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Chromatic Numbers of Hypergraphs

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ABSTRACT

Positive integers m1, m2, . . . , mk are studied in this work. If and merely if G can be articulated as the edge-disjoint union of Subgraphs (SGs) F_i, fulfilling $\chi(F_i) \le m_i$, any graph G possess $\chi(G) \le \prod_{i=1}^{k} m_i$. By appropriate interpretations, the subsequent theorem is generalized to Hypergraphs (HGs) to infer propositions on the graph's coverings.

KEYWORDS: Hyper graph, Bipartite Graph, Edge Cut, Node Cut, Chromatic Number and k-colourable

1. INTRODUCTION

In an edge-disjoint factorization of G, the chromatic number $\chi(G)$ number $\chi(G)$ of a graph G to the chromatic numbers $\chi(F_i)$ of the SGs F_i occurs. This work has a double purpose. Firstly, Burr's theorem is generalized to HGs [3]. Then, to acquire outcomes on graph's covering this generalization is employed by SGs having particular properties.

1.1. Theorem: Consider G as a graph. The positive integers are $m_1, m_2, \ldots, m_k$. Next, the edge-disjoint union is G, $G = \bigcup_{i=1}^{k} F_i$, of SGs F_i with $\chi(F_i) \le m_i$, $1 \le i \le k$, if and merely if $\chi(G) \le \prod_{i=1}^{k} m_i$.

Proof: A few terminologies of HGs are recalled [1]. A finite, non-empty group X of nodes along with a finite group E of edges are encompassed in a finite HG $H = (\chi, E)$, in which a subset of the power set of X is E. The least number of colors required to color the H's nodes is delineated as the chromatic number [4] $\chi(H)$ in order that no edge with above '1' element contains the entire of its nodes of similar colors. A partial HG of H is an HG $H' = (X', E')$ with $X' = X$ together with $E' \subseteq E$.

1.2. Theorem: Consider H as an HG. Let $m_1, m_2, \ldots, m_k$ be the positive integers. Next, the edge-disjoint union is H, where $H = \bigcup_{i=1}^{k} H_i$ of HGs H_i with $\chi(H_i) \le m_i$, $1 \le i \le k$, if and only if $\chi(H) \le \prod_{i=1}^{k} m_i$.

Proof: Consider $H_1, H_2, \ldots, H_k$ be partial HGs[6] enclosing the edges of H, $\chi(H_i) \le m_i$. Put $N = \prod_{i=1}^{k} m_i$. An N-coloring of $H = (X, E)$ has been delineated as given below. Take into account the group of k-tuples $T = \{(r_1, r_2, \ldots, r_k) \mid 1 \le r_i \le m_i\}$, as well as allot to every node $x \in X$ a k-tuple $(r_1(x), \ldots r_k(x))$ in T by allowing $r_i(x)$ be the color of x in an m_i-coloring of H_i. It is exhibited that no edge $E \in E$ with $|E| \ge 2$ has the entire of its nodes allocated a similar k-tuple[7] to reveal that this is an H's N-coloring. Consider $E \in E$ to be random, as well as examine that E is an edge of H, for a few i, $1 \le i \le k$. So, no nodes $x_1, x_2 \in E$ possess distinctive i^{th} coordinates

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in their k-tuples. Especially, the k-tuples of x_1 together with x_2 is distinctive. Therefore, N-coloring of H is possessed as preferred. Contemplate again the group T of k-tuples, where H = (X, E) with $\chi(H) \le \prod_{i=1}^{k} m_i$.

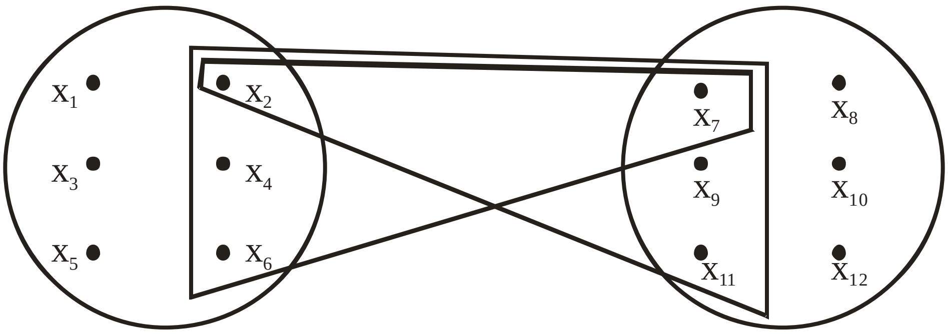
Any N-coloring of it may be noticed as the k-tuple's assignment in T to the nodes of H because | T | = N = $\prod_{i=1}^{k} m_i$, in order that no edge E ∈ E, | E | ≥ 2, has the entire of its nodes allocated to a similar k-tuple. The partial HGs are built H_i, $1 \le i \le k$, with $\chi(H_i) \le m_i$ as given below. Consider E ∈ E, | E | ≥ 2. Next, there prevails a pair of nodes $x_1, x_2 \in E$ in such a manner that the k-tuples of x_1 as well as x_2 vary. The 1st coordinate be $c(x_1, x_2)$ where these k-tuples vary, along with provide E the color c(E), in which $c(E) = \min\{ c(x_1, x_2) \mid x_1, x_2 \in E\}$. For E ∈ E, | E | = 1, set c(E) = 1. The group of integers $S = \{c(E) \mid E \in E\} \subseteq \{1, \ldots, k\}$are regarded. At this time, if and merely if c(E) = s, fulfils $\chi(H_s) \le m_s$ the partial HG $H_s = (X_s, E_s)$ of H, with $E \in E_s$ for $s \in S$. Let $H_s = (X_s, E_s)$ with Es = ϕ for $s \notin S$, $1 \le s \le k$. Next, H's edge cover with the needed property is formed by partial HGs $H_1, H_2, \ldots, H_k$.

2. CONSEQUENCES FOR GRAPHS

An Edge Cut (EC) of H comprises erasing the entire edges in H, which encompass nodes in both of the '2' sets in a presented partition of X (S_o, $S_1 \neq \phi$, $S_o \cup S_1 = X$, $S_o \cap \phi$) provided an HG H = (X, E). An H's node cut encompasses in 1st substituting every node x of H that is comprised in an edge [2], by '2' nodes x°, x^1 then substituting every edge E = $\{x_1, x_2, \ldots, x_k\}$ of H by either $E^o = \left\{x_1^0, \ldots, x_m^0\right\}$ or $E^1 = \left\{x_1^1, \ldots, x_m^1\right\}$, together with lastly removing the entire nodes of the construct x^o or x^l which are not encompassed in an edge. In this, the node cuts not possessing the entire edges with a similar upper index [5] are only regarded.

Figures 1 and 2 evince the notion of EC and node cut.

a. HG and point partition are given



$s_0 = \{x_1, x_2, x_3, x_4, x_5, x_6\}$

$s_1 = \{x_7, x_8, x_9, x_{10}, x_{11}, x_{12}\}$

$E = \{\{x_1, x_2, x_3, x_4, x_5, x_6\}, \{x_7, x_8, x_9, x_{10}, x_{11}, x_{12}\},$
$\{x_2, x_4, x_6, x_7\}, \{x_2, x_7, x_9, x_{11}\}\}$

b. Subsequent to EC the resultant HG.

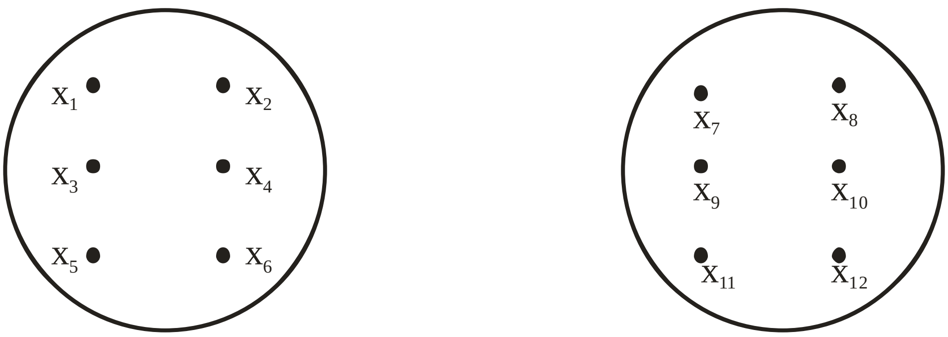
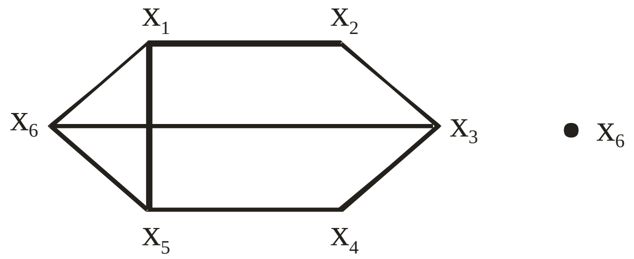


Figure 1: An edge cut applied to a hypergraph

a. Given HG.



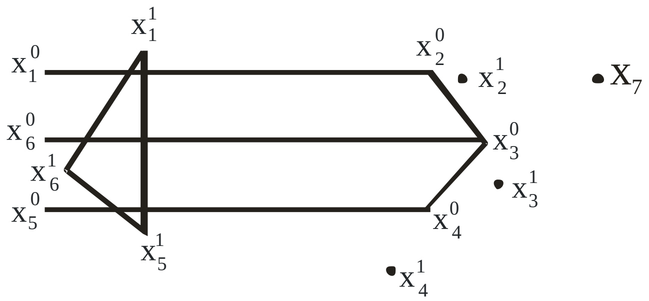
b. The nodes are duplicated and the edges are replaced.

$\{x_1, x_2\} \to \{x_1^0, x_2^0\}$ $\{x_1, x_6\} \to \{x_1^1, x_6^1\}$

$\{x_2, x_3\} \to \{x_2^0, x_3^0\}$ $\{x_1, x_3\} \to \{x_1^1, x_5^1\}$

$\{x_3, x_4\} \to \{x_3^0, x_4^0\}$ $\{x_3, x_6\} \to \{x_3^0, x_6^0\}$

$\{x_5, x_6\} \to \{x_5^1, x_6^1\}$ $\{x_4, x_5\} \to \{x_4^0, x_5^0\}$



c. Removing nodes with upper index to obtain resultant HG.

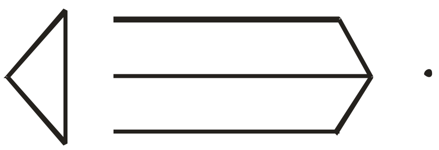


Figure 2: A. node cut applied to a hypergraph

2.1. Theorem: Assume the graph as G. P is a graph property fulfilling,
(1) Hereditary on Induced SGs (IGSs) is P;
(2) The complete graph has the property P if every linked component of this property.
(3) A single node encompasses property P.

The least number of ISGs possessing property P is $X_p^n(G)$ which enclose the nodes of G, $c_p^E(G)$ the least number of ECs, which are needed in order that the resultant graph to contain property P. Afterward,

$$c_p^E(G) = \{\log_2 X_p^n(G)\}$$

Proof: Provided a graph G = (X, E) along with a property P. Assume H′ = (X′, E′) be the HG with X′ = X along with E′ = | E′ | E′ $\subseteq$ X′, as well as E′ induces the SG of G, which is minimum in not possessing property P. Every minimum graph not possessing property P encompasses at least ‘2’ nodes by (3), and by (1), if and only if it has no minimum ISG not possessing property P, an ISG of G has property P. Therefore, $\chi(H') = X_p^n(G)$. To remove the entire edges of H′, assume c(H′) as the least number of ECs essential. Next, c(H′) = β(H′) is obtained for the edges removed by an EC in H′ to induce a 2-colorable partial HG of H′, together with for every 2-colorable partial HG, there is an EC in H′ removing at least its edges. c(H′) = $\{\log_2 X_p^n(G)\}$ is acquired. Then, it suffices to exhibit $c_p^E(G)$ = c(H′).

Initially, it is noticed that a minimum graph Ga not possessing property P is linked regarding (2).

Therefore, provided a series of c(H′) ECs in H′, so that the entire edges are removed, the series of ECs in G possessing the node set's similar partitions generates a graph that has no SG not possessing property P. Thus, c(H′) ≥ $c_p^E(G)$. Offered a series of $c_p^E(G)$ ECs in G, so that the resultant graph encompasses property P, the series of ECs in H′ possessing similar partitions of the node set removes the entire edges in H′, so $c_p^E(G) \geq$ c(H′). Therefore, preferred $c_p^E(G)$ = c(H′) is obtained.

2.2. Theorem: Assume a graph as G, P′ a graph property fulfilling

(1) Hereditary on SGs is P′,

(2) The full graph has property P′ if every linked component has it,

(3) A single edge contains property P′.

The least number of SGs possessing property P′ is $X_p^n(G)$ that is essential to wrap the edges of G, and the least number of nodes that are essential is $c_p^n(G)$ in order that the resultant graph possesses the property P′. After that, $c_p^n(G)$ = {$\log_2 X_p^n(G)$ }.

Proof: Presented a graph G = (X, E) along with property P′. Assume H′ = (X′, E′) be the HG with X′ = E as well as E′ = {E′ | E′ ⊆ X′, together with the SG of G induced by the edges in E′ are minimum in not possessing property P′}. c(H′) = β(H′) = {$\log_2 X_p^n(G)$ } is exhibited as in the proof of Theorem 3. It can be noticed from (2) that a minimum graph G_o not possessing property P′ is linked, and from (3) that G_o contains at least '2' edges. Thus, every node cut in G_o generates a graph whose elements are SGs of G_o not equivalent to G_o. So, this graph contains property P′. By substituting E ∈ E by E°, there corresponds a node cut of G is delineated to every EC in H′ with node set partition S_0, S_1. Therefore, the series of the corresponding node cuts in G yields a graph that has no SG not possessing property P′ to a provided series of c(H′) ECs, in H′ removing the entire edges in H′. So, c(H′) ≥ $c_p^n(G)$. To every node cut in G, there corresponds an EC in H′ delineated by the partition S_o, S_1, with S_o = | E | E ∈ E. An edge in the node cut's ensuing graph, S_1 = X′ - S_o is E°. Thus, presented a series of $c_p^n(G)$ node cuts in G, such that the ensuing graph contains property P′, the series of the corresponding ECs in H′ removes the entire edges, in H′. Thus, $c_p^n(G) \geq$ c(H′) is acquired. So, $c_p^n(G)$ = c(H′) as preferred.

3. CONCLUSION

Thus, it can be deduced that the aforesaid consequence for an HG H delineates β(H) to be the least number of 2-colorable partial HGs of H that enclose the H's edges. The EC and node cut in the HG p and p^1 properties must be understood.

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