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Antimagic labeling of Triangular book graph and double fan graph *

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Abstract In 1990, Hartsfield and Ringel (N. Hartsfield and G. Ringel, Pearls in Graph Theory, Academic Press, San Diego, 1990) introduced the concept of antimagic labeling. Antimagic labeling of a graph G is a one-one correspondence between G and $\{1, 2, \ldots, |E|\}$ such that the sums of labels assigned to the edges incident to distinct vertices are different. In this paper we prove that triangular book graph and double fan graph admit antimagic labeling.

Key words Antimagic labeling, Triangular book graph, Double fan graph.

2020 Mathematics Subject Classification 05C78.

1 Introduction

Graph labelings trace their origins to labelings presented by Alexander Rosa [6] in 1967 who identified three types of labelings, which he called α, β and ρ-labelings. In 1990, Hartsfield and Ringel cite5 introduced the concept of antimagic labeling. Anantimagic labeling of a graph $G = (V, E)$ is a bijection from the set of edges of G to $\{1, 2, \ldots, |E(G)|\}$ and such that any two vertices of G have distinct vertex sums where the vertex sum of a vertex v in $V(G)$ is nothing but the sum of all the incident edge labelings of G. A graph is called antimagic if it admits an antimagic labeling. Hartsfield and Ringel [5] showed that paths, cycles, complete graphs K_n $(n = 3)$ are antimagic. They conjectured that all connected graphs besides K_2 are antimagic, which remains unsettled. Alon et. al [1] proved that large dense graphs are antimagic. Cranston et al. [3] proved the antimagic labeling of all odd regular graphs and Chang et al. [2] in 2016 verified that even regular graphs are antimagic. For more results on antimagic labeling, one can refer to the dynamic survey on graph labeling by Gallian [4].
In this paper, we prove that triangular book graph and double fan graph admit antimagic labeling.

2 Preliminary definitions

In this section we give below two definitions which are used by us for proving our results in this paper.

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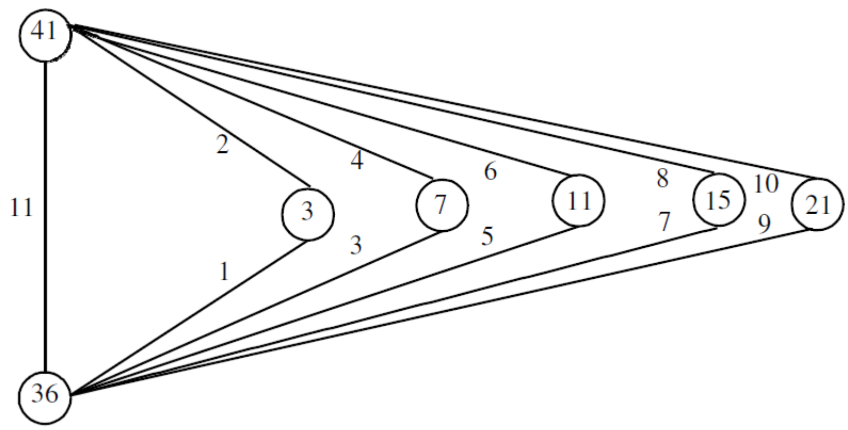

Fig. 1: The antimagic labeling of $B(3,5)$.

Definition 2.1. The*Triangular book* with n pages is defined as copies of cycle C_3 sharing a common edge. The common edge is called the spine or base of the book. This graph is denoted by $B(3,n)$. In other words it is the complete tripartite graph $K_{1,1,n}$.

Definition 2.2. The *Double fan* $F_{2,n}$ consists of two fan graphs that have a common path. It consists of two cycles of size n where the vertices of two cycles are all connected to a central vertex.

3 Main results

In this section we prove that the Triangular book graph $B(3,n)$ and the Double fan graph $F_{2,n}$ admit antimagic labeling.

Theorem 3.1. *The Triangular book graph $B(3,n)$ admits antimagic labeling.*

Proof. Let $B(3,n)$ denote the triangular book. Denote the spine vertices of $B(3,n)$ by u and v. The remaining vertices are denoted as $w_1, w_2, \ldots, w_n$. The edge sets are defined as $E(B(3,n))$=$\{uw_\lambda, 1 \leq \lambda \leq n\} \cup \{vw_\lambda, 1 \leq \lambda \leq n\} \cup \{uv\}$. Thus the vertex set of $B(3,n)$ is $V(B(3,n)) = \{w_\lambda, 1 \leq \lambda \leq n\} \cup \{u\} \cup \{v\}$. Note that $B(3,n)$ is a graph on $n+2$ vertices and $2n+1$ edges.
The edge labelings are defined by $\zeta :\ E(G) \rightarrow \{1, 2, \ldots, n+1\}$

$$\zeta\,(uv) = 2n+1,$$

$$\zeta\,(uw_\lambda) = \lambda,\ \lambda = 1, 2, \ldots, n,$$
$$\zeta\,(vw_\lambda) = 2\lambda+1,\ \lambda = 1, 2, \ldots, n.$$

We observe that the edge labelings defined above generate distinct edge labels for all the $2n+1$ edges satisfying the condition for antimagic labeling.
The induced vertex labeling are $\xi : V\,(G) \rightarrow \mathbb{Z}^+$:

$$\xi\,(u) = n^2 + 3n + 1,$$

$$\xi\,(v) = n^2 + 2n + 1,$$

$$\xi\,(w_\lambda)\ = 4\lambda - 1,\ \lambda = 1, 2, \ldots, n.$$

It is observed that the vertex labels are distinct. Thus the Triangular book graph $B(3,n)$ admits antimagic labeling. An illustration of antimagic labeling of triangular book graph $B(3,5)$ is shown in Fig. 1.

□

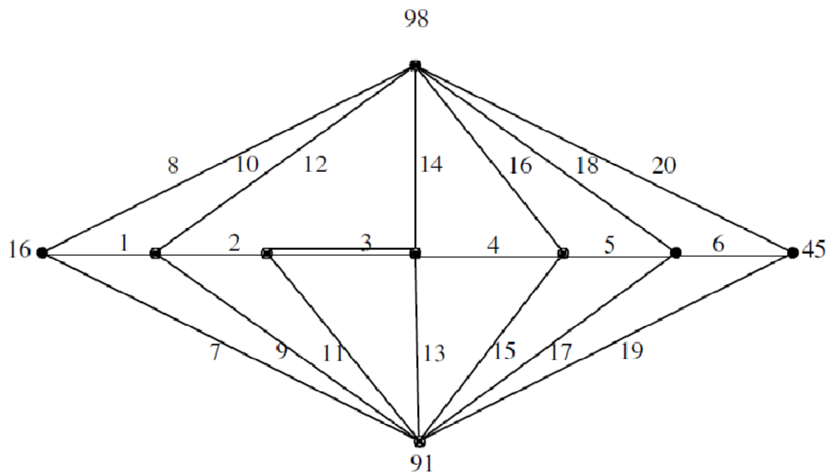

Fig. 2: The antimagic labeling of $F_{(2,7)}$.

Theorem 3.2. *The Double fan graph $F_{2,n}$ admits antimagic labeling.*

Proof. Let $F_{2,n}$ be a double fan graph. Denote the vertices on the common path as $w_1, w_2, \ldots, w_n$. The remaining vertices of the two cycles are defined as u and v. The edge set of $F_{2,n}$ is given by $E(F_{2,n}) = \{uw_\lambda,\ 1 \le \lambda \le\ n\} \cup \{vw_\lambda,\ 1 \le \lambda \le n\} \cup \{w_\lambda w_{\lambda+1},\ 1 \le \lambda \le n\}$. The vertex set of $F_{2,n}$ is given by $V\ (F_{2,n}) = \{w_\lambda,\ 1 \le \lambda \le n\} \cup \{u\} \cup \{v\}$. Note that $F_{2,n}$ has $n+2$ vertices and $3n-1$ edges. The edgelabeling is defined as $\zeta : E(G) \to \{1, 2, \ldots, 3n-1\}$.

Case (i): When n is odd,

$$\zeta\left(uw_\lambda\right) = n + 2\lambda - 1,\ 1 \le \lambda \le n,$$
$$\zeta\left(vw_\lambda\right) = n + 2\lambda - 2\ ,\ 1 \le \lambda \le n,$$
$$\zeta\left(w_\lambda w_{\lambda+1}\right) = \lambda,\ 1 \le \lambda \le n-1.$$

Case (ii): When n is even,

$$\zeta\left(uw_\lambda\right) = n + 2\lambda - 2,\ 1 \le \lambda \le n,$$
$$\zeta\left(vw_\lambda\right) = n + 2\lambda - 1,\ 1 \le \lambda \le n,$$
$$\zeta\left(w_\lambda w_{\lambda+1}\right) = \lambda,\ 1 \le \lambda \le n-1.$$

We observe that the edge labeling defined above generates distinct edge labels for all the $3n\ -1$edges satisfying the condition for antimagic labeling.
The induced vertex labelings are $\xi : V\left(F_{2,n}\right) \to \mathbb{Z}^+$:

Case (i): When n is odd,

$$\xi\left(u\right) = 2n^2 - n,$$
$$\xi\left(v\right) = 2n^2,$$
$$\xi\left(w_\lambda\right) = 6\lambda + 2n - 4,\ 1 \le \lambda \le n-1,$$
$$\xi\left(w_n\right) = 7n - 4.$$

Case (ii): When n is even,

$$\xi\left(u\right) = 2n^2,$$
$$\xi\left(v\right) = 2n^2 - n,$$
$$\xi\left(w_\lambda\right) = 6\lambda + 2n - 4,\ 1 \le \lambda \le n-1,$$
$$\xi\left(w_n\right) = 7n - 4.$$

The entire $n+2$ vertices labeled are distinct. Thus the Double fan graph $F_{2,n}$admits antimagic labeling. An illustration is shown in Fig. 2.

□

4 Conclusion

In this paper we showed the antimagic labeling for the Triangular book graph and the Double fan graph.

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