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FUZZY CRITICAL PATH ON TYPE-2 TRIANGULAR FUZZY NUMBERS

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Abstract

Network diagram plays a vital role in determining the project completion time. Network analysis is a technique which determines the various sequences of activities concerning a project and the project completion time. The popular method of this technique, that is widely used, is the critical path method. In this paper, a fuzzy critical path in an acyclic project network is found using type-2 triangular fuzzy numbers and its complement. An example is discussed to demonstrate our proposed approach.

Keywords: fuzzy critical path, type-2 triangular fuzzy numbers, complement of type-2 triangular fuzzy numbers, acyclic project network.

2010 Mathematics Subject Classification: 05C72.

1. INTRODUCTION

The critical path method (CPM), worked out at the beginning of 1960's has become one of the most widely applied tools in the planning and control of the realization of complex projects. The main purpose of the critical path method is to evaluate the project performance and to identify the critical activities on the critical path so that the available resources could be utilized on these activities in the project network in order to reduce the project completion time. With the help of the critical path, the decision maker can adopt a better strategy of optimizing time and the available resources to ensure the earlier completion and the quality of the project.

This paper analyzes the critical paths in a general project network with fuzzy activity times. We propose a ranking method for fuzzy numbers in a critical path method for a fuzzy project network, where the duration time of each activity in the fuzzy project network is represented by a type-2 triangular fuzzy number. Dubois et al. [5] extended the fuzzy arithmetic operations to compute the latest starting time of each activity in a project network.

The organization of the paper is as follows. In section 2 some basic concepts are discussed. The section 3 gives some properties of total slack fuzzy time. Section 4 gives the network terminology. An algorithm for finding the critical path combined with type-2 triangular fuzzy number using ranking function is given in Section 5.The proposed algorithm is illustrated through a numerical example in section 6 and the conclusions are given in section 7.

2. PRELIMINARIES

2.1 Definition

A fuzzy number $\widetilde{A} = ((a_1, a_2, a_3), (b_1, b_2, b_3), (c_1, c_2, c_3))$ is said to be a type-2 triangular number if its membership function is defined by $\widetilde{A} = ((\mu_a(x), \mu_b(x), \mu_c(x))$ where

$$\mu_a(x) = \begin{cases} \dfrac{(x-a_1)}{(a_2-a_1)}, & a_1 \le x \le a_2 \\ 1, & a_1 = a_2 \\ \dfrac{(x-a_3)}{(a_2-a_3)}, & a_2 \le x \le a_3 \end{cases}, \qquad \mu_b(x) = \begin{cases} \dfrac{(x-b_1)}{(b_2-b_1)}, & b_1 \le x \le b_2 \\ 1, & b_1 = b_2 \\ \dfrac{(x-b_3)}{(b_2-b_3)}, & b_2 \le x \le b_3 \end{cases}$$

$$\mu_c(x) = \begin{cases} \dfrac{(x-c_1)}{(c_2-c_1)}, & c_1 \le x \le c_2 \\ 1, & c_1 = c_2 \\ \dfrac{(x-c_3)}{(c_2-c_3)}, & c_2 \le x \le c_3 \end{cases}$$

2.2 Addition on type-2 fuzzy numbers

If $\tilde{A}$ and $\tilde{B}$ are two type-2 triangular fuzzy numbers then their addition is defined as follows:

$$\tilde{A} = \left(A^L, A^M, A^N\right)$$
$$= \left(\left(a_1^L, a_2^L, a_3^L\right), \left(a_1^M, a_2^M, a_3^M\right), \left(a_1^N, a_2^N, a_3^N\right)\right)$$

and

$$\tilde{B} = \left(B^L, B^M, B^N\right)$$
$$= \left(\left(b_1^L, b_2^L, b_3^L\right), \left(b_1^M, b_2^M, b_3^M\right), \left(b_1^N, b_2^N, b_3^N\right)\right)$$

be two normal type-2 fuzzy numbers. Then we have

$$\tilde{A} + \tilde{B} = \left(A^L + B^L, A^M + B^M, A^N + B^N\right)$$
$$= \left(\left(a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L\right), \left(a_1^M + b_1^M, a_2^M + b_2^M, a_3^M + b_3^M\right), \left(a_1^N + b_1^N, a_2^N + b_2^N, a_3^N + b_3^N\right)\right)$$

2.3 Definition

Let $\widetilde{A} = ((a_1, a_2, a_3), (b_1, b_2, b_3), (c_1, c_2, c_3))$ be a triangular type-2 fuzzy number. Then its ranking function R is defined by

$$R(\tilde{A}) = \frac{1}{16}(a_1 + 2a_2 + a_3 + 2b_1 + 4b_2 + 2b_3 + c_1 + 2c_2 + c_3).$$

2.4 Notations

t_{ij}= The activity between node i and j.

ESF_j = The earliest starting fuzzy time of node *j*.

LFF_i = The latest finishing fuzzy time of node *i*.

SFT_{ij} = The total slack fuzzy time of t_{ij}.

p_n= the n^{th} fuzzy path.

P = the set of all fuzzy paths in a project network.

$F(p_n)$= The total slack fuzzy time of path p_n in a project network.

3. PROPERTIES OF TOTAL SLACK FUZZY TIME

Property 3.1: (Forward pass calculation)

To calculate the earliest starting fuzzy time in the project network, set the initial node to zero for starting i.e.,

$ESF_1 = (0.0,0.0,0.0,0.0)$

$ESF_j = \max_i \{ESF_j + TSF_{ij}\},\ j \neq i,\ j \in N$, *i*=number of preceding nodes.(ESF_j =the earliest starting fuzzy time of node *j*). The ranking value is utilized to identify the maximum value.

Earliest finishing fuzzy time = Earliest starting fuzzy time + Fuzzy activity time.

Property 3.2: (Backward pass calculation)

To calculate the latest finishing time in the project network set $LFF_n = ESF_n$.

$LFF_j = \min_j \{LFF_j (-) SET_{ij}\}, i \neq n, i \in N$, j = number of succeeding nodes. Ranking value is utilized to identify the minimum value.

Latest starting fuzzy time= Latest finishing Fuzzy time (-) Fuzzy activity time.

Property 3.3:

For the activity t_{ij}, $i<j$

Total fuzzy slack:

$SFT_{ij} = LFF_j (-)(ESF_i (+) SFT_{ij})$ or, $(LFF_j (-) SFT_{ij})(-) ESF_i$, $1 \leq i \leq j \leq n;\ i, j \in N,$

Property 3.4:

$F(p_n) = \sum_{\substack{1 \leq i \leq j \leq n \\ i, j \in p_k}} SFT_{ij}$, $p_k \in P$, p_n denotes the number of possible paths in a network from source node to the destination node, $k=1$ to m.

4. NETWORK TERMINOLOGY

A directed acyclic project network consisting of six nodes and seven edges is considered. Each edge in this network is assigned by type-2 discrete fuzzy numbers. The set of all possible paths is denoted by *P*. The fuzzy critical path is identified from the set *P*.

5. ALGORITHM (FOR FINDING CRITICAL PATH)

Step 1: Estimate the fuzzy activity time with respect to each activity.

Step 2: Let $ESF_1 = (0.0,0.0,0.0,0.0)$ and calculate ESF_j, $j = 2,3,\ldots,n$ using the property 1.

Step 3: Let $LFF_n = ESF_n$ and calculate LFF_i, $i = n-1, n-2, \ldots, 2$, using the property 2.

Step 4: Calculate SFT_{ij} with respect to each activity in a project network using the property 3.

Step 5 : Calculate all the possible paths using property 4.

Step 6: Identify the critical path using the Ranking function.

6. NUMERICAL EXAMPLE

The problem is to find the fuzzy critical path in an acyclic project network whose edges are assigned with type-2 triangular fuzzy numbers.

Solution: The edge lengths are

$\tilde{E}_1 = ((2.5,2.8,3.0),(2.2,2.5,2.8),(2.1,2.3,2.5))$

$\tilde{E}_2 = ((2.6,2.7,2.8),(2.5,2.6,2.9),(2.3,2.5,2.7))$

$\tilde{E}_3 = ((2.8,2.9,3.0),(2.6,2.9,3.2),(2.2,2.7,3.1))$

$\tilde{E}_4 = ((2.3,2.5,2.9),(2.3,2.4,2.5),(2.2,2.6,2.8))$

$\tilde{E}_5 = ((2.8,2.9,3.3),(2.6,2.8,3.2),(2.5,2.7,3.0))$

$\tilde{E}_6 = ((2.5,2.7,3.1),(2.3,2.6,3.2),(2.1,2.5,2.9))$

$\tilde{E}_7 = ((2.9,3.1,3.3),(2.8,3.2,3.4),(2.4,2.7,3.0))$

The complements of the edge lengths are

$\overline{\tilde{E}_1} = ((-1.5,-1.8,-2.0),(-1.2,-1.5,-1.8),(-1.1,-1.3,-1.5))$

$\overline{\tilde{E}_2} = ((-1.6,-1.7,-1.8),(-1.5,-1.6,-1.9),(-1.3,-1.5,-1.7))$

$\overline{\tilde{E}_3} = ((-1.8,-1.9,-2.0),(-1.6,-1.9,-2.2),(-1.2,-1.7,-2.1))$

$\overline{\tilde{E}_4} = ((-1.3,-1.5,-1.9),(-1.3,-1.4,-1.5),(-1.2,-1.6,-1.8))$

$\overline{\tilde{E}_5} = ((-1.8,-1.9,-2.3),(-1.6,-1.8,-2.2),(-1.5,-1.7,-2.0))$

$\overline{\tilde{E}_6} = ((-1.5,-1.7,-2.1),(-1.3,-1.6,-2.2),(-1.1,-1.5,-1.9))$

$\overline{\tilde{E}_7} = ((-1.9,-2.1,-2.3),(-1.8,-2.2,-2.4),(-1.4,-1.7,-2.0))$

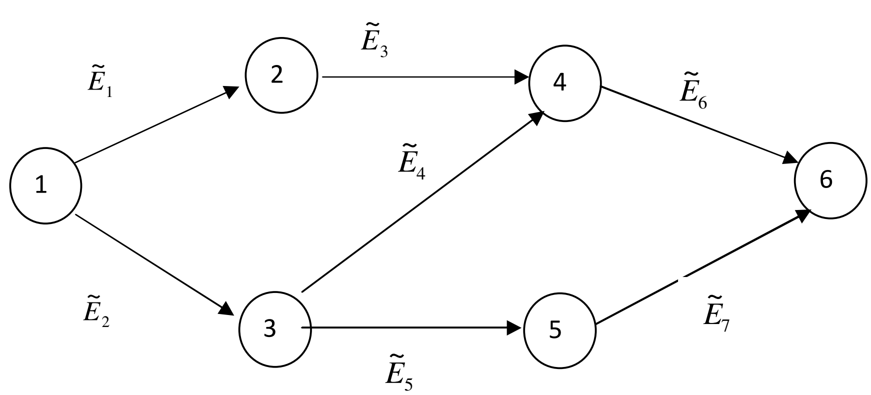

Fig. 6.1: Fuzzy acyclic project network

Table 6.1:

Activities, fuzzy durations and total slack time for each activity for type-2 triangular fuzzy number and its

Activity(*i*-*j*)*i*<*j*	Fuzzy activity time	$F\tilde{T}S_{ij}$
1-2	$((2.5,2.8,3.0),(2.2,2.5,2.8),(2.1,2.3,2.5))$	((-0.8,0.3,1.6),(-1.3,0.6,2.4),(-1.3,0.4,2.3))
1-3	$((2.6,2.7,2.8),(2.5,2.6,2.9),(2.3,2.5,2.7))$	((-1.1,0,1.1),(-1.6,0,1.6),(-1.5,0,1.5))
2-4	$((2.8,2.9,3.0),(2.6,2.9,3.2),(2.2,2.7,3.1))$	((-0.8,0.3,1.6),(-1.3,0.6, 2.4),(-1.3,0.4,2.3))
3-4	$((2.3,2.5,2.9),(2.3,2.4,2.5),(2.2,2.6,2.8))$	((-0.5,0.8,2.0),(-0.7,1.0,2.4),(-1.2,0.3,2.1))
3-5	$((2.8,2.9,3.3),(2.6,2.8,3.2),(2.5,2.7,3.0))$	((-1.1,0,1.1),(-1.6,0,1.6),(-1.5,0,1.5))
4-6	$((2.5,2.7,3.1),(2.3,2.6,3.2),(2.1,2.5,2.9))$	((-1.1,0,1.1),(-1.6,0,1.6),(-1.5,0,1.5))
5-6	$((2.9,3.1,3.3),(2.8,3.2,3.4),(2.4,2.7,3.0))$	((-1.1,0,1.1),(-1.6,0,1.6),(-1.5,0,1.5))

complement are given in the Tables 6.1 and 6.2.

Table 6.2:

Activity(*i*-*j*)*i*<*j*	Fuzzy activity time	$F\tilde{T}S_{ij}$
1-2	$((-1.5,-1.8,-2.0),(-1.2,-1.5,-1.8),(-1.1,-1.3,-1.5))$	((1.7,0.5,-1.0),(2.1,0.4,-1.5),(1.9,-0.1,-1.1))
1-3	$((-1.6,-1.7,-1.8),(-1.5,-1.6,-1.9),(-1.3,-1.5,-1.7))$	((1.4,0,-1.4),(1.5,0,-1.5),(1.8,0,-1.8))
2-4	$((-1.8,-1.9,-2.0),(-1.6,-1.9,-2.2),(-1.2,-1.7,-2.1))$	((1.7,0.5,-1.0),(2.1,0.4,-1.5),(1.9,-0.1,-1.1))
3-4	$((-1.3,-1.5,-1.9),(-1.3,-1.4,-1.5),(-1.2,-1.6,-1.8))$	((1.4,0,-1.4),(1.5,0,-1.5),(1.8,0,-1.8))
3-5	$((-1.8,-1.9,-2.3),(-1.6,-1.8,-2.2),(-1.5,-1.7,-2.0))$	((2.0,0.8,-0.5),(2.4,1.0,-0.7),(2.1,0.3,-1.2))
4-6	$((-1.5,-1.7,-2.1),(-1.3,-1.6,-2.2),(-1.1,-1.5,-1.9))$	((1.4,0,-1.4),(1.5,0,-1.5),(1.8,0,-1.8))
5-6	$((-1.9,-2.1,-2.3),(-1.8,-2.2,-2.4),(-1.4,-1.7,-2.0))$	((2.0,0.8,-0.5),(2.4,1.0,-0.7),(2.1,0.3,-1.2))

All the possible paths P = {(1-2-4-6),(1-3-5-6),(1-3-4-6)} are found in a given acyclic project network using properties of network. The path 1-3-5-6 is identified as the fuzzy critical path by using ranking function.

7. CONCLUSION

In this paper, an attempt is made to find the fuzzy critical path in an acyclic project network using type-2 triangular fuzzy numbers and its complement with the help of ranking function. The fuzzy critical path is the same for both type-2 triangular fuzzy number and its complement.

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