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Reliability properties of compound Rayleigh lifetime distribution-I *

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Abstract Sirisha and Jayasree (Sirisha, G. and Jayasree, G., "Compound Rayleigh lifetime distribution-I" EPH - International Journal of Mathematics and Statistics, 4(2), 43-52, (2018)) studied the basic assumptions, properties and derivation of the compound Rayleigh lifetime distribution-I. In this paper some special reliability properties of this compound Rayleigh lifetime distribution-I are derived. These properties can be used in statistical quality control and reliability analysis. The derived results are illustrated with a practical example.

Key words Reliability function, Hazard function, Rayleigh distribution, Compound Rayleigh lifetime distribution-I.

2020 Mathematics Subject Classification 60K20, 62N05.

1 Introduction

The compound Rayleigh lifetime distribution-I has many applications in our real scenario and it represents a useful model in reliability analysis or life testing studies. In many industrial experiments involving lifetimes of machines or products, experiments often terminate before all units fail due to variety of circumstances such as budget constraints, demand for rapid testing, etc., and in such and other similar types of situations the compound Rayleigh lifetime distribution finds rich applications. Continuing an earlier study of the first author [2] we augment the theoretical basis of the compound Rayleigh lifetime distribution in this paper and derive some interesting reliability properties like the reliability function, the Hazard function and the cumulative hazard function of this distribution and justify these properties for a practical application which is taken from the previous work [2] of the first author.

2 Properties of the distribution

Some important properties of the compound Rayleigh lifetime distribution-I are listed below which will be used in the section 3 for deriving the reliabilty properties of this distribution.

1. Let T be the lifetime of the product which has the compound Rayleigh lifetime distribution –I (CRLD-I), then the density function of T is

$$f^*(t) = \begin{cases} (m\delta)^{-1} \int_c^{c+m\delta} \pi t v^2 2^{-1} e^{-(t^2 v^2 \pi 4^{-1})} dv & ; \quad 0 < t < \infty, \\ 0 & ; \quad \text{elsewhere.} \end{cases} \tag{2.1}$$

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Note 2.1. The probability density function (p.d.f.) $f^*(t)$ can also be expressed in terms of incomplete gamma distribution function Γ as follows:

$$f^*(t) = \begin{cases} 2(m\delta)^{-1} t^{-2}\pi^{-1/2}\left[\Gamma(3/2,\omega_1) - \Gamma(3/2,\omega_2)\right]; 0 < t < \infty, \\ 0; \quad \text{elsewhere.} \end{cases} \tag{2.2}$$

where,

$$\omega_1 = t^2 c^2 \pi 4^{-1} \text{ and } \omega_2 = t^2 (c + m\delta)^2 \pi 4^{-1}. \tag{2.3}$$

2. The expected lifetime is given by

$$E^*(T) = (m\delta)^{-1} \log\left[1 + m\delta c^{-1}\right]. \tag{2.4}$$

For all $m, c > 0$ the expected lifetime of the CRLD-I is less than that of the Rayleigh distribution.

3. The variance of T is

$$V^*(T) = 4(\pi c(c + m\delta))^{-1} - (m\delta)^{-2}\left(\log\left(1 + m\delta c^{-1}\right)\right)^2 \tag{2.5}$$

4. The distribution function $F^*(t)$ is

$$F^*(t) = 1 - (m\delta t)^{-1}\pi^{-1/2}\int_{\omega_1}^{\omega_2} e^{-\omega}\omega^{1/2-1}d\omega \tag{2.6}$$

where ω_1, ω_2 are as in (2.3). It may also be noted that $F^*(t)$ can also be expressed as

$$F^*(t) = 1 - (m\delta t)^{-1}\pi^{-1/2}\left[\Gamma(1/2,\omega_1) - \Gamma(1/2,\omega_2)\right] \tag{2.7}$$

3 Reliability properties of CRLD-I

Lemma 3.1. *The reliability function $R^*(t)$ of CRLD-I is given by*

$$R^*(t) = (m\delta t)^{-1}\pi^{-1/2}\left[\Gamma(1/2,\omega_1) - \Gamma(1/2,\omega_2)\right]. \tag{3.1}$$

Proof. The proof follows immediately from (2.7) as $R^*(t) = 1 - F*(t)$. □

Lemma 3.2. *The Hazard function $h^*(t)$ of the distribution is given by*

$$h^*(t) = t^{-1} + 2t^{-1}\left[\omega_2^{1/2}e^{-\omega_2} - \omega_1^{1/2}e^{-\omega_1}\right]\left[\Gamma(1/2,\omega_1) - \Gamma(1/2,\omega_2)\right]^{-1}. \tag{3.2}$$

Proof. As the Hazard function $h^*(t)$ is given by $h^*(t) = F^*(t)/R^*(t)$, thus, (2.2) and (3.1) give

$$h^*(t) = \frac{2(m\delta)^{-1}t^{-2}\pi^{-1/2}\left[\Gamma(3/2,\omega_1) - \Gamma(3/2,\omega_2)\right]}{(m\delta t)^{-1}\pi^{-1/2}\left[\Gamma(1/2,\omega_1) - \Gamma(1/2,\omega_2)\right]}.$$

By observing that

$$\Gamma(3/2,\omega_1) = \int_{\omega_1}^{\infty} e^{-\omega}\omega^{3/2-1}d\omega = \frac{1}{2} - \omega_1^{1/2}e^{-\omega_1} + \frac{1}{2}\Gamma(1/2,\omega_1) \tag{3.3}$$

the last expression of $h^*(t)$ reduces to (3.2). □

Lemma 3.3. *The cumulative hazard function $H^*(t)$ is given by*

$$H^*(t) = \int_0^t \left(\frac{1}{x} + \frac{2}{x}\left(\omega_2^{1/2}e^{-\omega_2} - \omega_1^{1/2}e^{-\omega_1}\right)\left[\Gamma(1/2,\omega_1) - \Gamma(1/2,\omega_2)\right]\right)^{-1} dx. \tag{3.4}$$

Proof. The proof is elementary as $H^*(t) = \int_0^t h^*(x)\,dx$. Since, the expression for $H^*(t)$ involves the incomplete Gamma integrals (right tail), therefore, for given parameters, these values can be obtained from the Handbook of Abramowitz and Stegun [1]. □

4 Practical application

We illustrate below the discussion of section 3 with the help of the example considered earlier in the paper of the first author [2].

Example 4.1. Considering the example pertaining to manufacturing of piston rings for an automotive engine as given in Sirisha and Jayasree [2], we see that the measurable quality characteristic X, is the inside diameter of the piston ring, which is assumed to be normally distributed with mean μ and variance σ^2. $U = 75$ mm and $L = 73$ mm are the upper and lower specification limits respectively, $\mu_0 = 74.001$ mm and the estimated value of $\sigma = 0.00989$ mm. Here, $U - L > 6\sigma$. Hence using the concept of modified control charts, one has

$$\mu_U = 74.97033 \text{ mm}, \ \mu_L = 73.02967 \text{ mm}, \text{ and } \delta = 0.97033 \text{ mm}.$$

For different values of c and m the values of the reliability function and hazard function represented by R^*, h^* and R, h respectively for the CRLD-I and the conventional Rayleigh Distribution are tabulated in Table 1. and their respective graphs are presented in the Figs. 1 and 2 below:

Table 1: Compound Rayleigh Life Distribution-I and conventional Rayleigh Distribution for $c = 0.500000$ and $m = 1.000000$; $c = 1.500000$ and $m = 1.500000$; $c = 2.500000$ and $m = 2.000000$.

t	R^*	h^*	R	h	R^*	h^*	R	h	R^*	h^*	R	h
0.1	0.9669	0.1384	0.998	0.039	0.9495	0.4384	0.9825	0.353	0.8654	1.894	0.9521	0.981
0.2	0.94	0.203	0.9922	0.079	0.9355	1.339	0.9318	0.707	0.6538	3.689	0.8218	1.963
0.3	0.88491	0.489	0.9825	0.118	0.711	2.222	0.853	1.06	0.4268	5.391	0.643	2.944
0.4	0.84249	0.5906	0.9691	0.157	0.516	3.0329	0.7538	1.413	0.2325	6.752	0.4561	3.925
0.5	0.78475	0.7478	0.9521	0.196	0.3814	3.5411	0.643	1.766	0.11966	7.635	0.2933	4.906
0.6	0.71849	0.9516	0.9318	0.236	0.262	4.057	0.5295	2.12	0.0488	8.688	0.171	5.888
0.7	0.6802	1.053	0.9083	0.275	0.179	4.373	0.4209	2.473	0.0208	9.36	0.0904	6.869
0.8	0.5944	1.159	0.882	0.314	0.115	4.666	0.3229	2.826	0.0082	10.02	0.0433	7.85
0.9	0.536	1.2735	0.853	0.353	0.071	4.918	0.2392	3.179	0.0026	11.15	0.0188	8.831
1	0.476	1.3126	0.8218	0.393	0.039	5.361	0.171	3.533	0.0009	11.222	0.0074	9.813
2	0.1049	1.6101	0.4561	0.785	0.00006	8.331	0.0009	7.065	0		0	19.63
3	0.01984	1.7777	0.171	1.178	0		0	10.6	0		0	29.44
4	0.0033	1.935	0.0433	1.57	0		0	14.13	0		0	39.25
5	0.00036	2.3277	0.0074	1.963	0		0	17.66	0		0	49.06
6	0.00003	2.7666	0.0009	2.355	0		0	21.2	0		0	58.88
7	0.000001	5.6	0.0001	2.748	0		0	24.73	0		0	68.69
8	0		0	3.14	0		0	28.26	0		0	78.5
9	0		0	3.533	0		0	31.79	0		0	88.31
10	0		0	3.925	0		0	35.33	0		0	98.13
15	0		0	5.888	0		0	52.99	0		0	147.2
20	0		0	7.85	0		0	70.65	0		0	196.3
25	0		0	9.813	0		0	88.31	0		0	245.3
30	0		0	11.78	0		0	106	0		0	294.4
35	0		0	13.74	0		0	123.6	0		0	343.4
40	0		0	15.7	0		0	141.3	0		0	392.5
60	0		0	23.55	0		0	212	0		0	588.8
90	0		0	35.33	0		0	317.9	0		0	883.1
200	0		0	78.5	0		0	706.5	0		0	1963

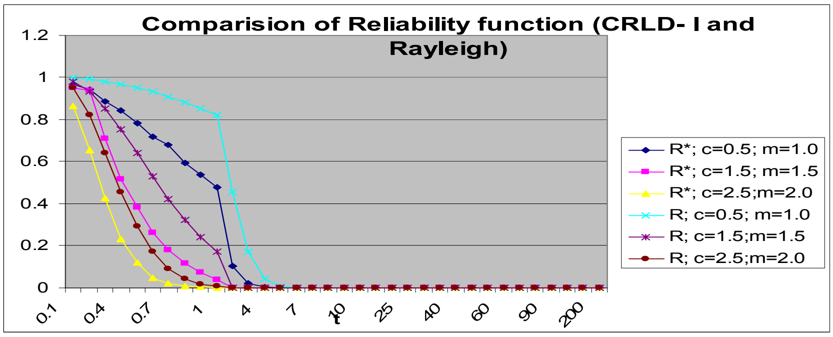

Fig. 1: Graph 1.

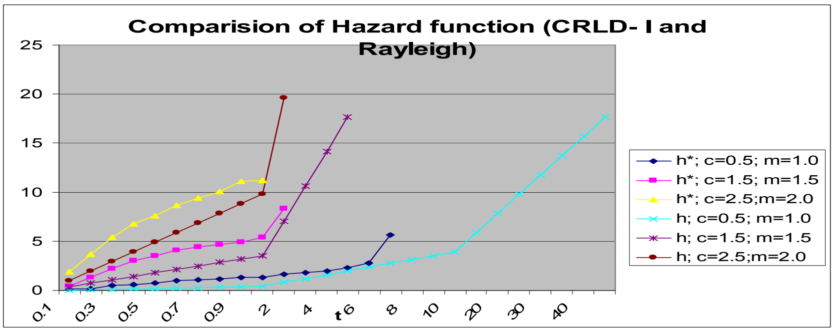

Fig. 2: Graph 2.

5 Conclusion

From the Figs. 1 and 2 and the Table 1 one can observe that CRLD-I has an increasing failure rate (IFR) distribution.

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