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Rainbow Antimagic Coloring Of Some Graphs

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ABSTRACT

Chartrand et al. introduced the idea of rainbow colors in 2008. If every edge on a path P in a graph has a different color, the path is called a rainbow path. If there is at least one rainbow path connecting each pair of vertices in a graph G, then the graph is said to be rainbow connected. An vertex antimagic-labeling of a graph G is a function $g: E(G) \rightarrow \{1,2,\ldots,|V|\}$ that assigns unique labels to the edges in such a way that the sum of labels (or weights) incident to each edge is different for all edges. When the edge $e = uv$ with weights, defined as $w(e) = g(u) + g(v)$, induce an edge coloring and guarantees that there is a rainbow path between every vertex pair, the graph is said to possess a rainbow antimagic coloring. The minimum number of colors necessary to establish rainbow connectivity under this edge weight assignment is $RACN(G)$ called the rainbow-antimagic-connection number. In this paper, we determine the rainbow-antimagic-connection number of triangular-snake-graph and double-triangular-snake-graph.

Keywords: RACN, Triangular-Snake-graph, Double-Triangular-Snake-graph

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1. INTRODUCTION

Graph theory, A fundamental field of mathematics, has continuously evolved with the introduction of new coloring techniques to analyze and optimize complex networks. One such concept is Rainbow Antimagic Coloring, which merges the principles of rainbow coloring and antimagic labeling. Chartrand et.al was the first to introduce the conception of Rainbow Coloring in 2008 [8]. It refers to an edge coloring where the path contains all different colours, ensuring diversity in the chosen path between all pair of vertices. The least colours used by following this procedure is called the rainbow connection number rc. This idea has since been applied in various network optimization and security problems.

Building upon this, the notion of Rainbow Antimagic Coloring which was proposed by Dafik et.al in 2014 [3] emerged as an extension of Antimagic Labeling, an idea initially introduced by Hartsfield and Ringel in 1990 [6]. Antimagic labeling [9] of a graph G is a bijective function $f: E(G) \to 1, 2, \ldots, |E|$ such that the vertex-sum for distinct vertices are different. Similarly, an vertex antimagic labeling of a graph G is a function $g: E(G) \to \{1,2,\ldots,|V|\}$ that assigns unique labels to the edges in such a way that the sum of labels (or weights) incident to each edge is different for all edges. The minimum number of colors that are needed in order to make G rainbow connected under the assignment of edge weight $w(e) = g(u) + g(v)$ of every edge is the rainbow-antimagic-connection number rc_A [13].

Rainbow antimagic coloring, introduced in recent years, enhances this by ensuring that every path in the graph remains both antimagic and rainbow-colored, adding an additional layer of complexity and utility to the labeling process. The study of rainbow antimagic coloring has gained attention due to its applications in cryptography, network security, and time series forecasting. As research progresses, this concept continues to reveal new theoretical and practical insights in combinatorial mathematics and computational sciences.

Intan Kusumawardani et.al (2019) [7] found the rainbow antimagic connection number of Flower Graph and Gear Graph. Sulistiyono B et.al (2020) [13] determined the rainbow antimagic connection number of Ladder Graph, Triangular Ladder Graph and Diamond Graph. Budi H S et.al (2021) [2] determined the rainbow antimagic connection numbers of Lollipop, Stacked Book, Dutch Windmill, Flowerpot and Dragonfly. Septory B J et.al (2022) [10] found the Rainbow Antimagic Connection Number of Comb Product of Friendship Graph and Tree. Dafik et.al (2023) [4] found the rainbow antimagic connection numbers of Octopus Graph, Sun Flower Graph, Volcano Graph and Semi Jahangir Graph. Septory B J et.al (2024) [11] found the value of the rainbow antimagic connection number of Comb Product of any Tree and Complete Bipartite Graph.

2. PRELIMINARIES

Definition 2.1
A triangular snake graph T_n [1] is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} to a new vertex w_i for $1 \leq i \leq n-1$.

Definition 2.2
Double triangular snake graph $D(T_n)$ [5] consists of two triangular snakes that have a common path.

The rainbow connection number of triangular snake graph T_n and double triangular snake graph $D(T_n)$ are $rc(T_n) = n-1, n \geq 2$ and $rc(D(T_n)) = n-1, n \geq 2$ which was found by Parmar. D. *et.al*. (2019) [5]

The lower bound of the *RACN* has been determined by Septory *et.al*. [8] for any connected graph G as $RACN(G) \geq \max\{rc(G), \Delta(G)\}$

3. MAIN RESUTS

Theorem 3.1: *RACN of a Triangular-Snake-graph T_n with $n > 4$ is* $n-1 \leq rc_A(T_n) \leq n+1$.
Proof. Let us denote the triangular-snake-graph by T_n. Let the vertices of the path P_n be denoted by $v_1, v_2, \ldots, v_n$. The vertex adjacent to v_i and v_{i+1} is denoted as $u_i, i = 1,2,\ldots,n-1$. The graph T_n is a connected-graph with $V(T_n) = \{u_\alpha, 1 \leq \alpha \leq n-1\} \cup \{v_\alpha, 1 \leq \alpha \leq n\}$ and $E(T_n) = \{v_\alpha v_{\alpha+1}, 1 \leq \alpha \leq n-1\} \cup \{u_\alpha v_\alpha, 1 \leq \alpha \leq n\} \cup \{u_\alpha v_{\alpha+1}, 1 \leq \alpha \leq n\}$. The graph T_n has 2n − 1 verices and 3n − 3 edges as shown in Figure 3.1.

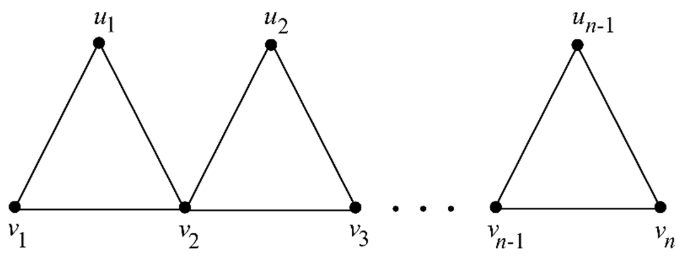

Figure 3.1: Triangular-Snake-Graph T_n

By [5], we get

$$rc(T_n) = n - 1$$

The maximum degree of T_n is 4. The lower bound is calculated by using [8] as

$$rc_A(T_n) \geq \max\{rc(T_n), \Delta(T_n)\}$$

$$rc_A(T_n) \geq \max\{n-1,\ 4\}$$

Since the maximum is $n - 1$, we get

$$rc_A(T_n) \geq n - 1 \qquad (3.1)$$

To find the upper bound, we define a function $j: V \to \{1,\ 2,\ \ldots, |V|\}$ as vertex function defined by

$$j(u_\alpha) = 2n - \alpha \qquad ,1 \leq \alpha \leq n-1$$

$$j(v_\alpha) = \alpha \qquad ,1 \leq \alpha \leq n$$

The following are the edge weights of T_n

$$w(v_\alpha v_{\alpha+1}) = 2\alpha + 1 \qquad ,1 \leq \alpha \leq n-1$$

$$w(u_\alpha v_\alpha) = 2n$$

$$w(u_\alpha v_{\alpha+1}) = 2n + 1$$

The number of different colors used are calculated by using the arithmetic formula

$$w(v_\alpha v_{\alpha+1}) :\ t_n = a + (N-1)d$$

$$2n - 1 = 3 + (N-1)(2)$$

$$N = n - 1$$

$$|w(v_\alpha v_{\alpha+1})| = n - 1$$

$$|w(u_\alpha v_\alpha)| = |w(u_\alpha v_{\alpha+1})| = 1$$

$$\sum |w| = n - 1 + 1 + 1 = n + 1$$

$$rc_A(T_n) \leq n + 1 \qquad (3.2)$$

From equations 3.1 and 3.2 we conclude that

$$n - 1 \leq rc_A(T_n) \leq n + 1$$

Hence the *RACN* of T_n is calculated

An illustration is given in Figure 3.2.

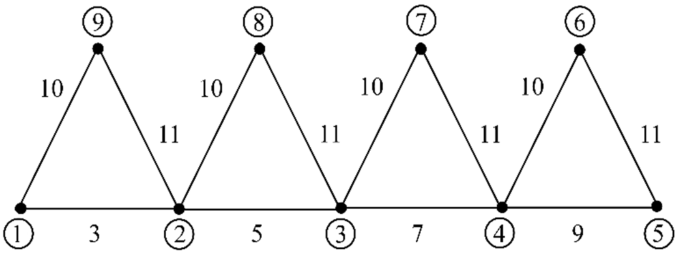

Figure 3.2: Triangular Snake Graph T_5

In the above illustration of triangular-snake-graph with $n = 5$, the vertices are labelled , subsequently edge weights are found out and given unique colors to distinct edge weights. The RACN of this graph is 6.

Theorem 3.2. *RACN of a Double-Triangular-Snake-graph $D(T_n)$ with $n > 6$ is $n - 1 \le RACN(D(T_n)) \le n + 3$.*
Proof. Let us denote the Double-Triangular-Snake-graph by $D(T_n)$. Let the vertices of the path P_n be denoted by $v_1, v_2, \dots, v_n$. Let the vertices lying above the path be denoted by $u_1, u_2, \dots, u_{n-1}$ and the vertices lying below the path be denoted by $w_1, w_2, \dots, w_{n-1}$. The graph $D(T_n)$ is a connected graph with $V(D(T_n)) = \{u_\alpha, 1 \le \alpha \le n-1\} \cup \{v_\alpha, 1 \le \alpha \le n\} \cup \{w_\alpha, 1 \le \alpha \le n-1\}$ and $E(D(T_n)) = \{u_\alpha v_\alpha, 1 \le \alpha \le n-1\} \cup \{u_\alpha v_{\alpha+1}, 1 \le \alpha \le n-1\} \cup \{v_\alpha v_{\alpha+1}, 1 \le \alpha \le n-1\} \cup \{v_\alpha w_\alpha, 1 \le \alpha \le n-1\} \cup \{v_{\alpha+1} w_\alpha, 1 \le \alpha \le n-1\}$. The graph $D(T_n)$ has $3n - 2$ verices and $5n - 5$ edges as shown in Figure 3.3.

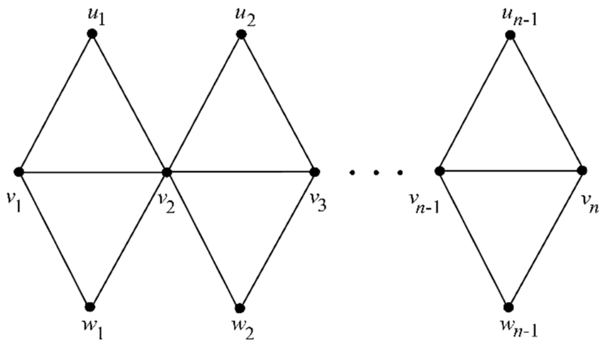

Figure 3.3: Double-Triangular-Snake-graph $D(T_n)$

By [5], we get

$$rc(D(T_n)) = n - 1$$

The maximum degree of $D(T_n)$ is 6. The lower bound is calculated by using [8] as

$$rc_A(D(T_n)) \ge \max\{rc(D(T_n)), \Delta(D(T_n))\}$$

$$rc_A(D(T_n)) \ge \max\{n - 1, 6\}$$

Since the maximum is $n - 1$, we get

$$rc_A(D(T_n)) \ge n - 1 \qquad (3.3)$$

To find the upper bound, we define a function $j: V \to \{1, 2, \dots, |V|\}$ as vertex function defined by

$$j(u_\alpha) = \alpha, \qquad 1 \le \alpha \le n - 1$$

$$j(v_\alpha) = 2n - \alpha, \qquad 1 \le \alpha \le n$$

$$j(w_\alpha) = 2n - 1 + \alpha, \quad 1 \le \alpha \le n - 1$$

The following are the edge weights of $D(T_n)$

$$w(v_\alpha v_{\alpha+1}) = 4n - 1 - 2\alpha, \quad 1 \le \alpha \le n - 1$$

$$w(u_\alpha v_\alpha) = 2n,$$

$w(u_\alpha v_{\alpha+1}) = 2n - 1,$
$w(v_\alpha w_\alpha) = 4n - 1,$
$w(v_{\alpha+1} w_\alpha) = 4n - 2,$

The number of different colors used are calculated by using the arithmetic formula

$$w(v_\alpha v_{\alpha+1}):\ t_n = a + (N-1)d$$

$$2n + 1 = 4n - 3 + (N-1)(-2)$$

$$N = n - 1$$

$$|w(v_\alpha v_{\alpha+1})| = n - 1$$

$$|w(u_\alpha v_\alpha)| = |w(u_\alpha v_{\alpha+1})| = |w(v_\alpha w_\alpha)| = |w(v_{\alpha+1} w_\alpha)| = 1$$

$$\sum |w| = n - 1 + 1 + 1 + 1 + 1 = n + 3$$

$$rc_A(D(T_n)) \leq n + 3 \qquad (3.4)$$

From equations 3.3 and 3.4 we conclude that

$$n - 1 \leq rc_A\big(D(T_n)\big) \leq n + 3$$

Hence the *RACN* of $D(T_n)$ is calculated
An illustration is given in Figure 3.4.

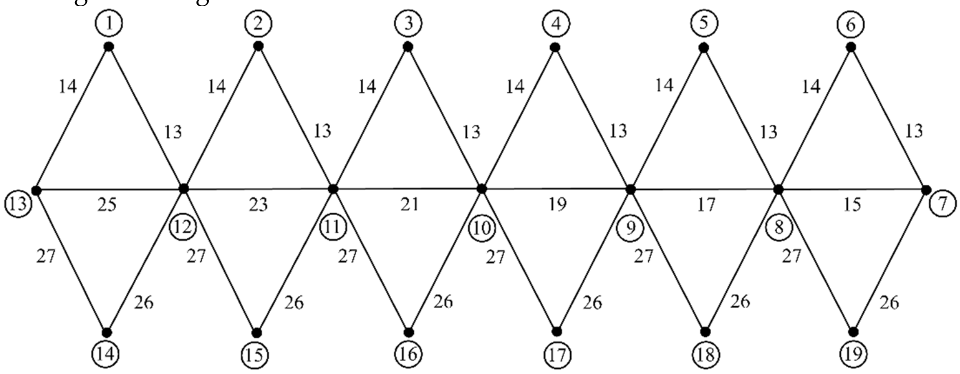

Figure 3.4: Double-Triangular-Snake-graph $D(T_7)$

In the above illustration of double triangular snake graph with $n = 7$, the vertices are labeled , subsequently edge weights are found out and given unique colors to distinct edge weights. The *RACN* of this graph is 10.

4. CONCLUSION

In this paper, we have determined the *RACN* of triangular snake graph T_n and double- triangular-snake-graph $D(T_n)$.

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