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Recognizing emotions using elementary hyperedges in intuitionistic fuzzy soft hypergraphs *

K.K. Myithili[1,†] and R.D. Beulah[2]

1. Department of Mathematics (CA), Vellalar College for Women,,
Erode-638012, Tamil Nadu, India.

2. Department of Mathematics, Sri G.V.G. Visalakshi College for Women (Autonomous),
Palani Road, Udumalpet, Tamil Nadu-642128, India.

1. E-mail: mathsmyth@gmail.com , 2. E-mail: beulahrd2019@yahoo.com

Abstract In this paper, we introduce the notion of intuitionistic fuzzy soft hypergraphs(IFSHGs), certain types of IFSHGs including core, simple, elementary, sectionally elementary, (μ, ν)-tempered (IFSHGs) are dealt with examples. Also this concept is analyzed in emotion recognition as an application too.

Key words Intuitionistic fuzzy soft hypergraphs(IFSHGs), Core, Simple, Support Simple, Elementary and Sectionally elementary IFSHGs, (μ, ν)-tempered IFSHG.

2020 Mathematics Subject Classification 03F55, 05C65, 05C72, 68R10.

1 Introduction

The concept of fuzzy set theory was first initiated by Zadeh [19] to deal with uncertainty and vagueness. Since then the theory of fuzzy sets is used by many researchers [1,2] to solve real life problems involving uncertainty. In 1986, Atanassov [3,4] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. He added a new component which determines the degree of non-membership in the definition of fuzzy set. The fuzzy sets give the degree of membership of an element in a given set, while intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership which are more or less independent from each other. In 1999, Molodtsov [11] initiated soft set theory as a general mathematical tool for dealing with uncertainty from the viewpoint of parameterization. Maji et al. [8,9] et al. studied soft set theory and introduced a new concept of fuzzy soft set by combining fuzzy set and soft set. Also by using the concept of soft set with intuitionistic fuzzy sets and he gave the noble concept of intuitionistic fuzzy soft set. The idea of graph theory was introduced by Euler. In order to expand the application base, the idea of graphs is generalized to a hypergraph, that is, a set V of vertices together with a collection of subsets of V. In 1975, Rosenfeld [16] introduced the concept of fuzzy graphs. In 1976, Berge [5] introduced the concept of fuzzy hypergraphs. Later in 2000, Moderson and Nair [12] et al. initiated the concept of fuzzy graphs and fuzzy hypergraphs. Karunambigai and Parvathi [6] discussed the concepts of intuitionistic fuzzy graphs. Further the concepts like degree, order and size in intuitionistic fuzzy graphs were proposed by Nagoorgani [14]. In 2009, Parvathi et

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†Corresponding author K.K. Myithili, E-mail: mathsmyth@gmail.com

al. [15] introduced and studied the concept of intuitionistic fuzzy hypergraphs with its application. Mythili et al. [13] defined certain types of intuitionistic fuzzy directed hypergraphs. In 2014, Thumbakara and George [18] discussed the concept of soft graphs in the specific way. The concept of fuzzy soft graphs introduced by Mohinta and Samanta [10] in 2015. Recently, intuitionistic fuzzy soft graphs were developed by many authors [7, 17] due to their application in Operations Research, Probability, Optimization, Complex Networks, Recognition of Objects, Images, etc. This paved a way to develop a new notion of IFSHGs. In this paper, section 2 deals with the basic definitions where we introduce the notion of intuitionistic fuzzy soft hypergraphs (IFSHGs) and the definitions of the order, size, degree and the strength of an IFSHG. In section 3, we introduce certain types of IFSHGs including core, simple, elementary, sectionally elementary, (μ, ν)-tempered (IFSHGs) are dealt with examples. An application for IFSHGs is identified in emotion recognition and is presented in section 4. Finally the conclusion is given in section 5.

2 Preliminaries

This section deals with the basic definitions like the fuzzy set, the soft set, the fuzzy soft set, the fuzzy soft graph, the intuitionistic fuzzy soft set and the intuitionistic fuzzy soft graph.

Notations List:

- Let U be the universe set and R be the set of all parameters.
- $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ is an intuitionistic fuzzy soft hypergraph (IFSHG).
- Order and size denoted by $\mathcal{O}(\tilde{H})$, $\mathcal{S}(\tilde{H})$ respectively.
- $\langle \mathfrak{N}_\mu, \mathfrak{N}_\nu \rangle$ or simply $\langle \mu_i, \nu_i \rangle$ denotes the degrees of membership and nonmembership of the vertex $v_i \in V$, such that $0 \leq \mathfrak{N}_\mu + \mathfrak{N}_\nu \leq 1$.
- $\langle \mathfrak{S}_\mu, \mathfrak{S}_\nu \rangle$ or simply $\langle \mu_{ij}, \nu_{ij} \rangle$ denotes the degrees of membership and nonmembership of the hyperedge $v_i, v_j \in V \times V$, such that $0 \leq \mathfrak{S}_\mu + \mathfrak{S}_\nu \leq 1$.
- $P(V \times V)$ is an intuitionistic fuzzy power set.
- $P(V)$ and $P(E)$ denote the sets of all intuitionistic fuzzy soft sets over V and E respectively.
- The support of an intutionistic fuzzy soft set V in $\mathfrak{S}$ is denoted by supp $\mathfrak{E}_j(a_i) = \{v_i/\mathfrak{S}_\mu(a_i) > 0$ and $\mathfrak{S}_\nu(a_i) > 0$, $a_i \in R\}$.

Definition 2.1. [19] A *fuzzy set* $\mathcal{F}$ on a set V is characterized by its membership function $\mu_{\mathcal{F}} : V \to [0, 1]$, where $\mu_{\mathcal{F}}(u)$ is the degree of membership of an element u in a fuzzy set $\mathcal{F}$ for $u \in V$.

Definition 2.2. [9, 11] Let $P(U)$ denotes the power set of U. An ordered pair (F, R) is said to be a *soft set* over U, where $F : R \to P(U)$.

Definition 2.3. [8] If $M \subseteq R$ and $\mathcal{F}^U$ be the collection of all fuzzy subsets of U. Then $(\tilde{F}, M)$ is called *fuzzy soft set*, where $\tilde{F} : M \to \mathcal{F}^U$ is a mapping called fuzzy approximate function of the fuzzy soft set $(\tilde{F}, M)$.

Definition 2.4. [8] If $M \subseteq R$ and $\mathcal{IF}^U$ denotes the set of all intuitionistic fuzzy sets of U. A pair (F, M) is called an *intuitionistic fuzzy soft set* over U, where intutionistic fuzzy approximation function is given by $F = (F_\mu, F_\nu) : M \to \mathcal{IF}^U$.

Definition 2.5. [10] A fuzzy soft graph $\tilde{G} = (\tilde{F}, \tilde{K}, R)$ is a 3-tuple, such that

- $(\tilde{F}, R)$ is a fuzzy soft set over V.
- $(\tilde{K}, R)$ is a fuzzy soft set over E.
- $(\tilde{F}(a), \tilde{K}(a))$ is a fuzzy subgraph of $\tilde{G}$ for all $a \in R$.

That is, $\tilde{K}(a)(xy) \leq \min\left\{\tilde{F}(a)(x), \tilde{F}(a)(y)\right\}$ for all $a \in R$ and $x,\ y \in V$.

Definition 2.6. [7, 17] An intuitionistic fuzzy soft graph (IFSG) on a nonempty set V is an ordered 3-tuple $G = (F, K, R)$ such that

- (F, R) is an intuitionistic fuzzy soft set over V.
- (K, R) is an intuitionistic fuzzy relation on V. That is, $K : R \to P(V \times V)$.
- $(F(a), K(a))$ is an intuitionistic fuzzy soft subgraph for all $a \in R$.

That is,

1. $K_\mu(a)(uv) \le \min\{F_\mu(a)(u), K_\mu(a)(v)\}$
2. $K_\nu(a)(uv) \le \max\{F_\nu(a)(u), K_\nu(a)(v)\}$,

such that $0 \le K_\mu(a)(uv) + K_\nu(a)(uv) \le 1$, for every $a \in R$ and $u,\ v \in V$.

Definition 2.7. An intuitionistic fuzzy soft hypergraph (IFSHG) $\tilde{H} = (H^*, \mathfrak{N}, \mathfrak{S}, R)$ is an ordered 4-tuple, such that

- $H^* = \langle V, E\rangle$ is a intutionistic fuzzy hypergraph.
- $(\mathfrak{N}, R)$ is an intuitionistic fuzzy soft set over V.
- $(\mathfrak{S}, R)$ is an intuitionistic fuzzy relation on V. That is $\mathfrak{S} : R \to P(V \times V)$.
- $(\mathfrak{N}(a), \mathfrak{S}(a))$ is an intuitionistic fuzzy soft subhypergraph for all $a \in R$.

That is,

1. $\mathfrak{S}_\mu(a)(x_1, ..., x_n) \le \max\{\mathfrak{N}_\mu(a)(x_1), \mathfrak{N}_\mu(a)(x_2), ..., \mathfrak{N}_\mu(a)(x_n)\}$,
2. $\mathfrak{S}_\nu(a)(x_1, ..., x_n) \le \min\{\mathfrak{N}_\nu(a)(x_1), \mathfrak{N}_\nu(a)(x_2), ..., \mathfrak{N}_\nu(a)(x_n)\}$,

such that $0 < \mathfrak{S}_\mu(a)(x_1, ..., x_n) + \mathfrak{N}_\nu(a)(x_1, ..., x_n) \le 1$, for all $a \in R$ and $x_1, ..., x_n \in V$, where, $\mathfrak{S}_\mu(a)(x_1, ..., x_n)$ denotes the degree of membership and $\mathfrak{S}_\nu(a)(x_1, ..., x_n)$ denotes the degree of non-membership of vertex to intuitionistic fuzzy soft hyperedge $\mathfrak{S}_j$.
An intuitionistic fuzzy soft hypergraph is denoted by $\tilde{H} = (\mathfrak{N}(a), \mathfrak{S}(a))$.

Example 2.8. Consider an IFSHG $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ such that $\mathfrak{N} = \{v_1, v_2, v_3, v_4, v_5\}$ and $\mathfrak{S} = \{v_1v_2v_3, v_2v_3v_4, v_3v_4v_5, v_1v_2, v_4v_5\}$. Let $R = \{a_1, a_2, a_3\}$ be a parameter set. Let $(\mathfrak{N}, R)$ be an intuitionistic fuzzy soft set over V with its approximate function $\mathfrak{N} : R \to P(V)$ defined by
$\mathfrak{N}(a_1) = \{v_1\langle 0.3, 0.4\rangle, v_2\langle 0.5, 0.2\rangle, v_3\langle 0.7, 0.3\rangle, v_4\langle 0.2, 0.8\rangle, v_5\langle 0.6, 0.4\rangle\}$
$\mathfrak{N}(a_2) = \{v_1\langle 0.5, 0.2\rangle, v_2\langle 0.4, 0.1\rangle, v_3\langle 0.3, 0.7\rangle, v_4\langle 0.8, 0.1\rangle\}$
$\mathfrak{N}(a_3) = \{v_1\langle 0.4, 0.4\rangle, v_2\langle 0.6, 0.2\rangle, v_3\langle 0.3, 0.7\rangle\}$
Let $(\mathfrak{S}, R)$ be an intuitionistic fuzzy soft set over E with its approximate function $\mathfrak{S} : R \to P(E)$ defined by
$\mathfrak{S}(a_1) = \{v_1v_2v_3\langle 0.7, 0.2\rangle, v_1v_2\langle 0.5, 0.2\rangle, v_3v_4v_5\langle 0.7, 0.3\rangle, v_2v_3v_4\langle 0.7, 0.2\rangle, v_4v_5\langle 0.6, 0.4\rangle\}$
$\mathfrak{S}(a_2) = \{v_1v_2v_3\langle 0.5, 0.1\rangle, v_1v_2\langle 0.5, 0.1\rangle, v_2v_3v_4\langle 0.8, 0.1\rangle\}$
$\mathfrak{S}(a_3) = \{v_1v_2v_3\langle 0.6, 0.2\rangle, v_1v_2\langle 0.6, 0.2\rangle\}$
Thus $\tilde{H}(a_1) = (\mathfrak{N}(a_1), \mathfrak{S}(a_1))$, $\tilde{H}(a_2) = (\mathfrak{N}(a_2), \mathfrak{S}(a_2))$, $\tilde{H}(a_3) = (\mathfrak{N}(a_3), \mathfrak{S}(a_3))$ are IFSHGs corresponding to the parameters a_1, a_2, a_3 respectively.

Table 1: The tabular representation of $\mathfrak{N}$ for IFSHGs.

$\mathfrak{N}$	v_1	v_2	v_3	v_4	v_5
a_1	$\langle 0.3, 0.4\rangle$	$\langle 0.5, 0.2\rangle$	$\langle 0.7, 0.3\rangle$	$\langle 0.2, 0.8\rangle$	$\langle 0.6, 0.4\rangle$
a_2	$\langle 0.5, 0.2\rangle$	$\langle 0.4, 0.1\rangle$	$\langle 0.3, 0.7\rangle$	$\langle 0.8, 0.1\rangle$	$\langle 0.0, 1.0\rangle$
a_3	$\langle 0.4, 0.4\rangle$	$\langle 0.6, 0.2\rangle$	$\langle 0.3, 0.7\rangle$	$\langle 0.0, 1.0\rangle$	$\langle 0.0, 1.0\rangle$

Hence, $\tilde{H} = (H^*, \mathfrak{N}, \mathfrak{S}, R)$ is an IFSHG (see Fig. 1, Table 1 and Table 2).

Definition 2.9. The order of an IFSHG is

$$\mathcal{O}(\tilde{H}) = \left\langle \sum_{a_i \in R}\left(\sum_{v \in V}\mathfrak{N}_\mu(v)\right), \sum_{a_i \in R}\left(\sum_{v \in V}\mathfrak{N}_\nu(v)\right)\right\rangle.$$

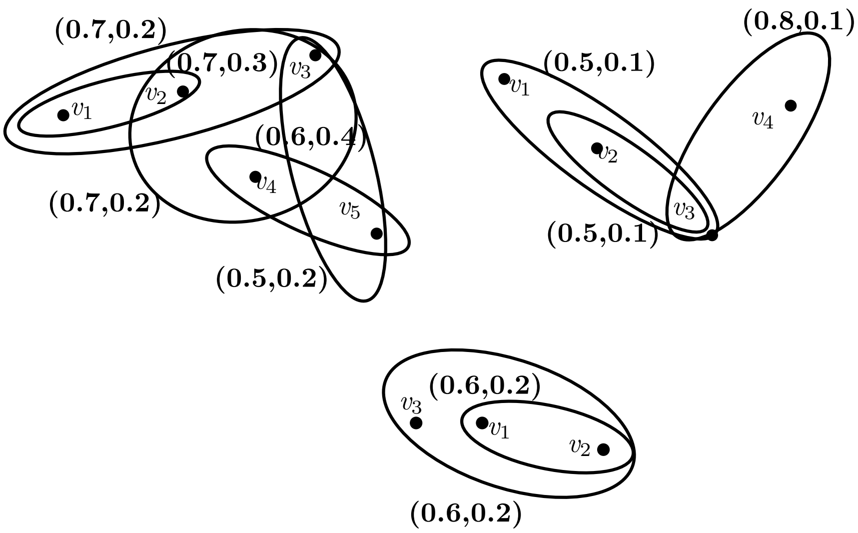

Fig. 1: Intuitionistic fuzzy soft hypergraphs.

Table 2: The tabular representation of $\mathfrak{S}$ for IFSHGs.

$\mathfrak{S}$	$v_1v_2v_3$	v_1v_2	$v_3v_4v_5$	$v_2v_3v_4$	v_4v_5
a_1	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.2\rangle$	$\langle 0.7, 0.3\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.6, 0.4\rangle$
a_2	$\langle 0.5, 0.1\rangle$	$\langle 0.5, 0.1\rangle$	$\langle 0.0, 1.0\rangle$	$\langle 0.8, 0.1\rangle$	$\langle 0.0, 1.0\rangle$
a_3	$\langle 0.6, 0.2\rangle$	$\langle 0.6, 0.2\rangle$	$\langle 0.0, 1.0\rangle$	$\langle 0.0, 1.0\rangle$	$\langle 0.0, 1.0\rangle$

Definition 2.10. The size of an IFSHG is

$$\mathcal{S}(\tilde{H}) = \Big\langle \sum_{a_i \in R}\Big(\sum_{v_1,\ldots,v_n \in \mathfrak{S}} \mathfrak{S}_\mu(a_i)(v_1 \ldots v_n)\Big), \sum_{a_i \in R}\Big(\sum_{v_1,\ldots,v_n \in \mathfrak{S}} \mathfrak{S}_\nu(a_i)(v_1 \ldots v_n)\Big)\Big\rangle.$$

Definition 2.11. The degree of an IFSHG is

$$\deg(v) = \Big\langle \deg\mathfrak{N}_\mu(a_i)(v_i), \deg\mathfrak{N}_\nu(a_i)(v_i)\Big\rangle,$$

where $\deg\mathfrak{N}_\mu(a_i)(v_i)$ denotes the sum of membership values of the hyperedge that contains the vertex v_i corresponding to the parameter $a_i \in R$ and $\deg\mathfrak{N}_\nu(a_i)(v_i)$ denotes the sum of non-membership values of the hyperedge that contains the vertex v_i corresponding to the parameter $a_i \in R$.

Example 2.12. Consider an IFSHG $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ such that $\mathfrak{N} = \{v_1, v_2, v_3, v_4\}$ and $\mathfrak{S} = \{v_1v_3, v_1v_4, v_2v_3v_4\}$. Let $R = \{a_1, a_2, a_3\}$ be a parameter set. Let $(\mathfrak{N}, R)$ be an intuitionistic fuzzy soft set over V with its approximate function $\mathfrak{N} : R \to P(V)$ defined by
$\mathfrak{N}(a_1) = \{v_1\langle 0.3, 0.4\rangle, v_2\langle 0.8, 0.2\rangle, v_3\langle 0.7, 0.2\rangle, v_4\langle 0.5, 0.1\rangle\}$
$\mathfrak{N}(a_2) = \{v_1\langle 0.5, 0.2\rangle, v_2\langle 0.9, 0.1\rangle, v_3\langle 0.6, 0.4\rangle, v_4\langle 0.8, 0.2\rangle\}$
$\mathfrak{N}(a_3) = \{v_1\langle 0.4, 0.4\rangle, v_2\langle 0.8, 0.2\rangle, v_3\langle 0.7, 0.3\rangle, v_4\langle 0.3, 0.6\rangle\}$
Let $(\mathfrak{S}, R)$ be an intuitionistic fuzzy soft set over E with its approximate function $\mathfrak{S} : R \to P(E)$ defined by
$\mathfrak{S}(a_1) = \{v_1v_3\langle 0.7, 0.2\rangle, v_1v_4\langle 0.5, 0.1\rangle, v_2v_3v_4\langle 0.8, 0.1\rangle\}$
$\mathfrak{S}(a_2) = \{v_1v_3\langle 0.6, 0.2\rangle, v_1v_4\langle 0.8, 0.2\rangle, v_2v_3v_4, \langle 0.9, 0.1\rangle\}$

$\mathfrak{S}(a_3) = \{v_1v_3\langle 0.7, 0.3\rangle, v_1v_4\langle 0.4, 0.4\rangle, v_2v_3v_4\langle 0.8, 0.2\rangle\}$
$\mathcal{O}(\tilde{H}) = \Big\langle \sum_{a_i\in R}\Big(\sum_{v\in V}\mathfrak{N}_\mu(v)\Big), \sum_{a_i\in R}\Big(\sum_{v\in V}\mathfrak{N}_\nu(v)\Big)\Big\rangle.$
$= [(0.3 + 0.8 + 0.7 + 0.5) + (0.5 + 0.9 + 0.6 + 0.8) + (0.4 + 0.8 + 0.7 + 0.3),$
$(0.4 + 0.6 + 0.2 + 0.1) + (0.2 + 0.1 + 0.5 + 0.4) + (0.4 + 0.2 + 0.5 + 0.6)]$
$= [(2.3 + 2.8 + 2.2), (1.3 + 1.2 + 1.7)]$
$= \langle 7.3, 4.2\rangle$
$\mathcal{S}(\tilde{H}) = \Big\langle \sum_{a_i\in R}\Big(\sum_{v_1\ldots v_n\in\mathfrak{S}}\mathfrak{S}_\mu(a_i)(v_1\ldots v_n)\Big), \sum_{a_i\in R}\Big(\sum_{v_1\ldots v_n\in\mathfrak{S}}\mathfrak{S}_\nu(a_i)(v_1\ldots v_n)\Big)\Big\rangle.$
$= [(0.7 + 0.5 + 0.8) + (0.6 + 0.8 + 0.9) + (0.7 + 0.4 + 0.8)$,
$(0.2 + 0.1 + 0.1) + (0.2 + 0.2 + 0.1) + (0.3 + 0.4 + 0.2)]$.
$= [(2.0 + 2.3 + 1.9), (0.4, 0.5, 0.9)]$
$= \langle 6.2, 1.8\rangle.$

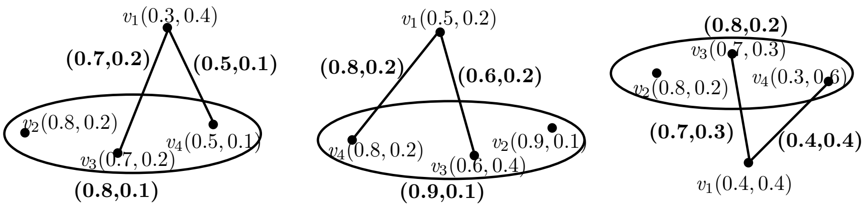

Fig. 2: IFSHGs for the parameters a_1, a_2, a_3.

The degree of the vertex v_i corresponding to the parameter a_1, $\deg(v_1) = \langle 1.2, 0.3\rangle$, $\deg(v_2) = \langle 0.8, 0.1\rangle$, $\deg(v_3) = \langle 1.5, 0.3\rangle$, $\deg(v_4) = \langle 1.3, 0.2\rangle$.
The degree of the vertex v_i corresponding to the parameter a_2, $\deg(v_1) = \langle 1.4, 0.4\rangle$, $\deg(v_2) = \langle 0.9, 0.1\rangle$, $\deg(v_3) = \langle 1.5, 0.3\rangle$, $\deg(v_4) = \langle 1.7, 0.3\rangle$.
The degree of the vertex v_i corresponding to the parameter a_3, $\deg(v_1) = \langle 1.1, 0.7\rangle$, $\deg(v_2) = \langle 0.8, 0.2\rangle$, $\deg(v_3) = \langle 1.5, 0.5\rangle$, $\deg(v_4) = \langle 1.2, 0.6\rangle$.
The degree of the vertex v_i differs in their values with respect to the parameters.

Definition 2.13. The strength (η) of a hyperedge $\mathfrak{S}_j$ of an IFSHG is the maximum of a membership value and minimum of a non-membership value.
$\eta\big(\mathfrak{S}_j(a_i)\big) = \big(\max_{v\in V}(\mathfrak{N}_\mu(v_i))(a_i), \min_{v\in V}(\mathfrak{N}_\nu(v_i))(a_i)\big)$, such that $\mathfrak{N}_\mu(v_i) > 0$ and $\mathfrak{N}_\nu(v_i) > 0$.

Example 2.14. In Fig. 2 the strength (η) of a hyperedge $\mathfrak{S}_j$ with respect to the corresponding parameters are as follows,
$\eta(\mathfrak{S}_1)(a_1) = \langle 0.7, 0.2\rangle, \eta(\mathfrak{S}_2)(a_1) = \langle 0.5, 0.1\rangle, \eta(\mathfrak{S}_3)(a_1) = \langle 0.8, 0.1\rangle$
$\eta(\mathfrak{S}_1)(a_2) = \langle 0.6, 0.2\rangle, \eta(\mathfrak{S}_2)(a_2) = \langle 0.8, 0.2\rangle, \eta(\mathfrak{S}_3)(a_2) = \langle 0.8, 0.1\rangle$
$\eta(\mathfrak{S}_1)(a_3) = \langle 0.7, 0.3\rangle, \eta(\mathfrak{S}_2)(a_3) = \langle 0.4, 0.4\rangle, \eta(\mathfrak{S}_3)(a_3) = \langle 0.8, 0.2\rangle$
The incidence matrix corresponding to the strength of a hyperedge is given below:

$$\begin{array}{cc} \eta & \begin{array}{ccc} \mathfrak{S}_1 & \mathfrak{S}_2 & \mathfrak{S}_3 \end{array} \\ \begin{array}{c} a_1 \\ a_2 \\ a_3 \end{array} & \begin{pmatrix} \langle 0.7, 0.2\rangle & \langle 0.5, 0.1\rangle & \langle 0.8, 0.1\rangle \\ \langle 0.6, 0.2\rangle & \langle 0.8, 0.2\rangle & \langle 0.8, 0.1\rangle \\ \langle 0.7, 0.3\rangle & \langle 0.4, 0.4\rangle & \langle 0.8, 0.2\rangle \end{pmatrix} \end{array}$$

Note: Among the strength of a hyperedges, the hyperedge which possess minimum of membership value and maximum of non-membership value in the corresponding parameters is said to be *stronger*. Thus the hyperedges $\mathfrak{S}_2$ in the parameter a_3 is stronger than the other hypergedges in a_1 and a_2.

3 Certain types of intuitionistic fuzzy soft hypergraphs

Definition 3.1. Let $\tilde{H}$ be an IFSHG. The height of $\tilde{H}$, is defined by

$$\mathtt{h}(\tilde{H}(a)) = \{(\max\mathfrak{E}_j, \min\mathfrak{E}_k) \,|\, \mathfrak{E}_j, \mathfrak{E}_k \in \mathfrak{S}\}.$$

where $\mathfrak{E}_j = \max\mathfrak{E}_\mu(a)(v_1...v_n)$ and $\mathfrak{E}_k = \min\mathfrak{E}_\nu(a)(v_1...v_n)$, $a \in R$, for all $j = 1, 2, ..., n$ and $k = 1, 2, ..., m$.

Definition 3.2. An IFSHG $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ is *simple* if hyperedge $\mathfrak{S}$ has no repeated intuitionistic fuzzy hyperedges, whenever $\mathfrak{E}_j, \mathfrak{E}_k \in \mathfrak{S}$ and $\mathfrak{E}_j(a_i) \subseteq \mathfrak{E}_k(a_i)$ then $\mathfrak{E}_j(a_i) = \mathfrak{E}_k(a_i)$ for all j and k, $a_i \in R$.

Example 3.3. Consider an IFSHG $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$, such that $\mathfrak{N} = \{v_1, v_2, v_3, v_4, v_5\}$ and $\mathfrak{S} = \{E_1, E_2, E_3, E_4\}$. Let $R = \{a_1, a_2\}$ be a parameter set. Let $(\mathfrak{N}, R)$ be an intuitionistic fuzzy soft set over V with its approximate function $\mathfrak{N} : R \to P(V)$.
$\mathfrak{N}(a_1) = \{v_1\langle 0.7, 0.3\rangle, v_2\langle 0.5, 0.4\rangle, v_3\langle 0.8, 0.3\rangle, v_4\langle 0.6, 0.2\rangle, v_5\langle 0.3, 0.3\rangle\}$
$\mathfrak{N}(a_2) = \{v_1\langle 0.5, 0.4\rangle, v_2\langle 0.4, 0.4\rangle, v_3\langle 0.6, 0.3\rangle, v_4\langle 0.6, 0.4\rangle, v_5\langle 0.5, 0.1\rangle\}$.
Let $(\mathfrak{S}, R)$ be an intuitionistic fuzzy soft set over E with its approximate function $\mathfrak{S} : R \to P(E)$.
$\mathfrak{S}(a_1) = \{E_1\langle 0.7, 0.3\rangle, E_2\langle 0.8, 0.2\rangle, E_3\langle 0.5, 0.3\rangle, E_4\langle 0.7, 0.2\rangle\}$
$\mathfrak{S}(a_2) = \{E_1\langle 0.5, 0.4\rangle, E_2\langle 0.6, 0.3\rangle, E_3\langle 0.5, 0.1\rangle, E_4\langle 0.6, 0.4\rangle\}$

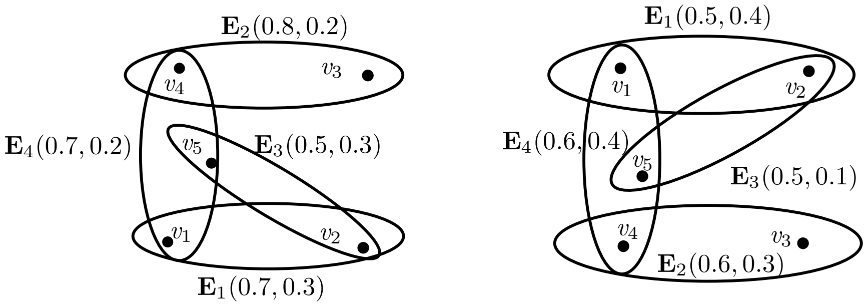

Fig. 3: Simple.

The incidence matrix corresponding to the parameters of a hyperedge is given below (Fig. 3):

$$\begin{array}{cc} \mathfrak{S} & \begin{array}{cccc} E_1 & E_2 & E_3 & E_4 \end{array} \\ \begin{array}{c} a_1 \\ a_2 \end{array} & \begin{pmatrix} \langle 0.7, 0.3\rangle & \langle 0.8, 0.2\rangle & \langle 0.5, 0.3\rangle & \langle 0.7, 0.2\rangle \\ \langle 0.5, 0.4\rangle & \langle 0.6, 0.3\rangle & \langle 0.5, 0.1\rangle & \langle 0.6, 0.4\rangle \end{pmatrix} \end{array}$$

Definition 3.4. An IFSHG $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ is *support simple*, whenever $\mathfrak{E}_j, \mathfrak{E}_k \in \mathfrak{S}$ and $\mathfrak{E}_j(a_i) \subseteq \mathfrak{E}_k(a_i)$ and $\mathrm{supp}\mathfrak{E}_j(a_i) = \mathrm{supp}\mathfrak{E}_k(a_i)$, then $\mathfrak{E}_j(a_i) = \mathfrak{E}_k(a_i)$ for all j and k, $a_i \in R$. Hence the hyperedges are supporting hyperedges.

Example 3.5. Consider an IFSHG $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$, such that $\mathfrak{N} = \{v_1, v_2, v_3, v_4, v_5\}$ and $\mathfrak{S} = \{E_1, E_2, E_3, E_4\}$. Let $R = \{a_1, a_2\}$ be a parameter set. Let $(\mathfrak{N}, R)$ be an intuitionistic fuzzy soft set over V with its approximate function $\mathfrak{N} : R \to P(V)$.
$\mathfrak{N}(a_1) = \{v_1\langle 0.8, 0.1\rangle, v_2\langle 0.5, 0.2\rangle, v_3\langle 0.6, 0.4\rangle, v_4\langle 0.8, 0.1\rangle, v_5\langle 0.8, 0.1\rangle\}$
$\mathfrak{N}(a_2) = \{v_1\langle 0.5, 0.3\rangle, v_2\langle 0.6, 0.4\rangle, v_3\langle 0.4, 0.4\rangle, v_4\langle 0.6, 0.3\rangle, v_5\langle 0.5, 0.4\rangle\}$
Let $(\mathfrak{S}, R)$ be an intuitionistic fuzzy soft set over E with its approximate function $\mathfrak{S} : R \to P(E)$.
$\mathfrak{S}(a_1) = \{E_1\langle 0.8, 0.1\rangle, E_2\langle 0.8, 0.1\rangle, E_3\langle 0.8, 0.1\rangle, E_4\langle 0.8, 0.1\rangle\}$
$\mathfrak{S}(a_2) = \{E_1\langle 0.6, 0.3\rangle, E_2\langle 0.6, 0.3\rangle, E_3\langle 0.6, 0.3\rangle, E_4\langle 0.6, 0.3\rangle\}$
The incidence matrix corresponding to the parameters of a hyperedge is given below (Fig. 4):

$$\begin{array}{cc} \mathfrak{S} & \begin{array}{cccc} E_1 & E_2 & E_3 & E_4 \end{array} \\ \begin{array}{c} a_1 \\ a_2 \end{array} & \begin{pmatrix} \langle 0.8, 0.1\rangle & \langle 0.8, 0.1\rangle & \langle 0.8, 0.1\rangle & \langle 0.8, 0.1\rangle \\ \langle 0.6, 0.3\rangle & \langle 0.6, 0.3\rangle & \langle 0.6, 0.3\rangle & \langle 0.6, 0.3\rangle \end{pmatrix} \end{array}$$

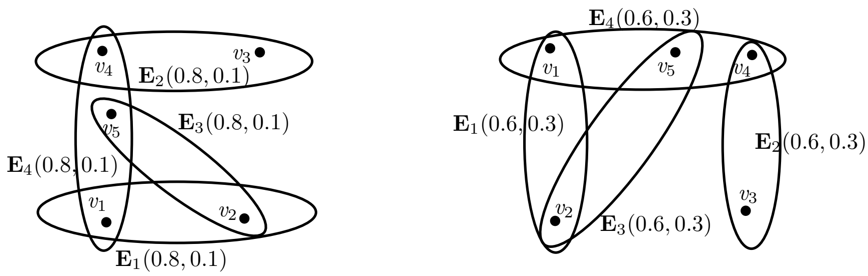

Fig. 4: Support simple.

Note: If the values of the hyperedges are same in the parameter then it is said to be uniform edge IFSHG.

Definition 3.6. Let $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ be an IFSHGs. Suppose $\mathfrak{E}_j, \mathfrak{E}_k \in \mathfrak{S}$ and $0 < (\alpha, \beta) < 1$. The (α, β) - level of the parameters $a_i \in R$ is defined by
$(\mathfrak{E}_j, \mathfrak{E}_k)^{\alpha,\beta}(a_i) = \left\{ v_i \in V / \max(\mu_{ij}^{\alpha}(v_i)(a_i) \geq \alpha), \min(\nu_{ij}^{\beta}(v_i)(a_i) \leq \beta) \right\}$.

Definition 3.7. Let $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ be an IFSHGs. For $0 < (s, r) < h(\tilde{H})$. Let $\tilde{H}_{s,r}$ be the (s-r) level intuitionistic fuzzy hypergraph of H^*. The sequence of real numbers $(s_1, r_1), (s_2, r_2), ..., (s_n, r_n)$, such that $0 \leq s_i \leq h\mu(\tilde{H})$ and $0 \leq r_i \leq h_\nu(\tilde{H})$, where $(s_n, r_n) = h(\tilde{H})$, satisfying the properties,

- If $s_1 < \alpha \leq 1$ and $0 \leq r_1$, then $\mathfrak{S}^{\alpha,\beta} \neq \phi$ for all $a_i \in R$.
- If $s_{i+1} \leq \alpha \leq s_i$, $r_{i+1} \leq \beta \leq r_i$, then $\mathfrak{S}^{\alpha,\beta}(a_i) = \mathfrak{S}^{s_i,r_i}(a_i)$ for all $a_i \in R$.
- $\mathfrak{S}^{s_i,r_i}(a_i) \subset \mathfrak{S}^{s_{i+1},r_{i+1}}(a_i)$ for all $a_i \in R$ is called the fundamental sequence of $\tilde{H}$ corresponding to the parameters and it is denoted by $F(\tilde{H})$.

The core set of $\tilde{H}$ is denoted by $\mathcal{C}(\tilde{H})$. It is defined by $\mathcal{C}(\tilde{H}) = \tilde{H}^{s_1,r_1}(a_i), \tilde{H}^{s_2,r_2}(a_i), ..., \tilde{H}^{s_n,r_n}(a_i)$. The corresponding set of (s_i, r_i)-level hypergraph $\tilde{H}^{s_1,r_1}(a_i) \subset \tilde{H}^{s_2,r_2}(a_i) \subset ... \subset \tilde{H}^{s_n,r_n}(a_i)$ is called the $\tilde{H}$ induced fundamental sequence and it is denoted by $\mathcal{I}(\tilde{H})$. The (s_n, r_n)-level is called the *support level* of $\tilde{H}$ and the $\tilde{H}^{s_n,r_n}(a_i)$ is called the *support* of $\tilde{H}$.

Definition 3.8. An IFSHG is said to be *elementary* if $\mu_{ij} : V \to [0, 1]$ and $\nu_{ij} : V \to [0, 1]$ are constant functions or has a range $\{0, e\}, e \neq 0$. If $|\text{supp}(\mu_{ij}, \nu_{ij})(a_i)| = 1$ then it is an intuitionistic fuzzy subsets with singleton support called a spike.

Example 3.9. Consider an IFSHG $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$, such that $\mathfrak{N} = \{v_1, v_2, v_3, v_4, v_5\}$ and $\mathfrak{S} = \{E_1, E_2, E_3, E_4\}$. Let $R = \{a_1, a_2\}$ be a parameter set. Let $(\mathfrak{N}, R)$ be an intuitionistic fuzzy soft set over V with its approximate function $\mathfrak{N} : R \to P(V)$.
$\mathfrak{N}(a_1) = \{v_1\langle 0.6, 0.4\rangle, v_2\langle 0.2, 0.5\rangle, v_3\langle 0.6, 0.4\rangle, v_4\langle 0.4, 0.3\rangle, v_5\langle 0.7, 0.1\rangle\}$
$\mathfrak{N}(a_2) = \{v_1\langle 0.8, 0.2\rangle, v_2\langle 0.2, 0.5\rangle, v_3\langle 0.7, 0.3\rangle, v_4\langle 0.4, 0.4\rangle, v_5\langle 0.6, 0.4\rangle\}$
Let $(\mathfrak{S}, R)$ be an intuitionistic fuzzy soft set over E with its approximate function $\mathfrak{S} : R \to P(E)$.
$\mathfrak{S}(a_1) = \{E_1\langle 0.6, 0.4\rangle, E_2\langle 0.4, 0.3\rangle, E_3\langle 0.6, 0.4\rangle, E_4\langle 0.7, 0.1\rangle\}$
$\mathfrak{S}(a_2) = \{E_1\langle 0.8, 0.2\rangle, E_2\langle 0.6, 0.4\rangle, E_3\langle 0.4, 0.4\rangle, E_4\langle 0.8, 0.2\rangle\}$
The incidence matrix corresponding to the parameters of a hyperedge is given below (Fig. 5):

$$\begin{array}{c} \mathfrak{S} \\ a_1 \\ a_2 \end{array} \begin{array}{c} \begin{array}{cccc} E_1 & E_2 & E_3 & E_4 \end{array} \\ \begin{pmatrix} \langle 0.6, 0.4\rangle & \langle 0.4, 0.3\rangle & \langle 0.6, 0.4\rangle & \langle 0.7, 0.1\rangle \\ \langle 0.8, 0.2\rangle & \langle 0.6, 0.4\rangle & \langle 0.4, 0.4\rangle & \langle 0.8, 0.2\rangle \end{pmatrix} \end{array}$$

Definition 3.10. If $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ be an IFSHG and $F(\tilde{H}) = s_n, s_{n-1}, ..., s_1, r_1, ..., r_n$. Then $\tilde{H}$ is called *sectionally elementary* if for each $\mu_{ij}, \nu_{ij} \in \mathfrak{S}$ and $s_i, r_i \in F(\tilde{H})$, $\mu_{ij}^{\alpha} = \mu_{ij}^{s_i}$ and $\nu_{ij}^{\beta} = \nu_{ij}^{r_i}$ for all $\alpha, \beta \in (s_{i+1}, r_i]$. Assume $s_{i+1} = 0$.

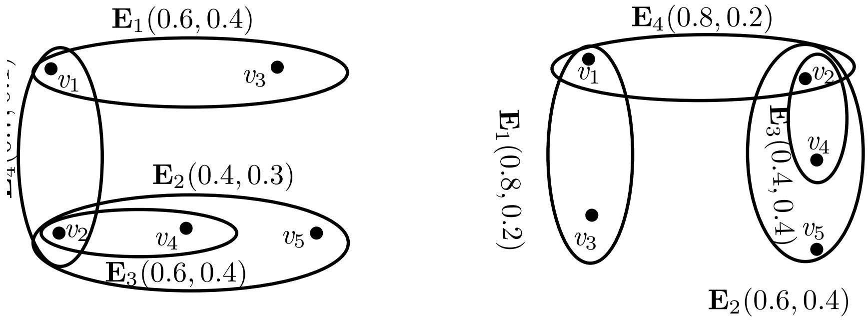

Fig. 5: Elementary IFSHGs.

Example 3.11. Consider an IFSHG $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$, such that $\mathfrak{N} = \{v_1, v_2, v_3, v_4, v_5\}$ and $\mathfrak{S} = \{E_1, E_2, E_3\}$. Let $R = \{a_1, a_2\}$ be a parameter set. Let $(\mathfrak{N}, R)$ be an intuitionistic fuzzy soft set over V with its approximate function $\mathfrak{N} : R \to P(V)$.
$\mathfrak{N}(a_1) = \{v_1\langle 0.6, 0.4\rangle, v_2\langle 0.5, 0.4\rangle, v_3\langle 0.5, 0.3\rangle, v_4\langle 0.5, 0.4\rangle, v_5\langle 0.5, 0.4\rangle\}$
$\mathfrak{N}(a_2) = \{v_1\langle 0.7, 0.2\rangle, v_2\langle 0.4, 0.3\rangle, v_3\langle 0.9, 0.1\rangle, v_4\langle 0.7, 0.2\rangle, v_5\langle 0.5, 0.4\rangle\}$
Let $(\mathfrak{S}, R)$ be an intuitionistic fuzzy soft set over E with its approximate function $\mathfrak{S} : R \to P(E)$.
$\mathfrak{S}(a_1) = \{E_1\langle 0.6, 0.3\rangle, E_2\langle 0.5, 0.4\rangle, E_3\langle 0.5, 0.3\rangle\}$
$\mathfrak{S}(a_2) = \{E_1\langle 0.7, 0.2\rangle, E_2\langle 0.7, 0.2\rangle, E_3\langle 0.7, 0.2\rangle\}$

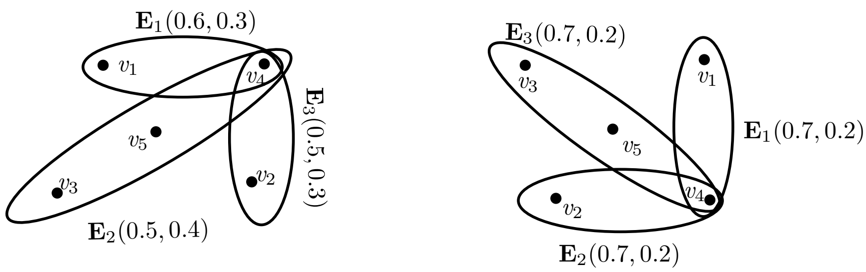

Fig. 6: Sectionally elementary IFSHGs.

The incidence matrix corresponding to the parameters of a hyperedge is given below (Fig. 6):

$$\begin{array}{cc} \mathfrak{S} & \begin{array}{ccc} E_1 & E_2 & E_3 \end{array} \\ \begin{array}{c} a_1 \\ a_2 \end{array} & \begin{pmatrix} \langle 0.6, 0.3\rangle & \langle 0.5, 0.4\rangle & \langle 0.5, 0.3\rangle \\ \langle 0.7, 0.2\rangle & \langle 0.7, 0.2\rangle & \langle 0.7, 0.2\rangle \end{pmatrix} \end{array}$$

Definition 3.12. An IFSHG $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ is called (μ, ν)-tempered intuitionistic fuzzy soft hypergraph (TIFSHG) if there exists an intuitionistic fuzzy subsets $\mu_{ij} : V \to [0, 1]$ and $\nu_{ij} : V \to [0, 1]$ such that $\mathfrak{S} = \{(\mu_{ij}(a_i)(v_i), \nu_{ij}(a_i)(v_i))/v_i \in \mathfrak{S}_i\}$,
where

$$\mu_{ij}(a_i)(v_i) = \begin{cases} \wedge \mu_j(y)(a_i)/y \in \mathfrak{S}, a_i \in R & \text{if } \ v_i \in \mathfrak{S}_i \\ 0, & \text{otherwise,} \end{cases}$$

and

$$\nu_{ij}(a_i)(v_i) = \begin{cases} \vee \nu_j(y)(a_i)/y \in \mathfrak{S}, a_i \in R & \text{if } v_i \in \mathfrak{S}_i \\ 0, & \text{otherwise.} \end{cases}$$

Example 3.13. Consider an IFSHG $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$, such that $\mathfrak{N} = \{v_1, v_2, v_3, v_4, v_5\}$ and $\mathfrak{S} = \{E_1, E_2, E_3, E_4\}$. Let $R = \{a_1, a_2\}$ be a parameter set. Let $(\mathfrak{N}, R)$ be an intuitionistic fuzzy soft set over V with its approximate function $\mathfrak{N} : R \to P(V)$. Let $(\mathfrak{S}, R)$ be an intuitionistic fuzzy soft set over E with its approximate function $\mathfrak{S} : R \to P(E)$. Define $\mu_{ij} : V \to [0, 1]$ and $\nu_{ij} : V \to [0, 1]$.

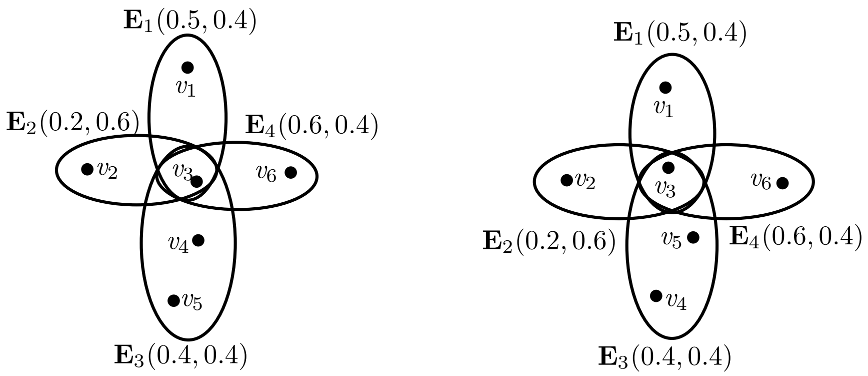

Fig. 7: (μ, ν)-tempered IFSHGs.

Considering for the parameter a_1,
$\mu_{1j}(v_1) = 0.5,\ \mu_{2j}(v_2) = 0.2,\ \mu_{3j}(v_3) = 0.6,\ \mu_{4j}(v_4) = 0.4,\ \mu_{5j}(v_5) = 0.4,\ \mu_{6j}(v_6) = 0.4$
$\nu_{1j}(v_1) = 0.4,\ \nu_{2j}(v_2) = 0.6,\ \nu_{3j}(v_3) = 0.4,\ \nu_{4j}(v_4) = 0.4,\ \nu_{5j}(v_5) = 0.4,\ \nu_{6j}(v_6) = 0.4.$
Note that
$\mu_{ij}(v_1) = \mu_1 j(v_1) \wedge \mu_3 j(v_3) = 0.5$
$\mu_{ij}(v_2) = \mu_1 j(v_2) \wedge \mu_3 j(v_3) = 0.2$
$\mu_{ij}(v_3) = \mu_1 j(v_3) \wedge \mu_3 j(v_1) = 0.4$
$\mu_{ij}(v_4) = \mu_1 j(v_3) \wedge \mu_3 j(v_4) = 0.4$
$\mu_{ij}(v_5) = \mu_1 j(v_4) \wedge \mu_3 j(v_5) = 0.4$
$\mu_{ij}(v_6) = \mu_1 j(v_3) \wedge \mu_3 j(v_6) = 0.6$
and
$\nu_{ij}(v_1) = \nu_1 j(v_1) \vee \nu_3 j(v_3) = 0.4$
$\nu_{ij}(v_2) = \nu_1 j(v_2) \vee \nu_3 j(v_3) = 0.6$
$\nu_{ij}(v_3) = \nu_1 j(v_3) \vee \nu_3 j(v_1) = 0.4$
$\nu_{ij}(v_4) = \nu_1 j(v_3) \vee \nu_3 j(v_4) = 0.4$
$\nu_{ij}(v_5) = \nu_1 j(v_4) \vee \nu_3 j(v_5) = 0.4$
$\nu_{ij}(v_6) = \nu_1 j(v_3) \vee \nu_3 j(v_6) = 0.4$

Table 3: The tabular representation of $\mathfrak{N}$ for (μ, ν)-tempered IFSHG.

$\mathfrak{N}$	v_1	v_2	v_3	v_4	v_5	v_6
a_1	$\langle 0.5, 0.3 \rangle$	$\langle 0.2, 0.6 \rangle$	$\langle 0.4, 0.1 \rangle$	$\langle 0.6, 0.4 \rangle$	$\langle 0.4, 0.2 \rangle$	$\langle 0.8, 0.2 \rangle$
a_2	$\langle 0.5, 0.4 \rangle$	$\langle 0.2, 0.6 \rangle$	$\langle 0.7, 0.3 \rangle$	$\langle 0.4, 0.4 \rangle$	$\langle 0.4, 0.4 \rangle$	$\langle 0.6, 0.4 \rangle$

Hence $\tilde{H}\,(a)$ is a (μ, ν)-tempered IFSHG (Fig. 7, Table 3 and Table 4).

Table 4: The tabular representation of $\mathfrak{S}$ for (μ,ν)-tempered IFSHG.

$\mathfrak{S}$	E_1	E_2	E_3	E_4
a_1	$\langle 0.5, 0.4\rangle$	$\langle 0.2, 0.6\rangle$	$\langle 0.4, 0.4\rangle$	$\langle 0.6, 0.4\rangle$
a_2	$\langle 0.5, 0.4\rangle$	$\langle 0.2, 0.6\rangle$	$\langle 0.4, 0.4\rangle$	$\langle 0.6, 0.4\rangle$

Theorem 3.14. *An IFSHG $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ is (μ,ν)-TIFSHG of some crisp hypergraph H if and only if $\tilde{H}$ is elementary, support simple and simply ordered.*

Proof. $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ be a (μ,ν)-TIFSHG, then in (μ,ν)-TIFSHG the membership values and non-membership values of an intuitionistic fuzzy soft hyperedges of $\tilde{H}$ are constant. Hence $\tilde{H}$ is elementary. It is obvious that the two intuitionistic fuzzy soft hyperedges of the (μ,ν)-TIFSHG are equal then the intuitionistic fuzzy soft hyperedges are equal. Hence it is support simple.
Let $\mathcal{C}(\tilde{H}) = \tilde{H}^{s_1,r_1}(a_i), \tilde{H}^{s_2,r_2}(a_i), ..., \tilde{H}^{s_n,r_n}(a_i)$ and since $\tilde{H}$ is elementary. $\tilde{H}$ is ordered. Let $a_i \in R$.
Claim(i): $\tilde{H}$ is simply ordered. Let $\mathfrak{S} \in H^{s_{i+1},r_{i+1}}(a_i) - H^{s_i,r_i}(a_i)$ then there exists $x_i \in \mathfrak{S}$ such that $\mu_{ij}(x_i)(a_i) = r_{i+1}$ and $\nu_{ij}(x_i)(a_i) = s_{i+1}$. Since $s_{i+1} < s_i$ and $r_{i+1} < r_i$, it follows that $(x_i)(a_i) \notin H^{s_i,r_i}(a_i)$ and $\mathfrak{E} \nsubseteq H^{s_i,r_i}(a_i)$.
Hence it is simply ordered.
Conversely, let $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ is elementary, support simple and simply ordered. By the core set definition $\tilde{H}^{s_i,r_i}(a_i) = \tilde{H}(\phi_i) = (\mathfrak{N}_i, \mathfrak{S}_i)$. Now we define $\mu_{ij} : \mathfrak{N}_i \to [0,1]$ and $\nu_{ij} : \mathfrak{N}_i \to [0,1]$ by

$$\mu_{ij}(x_i)(a_i) = \begin{cases} s_1 & \text{if } (x_i) \in \mathfrak{E}_1 \\ s_i & \text{if } (x_i) \in \mathfrak{E}_i \backslash \mathfrak{E}_{i-1}, i = 1,2,...,n \end{cases}$$

and

$$\nu_{ij}(x_i)(a_i) = \begin{cases} r_1 & \text{if } (x_i) \in \mathfrak{E}_1 \\ r_i & \text{if } (x_i) \in \mathfrak{E}_i \backslash \mathfrak{E}_{i-1}, i = 1,2,...,n \end{cases}$$

To prove: $\mathfrak{S}\left\{(\mu_{ij}(x_i)(a_i), \nu_{ij}(x_i)(a_i))\right\}|x_i \in \mathfrak{E}$, where

$$\mu_{ij}(x_i)(a_i) = \begin{cases} \wedge \mu_i(v)(a_i)/v \in \mathfrak{E} & \text{if } x_i \in \mathfrak{E} \text{ and } a_i \in R \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\nu_{ij}(x_i)(a_i) = \begin{cases} \vee \nu_i(v)(a_i)/v \in \mathfrak{E} & \text{if } x_i \in \mathfrak{E} \text{ and } a_i \in R \\ 0 & \text{otherwise.} \end{cases}$$

Let $\mathfrak{E} \in \mathfrak{E}_i$. Since $\tilde{H}$ is elementary and support simple there is a unique intuitionistic fuzzy soft hyperedge $\mathfrak{E}_s$ in $\mathfrak{S}$ having support $\mathfrak{S}_i$.
To prove: $\mathfrak{E} \in \mathfrak{E}_i$. As all the intuitionistic fuzzy soft hyperedges are elementary and $\tilde{H}$ is support simple, then the different hyperedges have different supports, from the definition of fundamental sequence $h\left(\mathfrak{E}_s\right)$ is equal to some member of (s_i, r_i) of $F(\tilde{H})$. Consequently, $\mathfrak{E} \subseteq \mathfrak{N}_i$ and if $i > 1$, then $\mathfrak{E} \in \mathfrak{E}_i \backslash \mathfrak{E}_{i-1}$.
From the definition of (μ,ν)-TIFSHG that for all $x_i \in \mathfrak{S}, \mu_{ij}(x_i)(a_i) \geq s_i$ and $\nu_{ij}(x_i)(a_i) \leq r_i$.
Claim(ii): $\mu_{ij}(x_i)(a_i) = s_i$ and $\nu_{ij}(x_i)(a_i) = r_i$ for some $x_i \in \mathfrak{S}$ and $a_i \in R$.
From the definition of (μ,ν)-TIFSHG, $\mu_{ij}(x_i)(a_i) \geq s_{i-1}$ and $\nu_{ij}(x_i)(a_i) \leq r_{i-1}$ for all $x_i \in \mathfrak{S}$ implies that $\mathfrak{S} \subseteq \mathfrak{N}_i$ and also $\mathfrak{E} \in \mathfrak{E}_i \backslash \mathfrak{E}_{i-1}$. Since H is simply ordered $\mathfrak{S} \nsubseteq \mathfrak{N}_i$, which is a contradiction. Thus from the definition of (μ,ν)-TIFSHG $\mathfrak{E} \in \mathfrak{E}_i$. Hence $\tilde{H}$ is (μ,ν)-TIFSHG. □

Theorem 3.15. *An IFSHG $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ is simply ordered and the fundamental sequence is $\{s_n, s_{n-1}, ..., s_1, r_1, ..., r_n\}$ and if $\tilde{H}^{s_n,r_n}$ is a simple hypergraph then there is a partial IFSHG $\tilde{H}' = (\mathfrak{N}, \mathfrak{S}')$ of $\tilde{H}$ such that the following statements hold:*

(i) $\tilde{H}'$ is a (μ,ν)-TIFSHG of $\tilde{H}$.
(ii) $F(\tilde{H}') = F(\tilde{H})$ and $\mathcal{C}(\tilde{H}') = \mathcal{C}(\tilde{H})$.

Proof. Since $\tilde{H}$ is simply ordered, then $\tilde{H}$ is an IFSHG. For obtaining the partial IFSHG $\tilde{H}^{'} = (\mathfrak{N}, \mathfrak{S}^{'})$ of $\tilde{H} = (\mathfrak{N}, \mathfrak{S})$ remove the hyperedges of $\tilde{H}$ that are properly contained in another hyperedges of $\tilde{H}$, where $\mathfrak{S}^{'} = \{\mu_{ij}(a_i), \nu_{ij}(a_i) \in \mathfrak{S} /$ if $\mu_{ij}(a_i) \leq \nu_{ij}(a_i)$ then $\mu_{ij}(a_i) = \nu_{ij}(a_i)\}$. Since $\tilde{H}^{s_n, r_n}$ is simple and all intuitionistic soft hyperedges are elementary, one hyperedge cannot be contained in another hyperedge of $\tilde{H}$ since then both of them have the same support. Hence $F(\tilde{H}^{'}) = F(\tilde{H})$ and $\mathcal{C}(\tilde{H}^{'}) = \mathcal{C}(\tilde{H})$.
From the definition, $\tilde{H}^{'}$ is support simple and satisfies all the conditions of Theorem 3.14. Thus $\tilde{H}^{'}$ is a (μ, ν)-TIFSHG of $\tilde{H}$. □

4 Application to emotion recognition

Emotion recognition is the task of classifying the emotion behind a piece of written text. The written text can be in the form of a sentence or sentences, a word or words, etc. and that written text identifies whether the user is happy or sad, or in fear or angered, or frustrated or disgusted and so on. Now we consider conversation between many users and their utterances (a piece of word or sentence) to recognize their emotions whether the users are happy or unhappy. We use hypergraph to represent between many users, by considering a set of utterances as vertices V. Here the vertices are utterances $v_1 =$ keep going, $v_2 =$ Are you okay, $v_3 =$ getting down, $v_4 =$ working together, $v_5 =$ exercise daily respectively, where as the hyperedge represent the relationship between two utterances based on emotions with respect to their parameters. The parameter set (R) may be k-users of different situation handlers, here $R = \{a_1, a_2, a_3\}$ where, $a_1 =$ players, $a_2 =$ unhealthy persons, $a_3 =$ business persons. Hence the membership and non-membership degree of a vertex denotes how much an utterance by user affects the emotions of the corresponding parameters. We consider an IFSHG $\tilde{H} = (\mathfrak{N}, \mathfrak{S}, R)$ where $(\mathfrak{N}, R)$ is an intuitionistic fuzzy soft set over V which describes the membership and non-membership values of the emotions based upon the given parameters. $(\mathfrak{S}, R)$ is an intuitionistic fuzzy soft set over $\mathfrak{S} \subseteq V \times V$ and it describes the membership and non-membership values of the relationship between two utterances corresponding to the given parameters a_1, a_2, a_3. For example, the utterance v_1 of a user holds 0.7 as a membership value) i.e., the utterance of a user makes 70% of happiness for the players. The hyperedge between the utterance v_1 and the utterance v_4 of a user can be 80% of happiness and 20% of unhappiness. An intuitionistic fuzzy soft hypergraph $\tilde{H} = \left\{\tilde{H}(a_1), \tilde{H}(a_2), \tilde{H}(a_3)\right\}$ corresponding to the parameter is given in Table 5 and Table 6.

Table 5: The tabular representation of $\mathfrak{N}$ for IFSHG.

$\mathfrak{N}$	v_1	v_2	v_3	v_4	v_5
a_1	$\langle 0.7, 0.1\rangle$	$\langle 0.3, 0.7\rangle$	$\langle 0.4, 0.5\rangle$	$\langle 0.8, 0.2\rangle$	$\langle 0.6, 0.4\rangle$
a_2	$\langle 0.7, 0.3\rangle$	$\langle 0.6, 0.4\rangle$	$\langle 0.8, 0.2\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.3\rangle$
a_3	$\langle 0.6, 0.2\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.3, 0.4\rangle$	$\langle 0.8, 0.1\rangle$	$\langle 0.7, 0.3\rangle$

Table 6: The tabular representation of $\mathfrak{S}$ for IFSHG.

$\mathfrak{S}$	E_1	E_2	E_3	E_4
a_1	$\langle 0.8, 0.1\rangle$	$\langle 0.7, 0.1\rangle$	$\langle 0.7, 0.1\rangle$	$\langle 0.6, 0.4\rangle$
a_2	$\langle 0.7, 0.3\rangle$	$\langle 0.8, 0.2\rangle$	$\langle 0.7, 0.3\rangle$	$\langle 0.8, 0.2\rangle$
a_3	$\langle 0.8, 0.1\rangle$	$\langle 0.6, 0.2\rangle$	$\langle 0.6, 0.2\rangle$	$\langle 0.7, 0.3\rangle$

The IFSHGs $\tilde{H}(a_1), \tilde{H}(a_2)$ and $\tilde{H}(a_3)$ corresponding to the parameters players, unhealthy persons, business persons respectively are shown in Fig. 8. By taking

$$\eta(\mathfrak{S}_j(a_i)) = \langle \max_{v \in V}(\mathfrak{N}_\mu(v_i))(a_i), \min_{v \in V}(\mathfrak{N}_\nu(v_i))(a_i), \rangle$$

such that $\mathfrak{N}_\mu(v_i) > 0$ and $\mathfrak{N}_\nu(v_i) > 0$, we obtain the strength of a hyperedge with respect to the corresponding parameters players, unhealthy persons, business persons and is given by the following incidence matrix:

$$\begin{array}{c} \eta \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{cccc} \quad E_1 \quad & \quad E_2 \quad & \quad E_3 \quad & \quad E_4 \quad \end{array} \\ \begin{pmatrix} \langle 0.8, 0.2\rangle & \langle 0.7, 0.1\rangle & \langle 0.7, 0.1\rangle & \langle 0.6, 0.4\rangle \\ \langle 0.7, 0.3\rangle & \langle 0.8, 0.2\rangle & \langle 0.7, 0.3\rangle & \langle 0.8, 0.2\rangle \\ \langle 0.8, 0.1\rangle & \langle 0.6, 0.2\rangle & \langle 0.6, 0.2\rangle & \langle 0.7, 0.3\rangle \end{pmatrix} \end{array}.$$

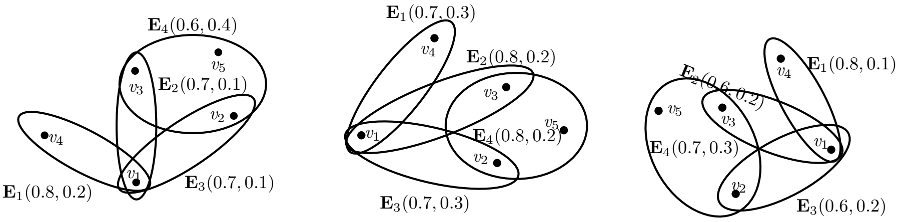

Fig. 8: IFSHGs for the parameters a_1, a_2, a_3.

The intuitionistic fuzzy hyperedges of an IFSHG corresponding to the parameters players, unhealthy persons, business persons respectively are elementary. Also, the hyperedge which possesses the minimum of membership value and the maximum of non-membership value in the corresponding parameters is said to be *stronger*. Thus the hyperedge E_4 in the parameter a_1 is stronger than the other hyperedges with respect to their parameters. The hyperedge E_4 has relationship between utterance v_2, utterance v_3 and utterance v_5, that the membership value and non-membership value of an utterances represent how much an utterance of a user is happy or unhappy based upon the parameter a_1 = players. For the parameter a_1, the utterance v_2, the utterance v_3 and the utterance v_5 holds $\langle 0.3, 0.7\rangle$, $\langle 0.4, 0.5\rangle$ and $\langle 0.6, 0.4\rangle$ as a membership value and a non-membership value respectively. That is, the utterance v_2 of a user may be 30% of happiness and 70% of unhappiness, the utterance v_3 of a user may be 40% of happiness and 50% of unhappiness and the utterance v_5 of a user may be 60% of happiness and 40% of unhappiness progressively. Since the hyperedge E_4 in the parameter a_1 is stronger and the hyperedge E_4 has relationship between the utterances v_2, v_3 and v_5 so that the user may be in the category of 60% of happiness and 40% of unhappiness for the parameter a_1 = players. Thus based on the utterances and their relationship for the parameter players, the emotion recognized by the user is to be happy.

5 Conclusion

Soft set theory plays a significant role as a mathematical tool for mathematical modeling, system analysis and computing of decision making problems with uncertainty. An intuitionistic fuzzy soft model is a generalization of the fuzzy soft model which gives more precision, flexibility, and compatibility to a system when compared with the fuzzy soft model. We applied the concept of intuitionistic fuzzy soft sets to hypergraphs in this paper. We presented certain types of intuitionistic fuzzy soft hypergraphs and also proved that an IFSHG is elementary, support simple and simply ordered. An application of IFSHGs to emotion recognition is also discussed and we recognized emotion based on the strength of an hyperedge among the parameters. In future, we intend to extend our research of fuzzification to interval-valued intuitionistic fuzzy soft hypergraphs and bipolar intuitionistic fuzzy soft hypergraphs.

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