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MAJORITY DOM- CHROMATIC SET OF A GRAPH

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Abstract:
This paper introduces majority dominating chromatic set of a graph G. The Majority Dom – Chromatic number $\gamma_{M\chi}(G)$ of G is investigated for some classes of graphs. Also Bounds of $\gamma_{M\chi}(G)$ and the relationship between other graph parameters are studied.

Keywords: Dom-Chromatic Set, Majority Dom – Chromatic number.

2010 Mathematics Subject Classification: 05C69.

1. INTRODUCTION

Domination is a rapidly developing area of research in graph theory. The concept of domination has existed for a long time and early discussion on the topic can be found in works of Ore[8] and Berge[1] in "Theory of Graphs and its Applications".

1.1 Basic Definitions

Let G be a finite and simple graph with p vertices and q edges. A subset D of vertices in a graph G= (V, E) is called a dominating set of G if every vertex in $(V - D)$ is adjacent to some vertex in D. A dominating set D is called a minimal dominating set if no proper subset of D is a dominating set. The domination number $\gamma(G)$ of a graph G is the minimum cardinality of a minimal dominating set in G. A dominating set D of a graph G such that $|D| = \gamma(G)$ is called a minimum dominating set of G.

A set $S \subseteq V(G)$ of vertices in a graph $G = (V, E)$ is called a majority dominating set of G[7] if at least half of the vertices of $V(G)$ are either in S or adjacent to the elements of S. A majority dominating set S is minimal if no proper subset of S is a majority dominating set of a graph G. The minimum cardinality of a minimal majority dominating set is called the majority domination number of G and is denoted by $\gamma_M(G)$. It is the minimum majority dominating set of G.

Janakiraman and Poobalaranjani [5] defined the dom-chromatic set of a graph. They established dom-chromatic numbers for some classes of graphs and also some main results in this area. Chaluvaraju and

Appajigowda [2] studied the bounds and characterization of dom-chromatic number. A dominating set $S \subseteq V(G)$ such that the induced subgraph $< S >$ satisfies the property $\chi(< S >) = \chi(G)$ is called the dominating chromatic set of a graph G. The minimum cardinality of a dominating chromatic set is called dom–chromatic number and it is denoted by $\gamma_{ch}(G)$ or $\gamma_{\chi}(G)$. A dom– chromatic set S of G such that $|S| = \gamma_{ch}(G)$ is the minimum dom–chromatic set of a graph G.

1.2 Results on some graphs

(i) [6] For a wheel $G = W_p, \gamma_M(G) = 1$.

(ii)[6] For any path P_p and any cycle C_p, $\gamma_M(G) = \left\lceil \frac{p}{6} \right\rceil$.

(iii) For any cycle C_p, $\chi(C_p) = \begin{cases} 2 & \text{, if } p \text{ is even} \\ 3 & \text{, if } p \text{ is odd.} \end{cases}$

(iv) For any tree T, $\chi(T) = 2$.

1.3 Definitions

i) A graph G is critical if $\chi(H) < \chi(G)$ for every proper subgraph H of G. G is said to be k – critical if G is critical and $\chi(G) = k$.

ii) A k- critical graph is a critical graph with chromatic number k. A graph G with $\chi(G) = k$ is called k – vertex critical if each of its vertex is a critical element.

iii) If the graph G is $(k - 1)$ - regular, then G is the complete graph K_k.

2. MAJORITY DOMINATING CHROMATIC SET OF A GRAPH

Definition: 2.1 A subset S of $V(G)$ is said to be a Majority Dominating Chromatic set if

i) S is a majority dominating set and

ii) $\chi(< S >) = \chi(G)$.

Definition: 2.2 The minimum cardinality of a majority dominating chromatic set of G is called a majority dominating chromatic number and is denoted by $\gamma_{M\chi}(G)$. It is also called the Majority Dom – Chromatic number of G.

Example: 2.3 Consider the following graph with p = 11 vertices.

G:

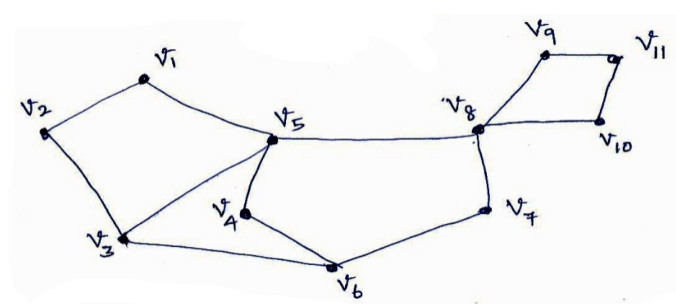

Figure (2.1)

The chromatic number of G is $\chi(G) = 3$ and $\gamma_{ch}(G) = 7$.

i) The sets $\{v_1, v_4, v_5, v_6, v_7, v_8\}$,$\{v_4, v_5, v_6, v_7, v_8\}$ and $\{v_4, v_5, v_6, v_7, v_8, v_{11}\}$ are majority dominating chromatic sets where as $D = \{v_8, v_{11}\}$ is a majority dominating set of G. Therefore $\gamma_M(G) = 2$.

ii) The set $\{v_4, v_5, v_6, v_7, v_8\}$ is the minimal majority dominating chromatic set of G. Hence $\gamma_{M\chi}(G) = 5$.

Observation: 2.4

i) Since $V(G)$ is the majority dominating set and $\chi(< V(G) >) = \chi(G)$, majority dominating chromatic set exists for all graphs.

ii) For a vertex χ- critical graph, the vertex set $V(G)$ itself is the only majority dominating chromatic set for G. For example, C_5, C_7, …, C_p, p is odd, $p \geq 5$.

Proposition: 2.5

For any graph G, $\gamma_{M\chi}(G) \leq \gamma_{ch}(G)$.

Proof: Since every dominating chromatic set of a graph G is a majority dominating chromatic set of G, $\gamma_{M\chi}(G) \leq \gamma_{ch}(G)$.

Proposition: 2.6

For any graph G, $\gamma_M(G) \leq \gamma_{M\chi}(G)$.

Proof: Since every majority dominating chromatic set of G is a majority dominating set of G, $\gamma_M(G) \leq \gamma_{M\chi}(G)$.

Corollary: 2.7 For any graph G, $\gamma_M(G) \leq \gamma_{M\chi}(G) \leq \gamma_{ch}(G)$.

Example: 2.8

i) The inequality $\gamma_{M\chi}(G) < \gamma_{ch}(G)$ holds for the graph G in Figure (2.1). Also, we have $\gamma_{M\chi}(G) = 5$ and $\gamma_{ch}(G) = 7$.

ii) For a star $G = K_{1,p-1}$, $\gamma_{M\chi}(G) = \gamma_{ch}(G) = 2$ and $\gamma_M(G) = 1$.

iii) For a graph G in Figure (2.1), $\gamma_M(G) = 2$, $\gamma_{M\chi}(G) = 5$, $\gamma_{ch}(G) = 7$. Hence, $\gamma_M(G) < \gamma_{M\chi}(G) < \gamma_{ch}(G)$.

3. MAJORITY DOM-CHROMATIC NUMBER OF SOME GRAPHS

The following results establish the majority dominating chromatic number $\gamma_{M\chi}(G)$ for some classes of graphs.

Proposition: 3.1

i) For a complete graph $G = K_p$, $p \geq 1$, $\gamma_{M\chi}(G) = p$.

ii) For a star $G = K_{1,p-1}$, $\gamma_{M\chi}(G) = 2$.

Proposition: 3.2

Let $G = C_p$ be a cycle of p vertices, $p \geq 3$. Then

$$\gamma_{M\chi}(G) = \begin{cases} \left(\frac{p}{6}\right) + 1 & , \text{ if } \quad p \equiv 0, 2 \pmod 6 \\ \left(\frac{p}{6}\right) + 2 & , \text{ if } \quad p \equiv 4 \pmod 6 \\ p & , \text{ if } \quad p \text{ is odd.} \end{cases}$$

Proof:

Let $\{v_1, v_2, v_3, \ldots, v_p\}$ be a set of vertices of C_p and $d(v_i) = 2$, for all $v_i \in V(G)$. By the result (1.1) (iii)

$$\chi(C_p) = \begin{cases} 2 & , \text{ if } \quad p \text{ is even} \\ 3 & , \text{ if } \quad p \text{ is odd.} \end{cases} \tag{1}$$

Case (i) Let $p \equiv 0{,}2 \pmod 6$. Let $D = \{v_2, v_3, v_6, \ldots\ldots v_{\gamma_{M\chi}(G)}\}$ be a majority dominating chromatic set of G such that $d(v_{2,} v_3) = 1$ and $d(v_i, v_j) = 3, i \neq j$ and $i, j = 3{,}6 \ldots, \gamma_{M\chi}$ and $v_i, v_j \in D$. So that the induced sub graph $< D >$ contains K_2 or, $K_2 \cup tK_1$, $t > 0$. Then $|N[D]| \geq \left\lceil\frac{p}{2}\right\rceil$, where $|D| = \gamma_{M\chi}(G)$ and by (1), since $\chi_{(K_2)} = 2$, $\chi(< D >) = \chi(G), p$ is even.

Then, $\quad |N[D]| \leq \sum_{i=1}^{|D|} d(v_i) + \gamma_{M\chi} - 1, \qquad \because \left\lceil\frac{p}{2}\right\rceil = 1 \pmod 3,$

$$|N[D]| \leq 3\,\gamma_{M\chi} - 1$$

$$\left\lceil\frac{p}{2}\right\rceil \leq |N[D]| \leq 3\gamma_{M\chi} - 1$$

$$\gamma_{M\chi} \geq \frac{1}{3}\left\lceil\frac{p}{2}\right\rceil + \left\lceil\frac{1}{3}\right\rceil$$

If $p = 2r$ and $2r + 1$, then $\frac{1}{3}\left\lceil\frac{p}{2}\right\rceil = \left(\frac{p}{6}\right)$. Therefore, $\gamma_{M\chi}(G) \geq \left(\frac{p}{6}\right) + 1$ (2)

Suppose the set $D = \{v_1, v_2, \ldots . v_t\} \subseteq V(G)$ with $d(v_i, v_j) = 3$, $i \neq j$ and exactly one pair $d(v_1, v_2) = 1$ and $t = \left(\frac{p}{6}\right) + 1$. Then

$$|N[D]| = 3\left(\frac{p}{6} + 1\right) - 2 \geq \left\lceil\frac{p}{2}\right\rceil.$$

Since $d(v_{1,} v_2) = 1$, the induced sub graph $< D >$ contains K_2. It implies that $\chi(< D >) = 2 = \chi(G)$, if p is even. Hence the set D is a majority dominating chromatic set of G.

Thus, $\gamma_{M\chi}(G) \leq |D| = \left(\frac{p}{6}\right) + 1$ (3)

On combining the results of (2) and (3), we obtain the result.

Case (ii) Let $p \equiv 4 \pmod 6$.

Let $D = \{v_2, v_3, v_6, \ldots . v_{\gamma_{M\chi}}\}$ be a majority dominating chromatic set of G with the same properties as in case (i). Now,

$|N[D]| \leq \sum_{i=1}^{\gamma_{M\chi}} d(v_i) + \gamma_{M\chi} - 4 \qquad \because \left(\frac{p}{2}\right) \equiv 2 \pmod 3$

$$\leq 3\gamma_{M\chi} - 4$$

$$\left\lceil\frac{p}{2}\right\rceil \leq |N[D]| \leq 3\gamma_{M\chi} - 4$$

$$\Rightarrow \gamma_{M\chi}(G) \geq \frac{1}{3}\left\lceil\frac{p}{2}\right\rceil + \left\lceil\frac{4}{3}\right\rceil = \left\lceil\frac{p}{6}\right\rceil + 2.$$

Applying the same argument as in case (i), we obtain $\gamma_{M\chi}(G) \leq \left(\frac{p}{6}\right) + 2$. Combining we get $\gamma_{M\chi}(G) = \left(\frac{p}{6}\right) + 2$, $if\ p \equiv 4 \pmod 6$.

Case (iii) Let $G = C_p, p$ is odd. Then by (1), $\chi(C_p) = 3, p$ is odd. By observation (ii) in (2.4), C_p is vertex χ - critical graph, and the vertex set $V(G)$ is a majority dom – chromatic set of G.

$\Rightarrow \gamma_{M\chi}(G) \leq p$. Since $|V(G)| = p$, $\gamma_{M\chi}(G) \geq p$.

Hence $\gamma_{M\chi}(G) = p$, if p is odd.

Corollary: 3.3

Let G be a path, then the majority dom - chromatic number is

$$\gamma_{M\chi}(P_p) = \begin{cases} \left\lceil \frac{P}{6} \right\rceil & \text{, if } p \equiv 1,2 \pmod 6 \\ \left\lceil \frac{P}{6} \right\rceil + 1 & \text{, if } p \equiv 0,4 \pmod 6. \end{cases}$$

Proposition: 3.4

For a complete bipartite graph $G = K_{m,n}, m \leq n$, $\gamma_{M\chi}(G) = 2$.

Proof: Let $G = K_{m,n}$, $m \leq n$. Then $\gamma_M(G) = 1$. Since $\chi(G) = 2$, $D = \{u_1, v_1\}$ is a majority dominating chromatic set of G such that $u_1 \epsilon V_1$ and $v_1 \in V_2$. Then $\chi(< D >) = \chi(G)$. Therefore, $\gamma_{M\chi}(G) = 2$.

Proposition: 3.5

Let $G = W_p = C_{p-1} \vee K_1$ be a wheel graph with p vertices, $p \geq 5$. Then

$$\gamma_{M\chi}(G) = \begin{cases} p, & \text{if } p \text{ is even} \\ 3, & \text{if } p \text{ is odd.} \end{cases}$$

Proof: Let $W_p = C_{p-1} \vee K_1$. From (i) of results of some graphs in (1.2),

$$\Upsilon_M(W_p) = 1 \text{ and } \chi\,(W_p) = \begin{cases} 3, & \text{if } p \text{ is odd} \\ 4, & \text{if } p \text{ is even.} \end{cases}$$

Since $C_{p-1}$is χ – vertex critical graph and $(p-1)$ is odd, the set VC_{p-1} is majority dominating chromatic set for the graph G.

$$\therefore \chi\,(C_{p-1}) = \begin{cases} p-1, & \text{if } (p-1) \text{ is odd} \\ 2, & \text{if } (p-1) \text{ is even.} \end{cases}$$

Then for a graph $G = W_p$ with p vertices, we obtain the result.

Proposition: 3.6

For a Fan graph with p vertices, $\gamma_{M\chi}(F_p) = 3$, $p \geq 3$.

Proof: Let $F_p = P_{p-1} \vee K_1$. Since $G = F_p$ has a full degree vertex, $\gamma_M(G) = 1$. Since G contains triangles, $\chi(G) = 3$. Hence $\gamma_{M\chi}(G) = 3$.

3.7 $\gamma_{M\chi}$ for some families of graphs

i) Let $G = mK_2$, $m \geq 1$ with $p = 2m$. Then $\gamma_{M\chi}(G) = \left\lceil \frac{p}{4} \right\rceil + 1$, $p \geq 2$.

ii) Let $G = \overline{K}_p$ be a totally disconnected graph of p vertices. Then $\gamma_{M\chi}(\overline{K}_p) = \left\lceil \frac{p}{2} \right\rceil$.

iii) For the Petersen graph $P(10,15)$, $\gamma_{M\chi}(P) = 5$.

iv) For a double star graph, $D_{r,s}$, $\gamma_{M\chi}(G) = 2$, if $r \leq s$.

(v) Let G be a caterpillar in which exactly one pendant is at each vertex. Then

Then, $\gamma_{M\chi}(G) = \begin{cases} \left\lceil \frac{p}{8} \right\rceil + 1, & if\ p \equiv 0,5,6,7 \pmod 8 \\ \left\lceil \frac{p}{8} \right\rceil, & if\ p \equiv 1,2,3,4 \pmod 8). \end{cases}$

4. CHARACTERIZATION THEOREM AND BOUNDS ON $\gamma_{M\chi}(G)$

The following theorem gives the characterization of a minimal majority dominating chromatic set of a graph G.

Theorem: 4.1: Let $G(p,q)$ be any graph. A majority dominating chromatic set S of G is minimal if and only if for each $u\epsilon S$, one of the following conditions hold

i) $\chi(< S - \{u\} >) < \chi(G)$,
ii) $S - \{u\}$ is not a majority dominating set of G.

Proof: Let S be a minimal majority dominating chromatic set for G. Then S is a majority dominating set and $\chi(< S >) = \chi(G)$. To prove that for each $u\epsilon S$, either (i) or (ii) holds. Suppose $\chi(< S - \{u\} >) = \chi(G)$, for any $u\epsilon S$. Then $S - \{u\}$ is a majority dominating chromatic set of G, which is a contradiction to the fact that S is minimal. Therefore condition (i) holds.

Suppose for any vertex $u\epsilon S$, $S - \{u\}$ is a majority dominating set of G. Then the induced subgraph $< S - \{u\} >$ such that $\chi(< S - \{u\} >) = \chi(G)$, it is a contradiction to the assumption that $\chi(< S >) = \chi(G)$. Hence condition (ii) holds.

Conversely suppose that S is not minimal majority dominating chromatic set, then there exists a vertex $u\epsilon S$ such that $S - \{u\}$ is a majority dominating chromatic set of G. It implies that, we get $S - \{u\}$ is a majority dominating set and $\chi(< S - \{u\} >) = \chi(G)$, for any vertex $u\epsilon S$, which is a contradiction to the conditions (i) and (ii). Hence the theorem.

Proposition: 4.2: For any graph G, $\max\{\chi(G), \gamma_M(G)\} \le \gamma_{M\chi}(G) \le p$. These bounds are sharp.

Proof: Since every majority dominating chromatic set is a majority dominating set of G, $\gamma_{M\chi}(G) \ge \gamma_M(G)$. Also since any majority dominating chromatic set of G contains at least one vertex from each color class $\gamma_{M\chi}(G) \ge \chi(G)$. Thus the lower bound follows.

For a vertex χ- critical graph, $V(G)$ is the only majority dominating chromatic set. Hence $\gamma_{M\chi}(G) \le p$. The lower bound is sharp for $G = K_p$ or $G = \overline{K}_p$ and the upper bound attains for $G = C_p$, when p is odd.

Proposition: 4.3: Let G be any graph with p vertices. Then $\gamma_{M\chi}(G) = 1$ if and only if $G = K_1$ or $\overline{K}_2$.

Proof: Assume that$\gamma_{M\chi}(G) = 1$. Then by proposition(4.2),max$\{\chi(G), \gamma_M(G)\} \le \gamma_{M\chi}(G) = 1$. It implies that $\gamma_M(G) = 1$ and $\chi(G) = 1$. Then there is no edge in G. Hence $G = \overline{K}_p$, which is totally disconnected. But by (ii) of $\gamma_{M\chi}\ for\ some\ families\ of\ graphs$ (3.7) ,$\gamma_{M\chi}(\overline{K}_p) = \left\lceil \frac{p}{2} \right\rceil$. So, when p=2, $\gamma_{M\chi}(\overline{K}_2) = 1$. It implies that $G = \overline{K}_2$ or K_1. The converse is obvious.

Proposition: 4.4:
Let G be any graph of order p. Then $\gamma_{M\chi}(G) = p$ if and only if G is vertex χ - critical.

Proof: Consider the graph G with p vertices with $\gamma_{M\chi}(G) = p$. It implies that $\chi(G) = p$ and $\gamma_M(G) \ge 1$. Then $S = \{v_1, v_2, \ldots, v_p\}$ is a majority dominating chromatic set for G and $|S| = p$. Hence, $\chi(< S >) = p = \chi(G)$. Clearly, G is either K_p or an odd cycle.

Claim $\chi(G-v) < \chi(G)$, for any $v \epsilon G$.
Let $G_1 = K_p$ or $G_2 = C_p$, p is odd. Then, by the result of Proposition (3.2), we have $\gamma_{M\chi}(G_1) = p$ and $\gamma_{M\chi}(G_2) = p$, p is odd. It implies that $\chi(G_1) = p$ and $\chi(G_2) = 3$, p is odd.
For a subgraph $H = (G_1 - v), \chi(< H >) = p - 1 < \chi(G_1)$,
$\Longrightarrow G_1 = K_p$ is vertex χ - critical graph.

Also, Let $H = (G_2 - v)$, then the induced subgraph $< H >$ is a path and its chromatic number $\chi(< H >) = 2 < \chi(G_2) >$. It implies that $G_2 = C_p$, (p is odd) is vertex χ - critical. Therefore, in both cases, G is a vertex χ-critical graph.

Conversely, assume that G is a vertex χ- critical graph. Then $\chi(G-v) < \chi(G)$, for any $v \epsilon G$. It implies that $\chi(G) = p$ and $\chi(G-v) = p-1$. Then $\gamma_M(G) \geq 1$. Since $\gamma_M(G) \geq 1$ and $\chi(G) = p$, the set $S = \{v_1, v_2, \ldots, v_p\}$ is the majority dominating chromatic set of G with $|S| = p$. $\Longrightarrow \gamma_{M\chi}(G) \leq |S| = p$. By the result of the Proposition (4.2), $\gamma_{M\chi}(G) \geq \max\{\gamma_M(G), \chi(G)\} \Longrightarrow \gamma_{M\chi}(G) \geq p$. Hence, $\gamma_{M\chi}(G) = p$.

Proposition: 4.5: Let G be a disconnected graph of order p. Then $\gamma_{M\chi}(G) = \left\lceil \frac{p}{2} \right\rceil$ if and only if the graph G is totally disconnected $\overline{K_p}$.
Proof: Let G be a disconnected graph with p vertices. Assume that $\gamma_{M\chi}(G) = \left\lceil \frac{p}{2} \right\rceil$. It implies that $\gamma_M(G) \leq \left\lceil \frac{p}{2} \right\rceil$ and $\chi(G) \geq 1$. Let $D = \left\{v_1, v_2, \ldots, v_{\left\lceil \frac{p}{2} \right\rceil}\right\}$ be a majority dominating chromatic set of G with $|D| = \left\lceil \frac{p}{2} \right\rceil$. It implies that $\gamma_M(G) = |D| = \left\lceil \frac{p}{2} \right\rceil$ and $\chi(< D >) \leq \left\lceil \frac{p}{2} \right\rceil$. Since G contains n components, say, G_1, G_2, … , G_n and let G_j be the component which used maximum number of colors in the χ - coloring of G. Since $\gamma_M(G) = \left\lceil \frac{p}{2} \right\rceil$, the majority dominating set D consists of only $\left\lceil \frac{p}{2} \right\rceil$ isolates and the maximum color used for this induced subgraph $\chi(< D >) = 1$. Hence $\chi(< D >) = \chi(G) = 1$ and $\gamma_M(G) = \left\lceil \frac{p}{2} \right\rceil$.
$\Longrightarrow$ The resulting graph G is totally disconnected graph $\overline{K_p}$.
Conversely, suppose $G = \overline{K_p}$. Then $\gamma_M(G) = \left\lceil \frac{p}{2} \right\rceil$ and $\chi(G) = 1$.
$\gamma_{M\chi}(G) = \max\left\{\left\lceil \frac{p}{2} \right\rceil, 1\right\} = \left\lceil \frac{p}{2} \right\rceil$. Hence the result.

Proposition: 4:6:
Let G be a graph of order p with $\chi(G) \geq 3$ and it has no triangles. Then $\gamma_{M\chi}(G) \geq 5$.
Proof: Let $\chi(G) \geq 3$ and G has no triangles. Then $G \neq K_p$ complete graph and G is not a tree. Therefore, G contains a cycle. If $\chi(G) \geq 3$, then G contains only odd cycles with at least $p \geq 5$. By the result of the Proposition 3.2, $\gamma_{M\chi}(C_p) = p$, p is odd, $p \geq 5$, and $\gamma_M(G) \geq 1$. Since $\chi(G) \geq 3$, $p \geq 5$ and from the argument just made in the preceding sentence we obtain that $\gamma_{M\chi}(G) \geq 5$.

5. CONCLUSION

As it is well known that the research problems in graph theory often come from its two subdomains the Graph Coloring and the Graph Domination which play a predominant role in many applications of graph theory. Therefore, in this research paper we study the combined effect of these two parameters. The new parameter majority dom-chromatic number of a graph G, defined by combining these two concepts, Majority Domination and Chromatic number of G is studied here. The majority dom-chromatic number $\gamma_{M\chi}(G)$ of G and the bounds of $\gamma_{M\chi}(G)$ are investigated for some classes of graphs. Some interesting results related with the three parameters such as $\chi(G)$, $\gamma_M(G)$ and $\gamma_{M\chi}(G)$ are proved.

REFERENCES

[1] Berge, C. (1962). Theory of graphs and its Applications, Methuen, London 1962.
[2] Chaluvaraju, B. and Appajigowda, C. (2016). The Dom – Chromatic Number of a Graph, *Malaya Journal of Matematik*, 4(1), 1-7.
[3] Harary, F. (1969). Graph Theory, Addison – Wesley, Reading Mass.
[4] Haynes, T.W., Hedetniemi Peter, S.T. and Slater, J. (1998). Fundamentals of Domination in Graphs, Marcel Dekker Inc, New York.
[5] Jankiraman, T.N. and Poobalaranjani, M. (2011). Dom – Chromatic sets of Graphs, *International*

Journal of Engineering Science, Advanced Computing and Bio –Technology, Vol. 2, No.2, April – June 2011, 88-103.

[6] Joseline Manora, J. and Swaminathan, V. (2011). Results on Majority Dominating Sets, *Scientia Magna* (Dept. of Mathematics, Northwest University, X'tian, P.R. China), Vol. 7, No. 3, 53-58.

[7] Joseline Manora, J. and Swaminathan, V. (2006). Majority Dominating Sets in Graphs – I, *Jamal Academic Research Journal*, Vol.3, No. 2, 75-82.

[8] Ore, O. (1962). Theory of graphs, Amer. Math. Soc. Colloq. Publ.38, American Mathematical Society, RI.