^{\alpha},\mu^{\alpha})$ be α- Cut diminish Fuzzy graph. If $0 \leq \alpha \leq \beta \leq 1$ then $G^{\beta}(\sigma^{\beta},\mu^{\beta})$ is β- Cut diminish Fuzzy sub graph of $G^{\alpha}(\sigma^{\alpha},\mu^{\alpha})$.

Proof:
Since $0 \leq \alpha \leq \beta \leq 1$, we must have the relation $\sigma^{\beta} \subseteq \sigma^{\alpha}$ and $\mu^{\beta} \subseteq \mu^{\alpha}$. Reference [6]

Illustration
Consider a fuzzy graph G(σ,μ)

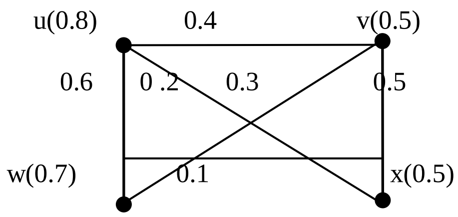
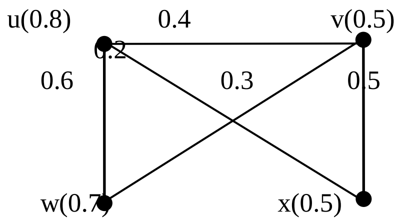
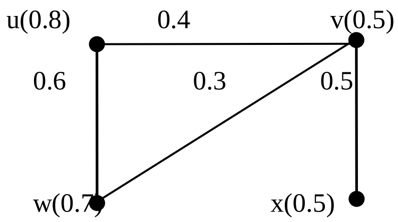


Figure: 3
Forα = 0.2



For β = 0. 3



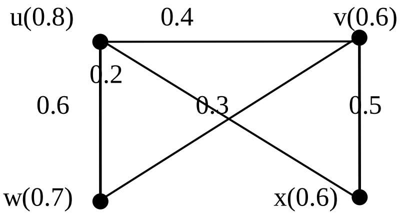
Clearly $\sigma^{\beta} \subseteq \sigma^{\alpha}$and $\mu^{\beta} \subseteq \mu^{\alpha}$.

4. COMPLEMENT OF α- CUT DIMINISH FUZZY GRAPH $G^{\alpha}(\sigma^{\alpha},\mu^{\alpha})$

Definition 4.1
For any threshold $\alpha \in [0,1]$, the complement of a α- cut diminish fuzzy graph $G^{\alpha}(\sigma^{\alpha},\mu^{\alpha})$ is a subgraph $\overline{G^{\alpha}}(\overline{\sigma^{\alpha}},\overline{\mu^{\alpha}})$ where $\overline{\sigma^{\alpha}} = \sigma^{\alpha}$ and $\overline{\mu^{\alpha}}(x,y) = \mu^{\alpha}(x,y) \in \mu^{\alpha}$ if $\sigma^{\alpha}(x) \wedge \sigma^{\alpha}(y) - \mu^{\alpha}(x,y) \leq \alpha$, for all $x, y in V$.

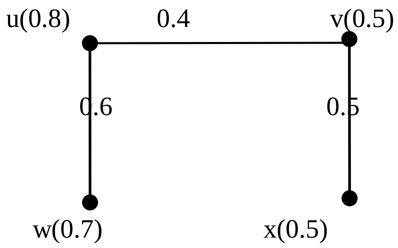
That is $\overline{\mu^{\alpha}} = \{\mu^{\alpha}(x,y) \in \mu^{\alpha}$ when $\sigma^{\alpha}(x) \wedge \sigma^{\alpha}(y) - \mu^{\alpha}(x,y) \in [0,\alpha]$ } where $0 \leq \alpha \leq 1$.

Example 4.2
Consider α- cut diminish fuzzy graph G^{α}
For $\alpha = 0.2$



Complement α- cut diminish fuzzy graph $\overline{G^{\alpha}}$ are drawn below

For $\alpha = 0.2$



Two arc (u,x) and (v,w) are removed since $\sigma^{\alpha}(u) \wedge \sigma^{\alpha}(x) - \mu^{\alpha}(u,x) \geq 0.2$
For the arc (u,x) , Min{ 0.6 ,0.8}- 0.3 =0.3 which is greater than $\alpha = 0.2$

(For the sake of inconvenient , use $G^{\alpha}(\sigma^{\alpha},\mu^{\alpha}) = G^{\alpha}$ and $\overline{G^{\alpha}}(\overline{\sigma^{\alpha}},\overline{\mu^{\alpha}}) = \overline{G^{\alpha}}$ is the complement of α- Cut diminish fuzzy graph G^{α})

5. GLOBAL DOMINATION IN FUZZY GRAPH BY USING STRONG ARC

A strong arc dominating set D of a fuzzy graph G, is called a global strong arc dominating set of G if D is also a strong arc dominating set of $\bar{G}$ of G. The global domination number $\gamma_{gs}(G)$ of G is the minimum fuzzy cardinality taken over all global strong arc dominating set of G and corresponding Global strong arc dominating set is called a minimum Global strong arc dominating set . The number of elements of global strong arc dominating set using strong is denoted by n $[\gamma_{gs}(G)]$.

5.1GLOBAL DOMINATING IN α- CUT DIMINISH FUZZY GRAPH $G^{\alpha}(\sigma^{\alpha},\mu^{\alpha})$

A strong arc dominating set D^{α} of a α- Cutdiminish fuzzy graph G^{α}, is called a global strong arc dominating set of G if D^{α} is also a strong arc dominating set of $\overline{G^{\alpha}}$ of G^{α}. The global domination number $\gamma_{gs}(G^{\alpha})$ of G^{α} is the minimum fuzzy cardinality taken over all global strong arc dominating set D^{α} of G^{α} and corresponding Globalstrong arc dominating set is called a minimum Global strong arc dominating set . The number of elements of global strong arc dominating set using strong is denoted by $n[\gamma_{gs}(G^{\alpha})]$.

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