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KAMAL DECOMPOSITION METHOD FOR SOLVING NONLINEAR DELAY DIFFERENTIAL EQUATIONS

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Abstract
In this paper, the Kamal Decomposition Method is proposed to solve nonlinear Delay Differential Equations. The proposed method is a combination of Kamal Transform and Adomian Decomposition Method. In this study, the solution is obtained as a series by first applying the Kamal Transform to the delay differential equations and then decomposing the nonlinear term by finding Adomian polynomial. Numerical examples are given to illustrate the effectiveness of our proposed method in solving nonlinear delay differential equations.

Keywords: Kamal decomposition, Adomian polynomial, Nonlinear delay differential equations.

2010 Mathematics Subject Classification: 34K17, 65L

1. INTRODUCTION

Delay Differential Equations (DDEs) are a type of differential equations in which the derivative of the unknown function at a certain time is given in terms of the values of the function at previous times. DDEs appear in chemical kinetics [1], population dynamics [2] and traffic models [3] and in several fields.

The general first order DDEs of the form

$$y'(t) = f(t, y, y(t-\tau)), \quad t > t_0 \tag{1}$$

$$y(t) = \emptyset(t), \quad t \le t_0$$

Here, $\emptyset(t)$ is the initial function, $\tau(t, y(t))$ is called the delay term. If the delay is a constant, then it is called constant delay. If it is function of time t, then it is called time dependent delay. If it is a function of time t and $y(t)$, then it is called the state dependent delay.

The general theory of DDEs have been widely developed by Bellman and Cooke [4], Hale [5], Driver [6]. Many numerical methods have been proposed by the researchers for the solution of DDEs. Some notable methods are Adomian Decomposition Method [7,12], Variational Iteration Method [8], Runge-Kutta Method [9], Modified Power Series Method [10].

In this paper, Kamal Decomposition method is proposed to solve the nonlinear DDEs. This method is the combination of Kamal transform and Adomian decomposition method which is capable to solve nonlinear differential equations. This paper has been organized as follows: In Section 2, the Kamal transform and its fundamental properties have been discussed. In Section 3, Kamal Decomposition Method has been proposed for nonlinear DDEs. In Section 4, nonlinear numerical examples have been provided to demonstrate the efficiency of the proposed method.

2. KAMAL TRANSFORM AND ITS FUNDAMENTAL PROPERTIES

Kamal transform was recently introduced by Abdelilah Kamal and H. Sedeeg [11] in 2016. This transform is derived from the classical Fourier integral. It is defined for functions of exponential order in the set A defined by:

$$A = \{f(t) \colon \exists M, k_1, k_2 > 0, |f(t)| < Me^{\frac{|t|}{k_j}}, if\ t \in (-1)^j \times [0, \infty)\}$$

For a given function in the set A, M is a constant and finite number, k_1, k_2 either finite or infinite. The Kamal transform is denoted by the operator $K(\cdot)$ defined by the integral equation:

$$K[f(t)] = G(v) = \int_0^\infty f(t) e^{\frac{-t}{v}} dt, \quad t \geq 0, \quad k_1 \leq v \leq k_2 \tag{2}$$

In this transform the variable v is used to factor the variable t in the argument of the function f. This transform has connection with the Fourier, Laplace and Elzaki transforms.

The Kamal transforms of simple functions are given below:

i. $K[1] = v$
ii. $K[t] = v^2$
iii. $K[t^n] = n!\, v^{n+1}$ where, n is a positive integer.
iv. $K[e^{at}] = \frac{v}{1-av}$
v. $K[\sin at] = \frac{av^2}{1+a^2v^2}$
vi. $K[\cos at] = \frac{v}{1+a^2v^2}$

Kamal Transform for derivatives are:

i. $K[f'(t)] = \frac{1}{v} G(v) - f(0)$
ii. $K[f''(t)] = \frac{1}{v^2} G(v) - \frac{1}{v} f(0) - f'(0)$
iii. $K[f^{(n)}(t)] = v^{(-n)} G(v) - \sum_{k=0}^{n-1} v^{k-n+1} f^{(k)}(0)$

3. KAMAL DECOMPOSITION METHOD FOR DDEs

The Adomian Decomposition Method (ADM) has been developed by Adomian [12]. Kamal Decomposition Method (KDM), which is the combination of Kamal Transform and ADM, is proposed here to solve the nonlinear DDEs. The solution is obtained as a series. First, we apply the Kamal transform to DDEs and then decomposing the nonlinear term by finding Adomian polynomials.

Consider the nonlinear delay differential equation in the form:

$$Ly + N(y, y_\tau) + R(y, y_\tau) = g, \text{ for } t > 0, \quad y = \emptyset, \quad \text{for } t \in [-\tau, 0]$$

where $y_\tau(t) = y(t - \tau)$, L is easily invertible, N is the nonlinear part and R is the remaining part and g is the source term.

The nonlinear term shall be decomposed into an infinite series of Adomian polynomials as follows:

$$N(y, y_\tau) = \sum_{n=0}^{\infty} A_n$$

where $A_n = A_n(y_0, y_1, \ldots, y_n)$ are called Adomian polynomials and A_n is classically suggested to the computed form

$$A_n(y_0, y_1, \ldots, y_n) = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty} \lambda^i y_i, \sum_{i=0}^{\infty} \lambda^i y_{\tau i}\right)\right]_{\lambda>0}$$

The Kamal Decomposition algorithm is implemented for the solution of the following first order nonlinear initial value problem

$$y' = f(t, y, y(t-\tau)), \quad y(0) = \alpha \tag{3}$$

Applying the Kamal transform on both sides of (3) and using initial condition we get,

$$K[y'] = K[f(t, y, y(t-\tau))]$$

$$\frac{1}{v}K[y(t)] - y(0) = K[f(t, y, y(t-\tau))]$$

$$K[y(t)] = v\alpha + vK[f(t, y, y(t-\tau))]$$

The Kamal decomposition technique now represents the solution as an infinite series,

$$K[\textstyle\sum_{n=0}^{\infty} y_n] = v\alpha + vK[\sum_{n=0}^{\infty} A_n] \tag{4}$$

where A_n is Adomian polynomials for nonlinear terms.

From . (4), we get the following recursive algorithm

$$K[y_0] = v\alpha$$

$$K[y_{n+1}] = vK[A_n], \; n > 0.$$

By taking inverse Kamal transform, we get $y_0, y_1, y_2, \ldots$. The analytical solution of nonlinear DDEs by KDM is given as an infinite series

$$y(t) = \textstyle\sum_{n=0}^{\infty} y_n(t).$$

By finding sufficient number of $y_n{}'s$ we get the numerical solution with good accuracy.

4. EXAMPLES

EXAMPLE 4.1:

Consider the first order nonlinear DDE

$$y'(t) = 1 - 2y^2\left(\frac{t}{2}\right), \quad y(0) = 0$$

The exact solution is $y(t) = \sin(t)$.

Applying Kamal Transform to the above first order nonlinear DDE, we obtain

$$K[y(t)] = v^2 - 2vK\left[y^2\left(\frac{t}{2}\right)\right]$$

Then using Adomian Decomposition Method, we obtain

$$K[\textstyle\sum_{n=0}^{\infty} y_n(t)] = v^2 - 2vK[\sum_{n=0}^{\infty} A_n] \tag{5}$$

From Eqn. (5), we get,

$$y_0(t) = t$$

$$y_1(t) = -\frac{t^3}{3!},$$

$$y_2(t) = \frac{t^5}{5!},$$

$$y_3(t) = -\frac{t^7}{7!},$$

$$\ldots\ldots$$

The infinite series solution becomes,

$$y = y_0 + y_1 + y_2 + y_3 + \cdots$$

$$y = t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \cdots$$

which converges to the exact solution $y(t) = \sin(t)$ as $n \to \infty$.

EXAMPLE 4.2:

Consider the third order nonlinear DDE

$$y'''(t) = -1 + 2y^2\left(\frac{t}{2}\right), \quad y(0) = 0, y'(0) = 1, y''(0) = 0$$

The exact solution is $y(t) = \sin(t)$.

Applying Kamal Transform to the above third order nonlinear DDE, we obtain

$$K[y(t)] = v^2 - v^4 + 2v^3 K\left[y^2\left(\frac{t}{2}\right)\right]$$

Then using Adomian Decomposition Method, we obtain

$$K[\textstyle\sum_{n=0}^{\infty} y_n(t)] = v^2 - v^4 + 2v^3 K[\sum_{n=0}^{\infty} A_n] \quad (6)$$

From Eqn. (6), we get

$$y_0(t) = t - \frac{t^3}{3!}$$

$$y_1(t) = \frac{t^5}{5!} - \frac{t^7}{7!} + \frac{5}{8}\frac{t^9}{9!},$$

$$y_2(t) = \frac{3}{8}\frac{t^9}{9!} - \frac{63}{2^6}\frac{t^{11}}{11!} + \frac{505}{2^{10}}\frac{t^{13}}{13!} - \frac{275}{2^{11}}\frac{t^{15}}{15!},$$

$$\dots\dots$$

The infinite series solution becomes,

$$y = y_0 + y_1 + y_2 + y_3 + \cdots$$

$$y = t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \frac{t^9}{9!} - \cdots$$

which converges to the exact solution $y(t) = \sin(t)$ as $n \to \infty$.

5. CONCLUSION

In this paper we proposed Kamal Decomposition Method to solve nonlinear delay differential equations. It is the combination of Kamal transform and Adomian decomposition method to produce exact/approximate solutions of the nonlinear DDEs. Two examples have been considered to demonstrate the applicability of the proposed method. This method is very efficient to solve nonlinear DDEs and gives results with good accuracy.

REFERENCES

[1]. Epstein, I. and Luo, Y. (1991). Differential delay equations in chemical kinetics: Non-linear models; the cross- shaped phase diagram and the Oregonator, *Journal of Chemical Physics,* 95, 244-254.

[2]. Kuang, Y. (1993). Delay Differential Equations with Applications in Population Biology, Academic Press, New York.

[3]. Davis, C.L. (2002). Modification of the optimal velocity traffic model to include delay due to driver reaction time, *Physica,* 319, 557-567.

[4]. Bellman, R. and Cooke, K.L. (1963). Differential Difference Equations, Academic Press: New York.

[5]. Hale, J.K. (1977). Theory of Functional Differential Equations, Springer, New York.

[6]. Driver, R.D. (1977). Ordinary and delay differential equations, Springer, New York.

[7]. Evans, D.J. and Raslan, K.R. (2004). The Adomian Decomposition Method for solving Delay Differential Equation, *International Journal of Computer Mathematics,* (2004), 1-6.

[8]. Syed Tauseef, Mohyud Din and Ahmet Yildirim (2010). Variational Iteration Method for Delay Differential Equations using He's Polynomials, *Z. Naturforsch.*, 65, 1045-1048.

[9]. Fudziah Ismail, Raed Ali Al Khasawneh, Aung San Lwin and Mohamed Suleiman (2002). Numerical Treatment of Delay Differential Equations by Runge-Kutta Method using Hermite Interpolation, *Mathematika*, 18,79-90.

[10]. Oladotun, Matthew, Ogunlaran, Adeyemi and Sunday, Olagunju (2015). Solution of Delay Differential Equations Using a Modified Power Series Method, *Applied Mathematics,* 6(2015), 670-674.

[11]. Kamal, Abdelilah and Sedeeg, H. (2016). The New Integral Transform Kamal Transform, *Advances in Theoretical and Applied Mathematics,* 11, 451-458.

[12]. Adomian, G. (1994). Solving Frontier Problems of Physics: The Decomposition Method, Kluwer Academic Publishers, Boston, MA.