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FUZZY GAMMA SEMI-PREOPEN AND FUZZY GAMMA SEMI-PRECLOSED SETS IN FUZZY BITOPOLOGICAL SPACES

A. Nagoor Gani[1,*], J. Rameeza Bhanu[2]

Authors Affiliation
[1]P.G. & Research Department of Mathematics, Jamal Mohamed College (Autonomous), Tiruchirappalli, Tamil Nadu 620020, India.
[2]P.G. & Research Department of Mathematics, Bishop Heber College (Autonomous), Tiruchirappalli, Tamil Nadu 620017, India.

***Corresponding Author**
A.Nagoor Gani, P.G. & Research Department of Mathematics, Jamal Mohamed College (Autonomous), Tiruchirappalli, Tamil Nadu 620020, India.

E-mail: ganijmc@yahoo.co.in

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Abstract:
This article proposes the concept of fuzzy gamma semi-preopen (respectively, fuzzy gamma semi-preclosed) sets in fuzzy bitopological spaces, which is weaker than the concept of fuzzy strongly semiopen (respectively, fuzzy strongly semiclosed) set, fuzzy semi-preopen (respectively, fuzzy semi-preclosed) set, fuzzy semiopen (respectively, fuzzy semiclosed) set, fuzzy preopen (respectively, fuzzy preclosed) set, fuzzy gamma open (respectively, fuzzy gamma closed) set, fuzzy gamma semiopen (respectively, fuzzy gamma semiclosed) set and fuzzy gamma preopen (respectively, fuzzy gamma preclosed) set in fuzzy bitopological spaces. This paper examines the aspects and attributes of fuzzy gamma semi-preopen (respectively, fuzzy gamma semi-preclosed) sets with examples and investigates the relationship between these concepts and relevant concepts in fuzzy bitopological spaces. This article introduces the definition of fuzzy gamma semi-pre neighbourhood and fuzzy gamma semi-pre q- neighbourhood. Further, it characterizes fuzzy gamma semi-preinterior and fuzzy gamma semi-preclosure and establishes their fundamental properties.

Keywords: Fuzzy bitopological spaces,(δ_i, δ_j) F-γ-open, (δ_i, δ_j) F-γ-semi-preopen, (δ_i, δ_j) F-γ- semi-preclosed, (δ_i, δ_j) F-γ-semi-pre neighbourhood, (δ_i, δ_j) F-γ-semi-pre q- neighbourhood, (δ_i,δ_j) F-γ-semi-preinterior, (δ_i,δ_j) F-γ-semi-preclosure.

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1.INTRODUCTION

The notion of semi-preopen sets in topological spaces was introduced by D. Andrijevic [1] in the year 1986. F.H. Khedr, S.M. Al-Areefi and T. Noiri [10] studied semi-preopen sets along with their properties in bitopological spaces. The concept of fuzzy semi-preopen sets was introduced by J.H. Park and B.Y. Lee [18]. They established that fuzzy semi-preopen sets are weaker than the concepts of fuzzy semiopen or fuzzy preopen sets. Fuzzy bitopological spaces (Fbts, in short) was introduced by A. Kandil and M.E. El-Shafee [9] in 1989, which is an extension of fuzzy topological spaces and generalization of bitopological spaces. The concept of fuzzy-γ-open sets and fuzzy-γ-continuity were introduced in fuzzy bitopological spaces, by F.S. Mahmoud, M.A. Fath Alla and M.M. Khalaf [15] and related properties were studied.

In this article, we extend the notion of semi-preopen sets to Fuzzy γ-open sets to obtain (δ_i, δ_j) Fuzzy γ semi-preopen sets in fuzzy bitopological spaces. This assists to examine the nature of (δ_i, δ_j) Fuzzy γ-semi-preopen sets in fuzzy bitopological spaces and to study their aspects and attributes. This is beneficial in developing Fuzzy γ-semi-preopen pairwise continuous functions and extending their applications.

In section 3, we introduce (δ_i, δ_j) Fuzzy γ-semi-preopen sets and establish their related characteristics along with examples and counter examples. The concept of (δ_i, δ_j) Fuzzy γ semi-preopen sets are weaker than the concept of (δ_i, δ_j) fuzzy strongly semiopen(respectively,(δ_i, δ_j) fuzzy strongly semiclosed) set,(δ_i, δ_j) fuzzy semi-preopen (respectively,(δ_i, δ_j) fuzzy semi-preclosed) set,(δ_i, δ_j) fuzzy semiopen(respectively,(δ_i, δ_j) fuzzy semiclosed) set,(δ_i, δ_j) fuzzy preopen (respectively,(δ_i, δ_j) fuzzy preclosed) set, (δ_i, δ_j)fuzzy γ-open (respectively,(δ_i, δ_j)fuzzy γ-closed) set, (δ_i, δ_j)fuzzy γ-semiopen (respectively,(δ_i, δ_j) fuzzy γ-semiclosed) set and (δ_i, δ_j) fuzzy γ-preopen (respectively,(δ_i, δ_j) fuzzy γ-preclosed) set.

In section 4, we define (δ_i, δ_j) fuzzy-γ-semi-preinterior and (δ_i, δ_j) fuzzy-γ-semi-preclosure along with their properties. Throughout this paper (X, δ_i, δ_j) (or simply X), denote fuzzy bitopological spaces (Fbts, in short). For a fuzzy set A in a fuzzy bitopological space X, δ_i-cl(A) and δ_i-int(A) denote the closure and interior with respect to the topology δ_i respectively.

2. PRELIMINARIES

Definition 2.1: Let X be a nonempty set and I= [0, 1]. A fuzzy set (briefly F-set) A in X is a mapping from X to I. A fuzzy set A of X is contained in a fuzzy set B of X denoted by A≤ B if and only if A(x)≤B(x) for each x∈X.

A fuzzy point [26] with singleton support x∈X and the value $\alpha\in[0,1]$ is denoted by x_α. The complement A' of a fuzzy set X is 1−A defined by (1−A) (x)= 1−A(x) for each x ∈X. A fuzzy point $x_\beta \in$ A if and only if β ≤ A(x).

Definition 2.2: [20]A fuzzy set A is the union of all fuzzy points which belong to A. A fuzzy point x_β is said to be quasicoincident with the fuzzy set A denoted by x_βqA if and only if β + A(x) > 1.

A fuzzy set A is said to be quasicoincident [20] with B denoted by AqB if and only if there exists x∈X such that A(x) + B(x) > 1. A ≤ B if and only if ˈI(AqB′).

Definition 2.3: Let λ be a fuzzy set of a fuzzy topological space (X, δ). Then λ is called
(a) a F semiopen (briefly FSO) set of X if $\lambda \leq$ cl (int (λ)) [2];
(b) a F preopen (briefly FPO) set of X if $\lambda \leq$ int (cl (λ)) [14];
(c) a F strongly semiopen (briefly FSSO) set of X if $\lambda \leq$ int (cl (int (λ))) [3];
(d) a F semi-preopen (briefly FSPO) set of X if $\lambda \leq$ cl (int (cl (λ))) [18];
The set of all F-so (resp. F-sc), F-po (resp. F-pc), F-sso (resp. F-ssc), F-spo (resp. F-spc) of a fuzzy topological space will be denoted by FSO(X) (resp. FSC(X)), FPO(X) (resp. FPC (X)), F-SSO(X) (resp. F-SSC(X)), F-SPO(X) (resp. F-SPC(X)).

Definition 2.4: [8]. Let (X, δ) be a fuzzy topological space. Then v is called a F - γ open (F - γ closed) set of Xif $v \leq$ int (cl (v)) ∨ cl (int (v)) ($v \geq$ cl (int (v)) ∧ int (cl (v))).
The family of all F-γ open (respectively F-γ closed) sets of X is denoted by F-γ O(X) (resp. F-γ C(X)).

Lemma 2.5: [2]. For a family {λ_α}of fuzzy sets of a Fts X, ∨ cl(λ_α) ≤ cl(∨λ_α) and ∨int(λ_α) ≤ int(∨λ_α)

Lemma 2.6: [2]. For a fuzzy set λ of a F-ts X, (i) (int (λ))′ = cl (λ′) and (ii) (cl(λ))′ = int(λ′)
Lemma 2.7: [2]. For a fuzzy set λ of a F-ts X, (a) 1–int λ=cl (1–λ) and (b) 1–cl λ=int (1–λ).

Definition 2.8: [9]. A set X on which are defined two (arbitrary) F-topologies δ_1 and δ_2 is called a F-bitopological space (briefly F-bts) and denoted by (X, δ_1,δ_2). As to the notions, we shall write δ_i-int(λ) and δ_i-cl(λ) to mean respectively the interior and closure of a F-set λ with respect to the F-topology δ_i in F-bts (X, δ_i, δ_j), with δ_i-F-o set and δ_i-F-c set, we mean respectively δ_i-F-open and δ_i-F-closed set. The indices i and j take values {1,2} throughout this paper and i ≠ j, i=j gives the known results in F-ts.

Definition 2.9: [23]. Let λ be a fuzzy set of a F-bts (X, δ_i, δ_j). Then λ is called
(a) a (δ_i, δ_j) F semiopen (briefly (δ_i, δ_j) F-so) set of X if $\lambda \leq \delta_j$-cl (δ_i-int (λ));
(b) a (δ_i, δ_j) F semiclosed (briefly (δ_i, δ_j) F-sc) set of X if $\lambda \geq \delta_j$-int (δ_i-cl (λ));
The set of all (δ_i, δ_j) F-so, (resp. (δ_i, δ_j) F-sc) sets of a F-bts X will be denoted by (δ_i, δ_j) FSO(X), (resp. (δ_i, δ_j) FSC(X)).

Definition 2.10: [22]. Let λ be a fuzzy set of a F-bts (X, δ_i, δ_j).Then λ is called
(a) a (δ_i, δ_j) F strongly semiopen (briefly (δ_i, δ_j) F-sso) set of X if $\lambda \leq \delta_i$-int $(\delta_j$-cl $(\delta_i$-int $(\lambda)))$;
(b) a (δ_i, δ_j)F strongly semiclosed (briefly(δ_i, δ_j) F-ssc) set of X if$\lambda \geq \delta_i$-cl $(\delta_j$-int $(\delta_i$cl$(\lambda)))$;
(c) a (δ_i, δ_j) F preopen (briefly (δ_i, δ_j) F-po) set of X if $\lambda \leq \delta_i$-int $(\delta_j$-cl $(\lambda))$;
(d) a (δ_i, δ_j) F preclosed (briefly (δ_i, δ_j) F-pc) set of X if $\lambda \geq \delta_i$-cl $(\delta_j$-int $(\lambda))$;
The set of all (δ_i, δ_j) F-sso, (δ_i, δ_j) F-ssc, (δ_i, δ_j) F-po, (δ_i, δ_j) F-pc sets of a F-bts X will be denoted by (δ_i, δ_j) FSSO(X), (δ_i, δ_j) FSSC(X), (δ_i, δ_j) FPO(X) and (δ_i, δ_j) FPC(X) respectively.

Definition 2.11: [19]. Let λ be a fuzzy set of a Fbts (X, δ_i, δ_j). Then λ is called
(a) a (δ_i, δ_j) F semi-preopen (briefly (δ_i, δ_j)F-spo) set of X if $\lambda \leq \delta_j$-cl $(\delta_i$-int $(\delta_j$-cl $(\lambda)))$;
(b) a (δ_i, δ_j) F semi-preclosed (briefly (δ_i, δ_j)F-spc) set of X if $\lambda \geq \delta_j$-int $(\delta_i$-cl $(\delta_j$-int $(\lambda)))$;
The set of all (δ_i, δ_j) F-spo, (respectively (δ_i, δ_j) F-spc) sets of a F-bts X will be denoted by (δ_i, δ_j) FSPO(X), (respectively (δ_i, δ_j) FSPC(X)).

Definition 2.12: [15]. Let λ be a fuzzy set of a F-bts (X, δ_i, δ_j). Then λ is called a (δ_i, δ_j) F γ open (respectively, (δ_i, δ_j) F γ closed), briefly (δ_i, δ_j) F-γo (respectively, (δ_i, δ_j) F-γ c) if
$\lambda \leq \delta_i$-int $(\delta_j$- cl $(\lambda)) \vee \delta_j$-cl $(\delta_i$-int $(\lambda))$, respectively $\lambda \geq \delta_i$-cl $(\delta_j$-int $(\lambda)) \wedge \delta_j$-int $(\delta_i$-cl $(\lambda))$.
The family of all (δ_i, δ_j) F-γ o (respectively (δ_i, δ_j) F-γ c) sets of X is denoted by (δ_i, δ_j) F-γ O(X) and (respectively (δ_i, δ_j) F-γ C(X)).

Remark 2.13: [15] (i) The union of (δ_i, δ_j) F-γo sets is a (δ_i, δ_j) F-γo set.
(ii) The intersection of (δ_i, δ_j) F-γc sets is a (δ_i, δ_j) F-γc set.

Definition 2.14: [15]. Let λ be a fuzzy set of a F-bts (X,δ_i, δ_j). Then the (δ_i, δ_j) γ-closure ((δ_i, δ_j) γ-cl for short) and (δ_i, δ_j) γ-interior ((δ_i, δ_j) γ-int for short) of λ are defined as
(δ_i, δ_j) γ-cl$(\lambda) = \wedge \{v : v$ is (δ_i, δ_j) F-γ closed and $\lambda \leq v\}$ and (δ_i, δ_j) γ-int$(\lambda) = \vee\{v : v$ is (δ_i, δ_j) F-γ open and $v \leq \lambda\}$

Remark 2.15: (i) (δ_i, δ_j) γ-cl (λ) is the intersection of all (δ_i, δ_j) F-γ c sets of X containing λ.
(ii) (δ_i, δ_j) γ-int (λ) is the union of all (δ_i, δ_j) F-γo sets of X contained in λ.
(iii) δ_i-γ cl (λ) is the intersection of all (δ_i, δ_j) F-γc sets of X containing λ with respect to δ_i.
(iv) δ_i-γint (λ) is the union of all (δ_i, δ_j) F-γo sets of X contained in $\lambda$with respect to the δ_i.

Definition 2.16: [16]. Let A be a fuzzy set of a F-bts (X, δ_i, δ_j). Then A is called a
(a) (δ_i, δ_j) F-γ-semiopen (briefly (δ_i, δ_j) F-γ-so) set if A $\leq \delta_j$-cl $(\delta_i$ -γ int(A)).
(b) (δ_i, δ_j) F-γ-semiclosed (briefly (δ_i, δ_j) F-γ-sc) set if A $\geq \delta_j$- int $(\delta_i$ -γ cl(A)).
The family of all (δ_i, δ_j) Fγ so (respectively (δ_i, δ_j) F-γ sc)sets of X is denoted by (δ_i, δ_j) F-γ SO(X) and respectively (δ_i, δ_j) F-γ SC(X)).

Definition 2.17: [17]. Let A be a fuzzy set of a fuzzy bitopological space (X, δ_i, δ_j). Then A is called a
(a) (δ_i, δ_j) F-γpreopen (briefly (δ_i, δ_j) F-γ po) set if A $\leq \delta_i$ int $(\delta_j\gamma$cl(A)).(b) (δ_i, δ_j) F-γ preclosed (briefly (δ_i, δ_j) F-γ pc set if A $\geq \delta_i$ cl $(\delta_j\gamma$int(A)).The family of all (δ_i, δ_j) F-γ po (respectively (δ_i, δ_j) F-γ pc) sets of X is denoted by (δ_i, δ_j) F-γ PO(X) and (respectively (δ_i, δ_j) F-γ PC(X)).

3. MAIN RESULTS

(δ_i, δ_j) F-γSemi preopen and (δ_i, δ_j) F-γ Semi preclosed set

Definition 3.1: Let A be a fuzzy set of a fuzzy bitopological space (X, δ_i,δ_j). Then A is called a

(a) (δ_i, δ_j) F-γ semi preopen (briefly (δ_i, δ_j) F-γspo) set if A $\leq \delta_j$cl $(\delta_i$ int $(\delta_j\gamma$cl(A))).
(b) (δ_i, δ_j) F-γ semi preclosed (briefly (δ_i, δ_j) F-γspc set if A $\geq \delta_j$int $(\delta_i$ cl $(\delta_j\gamma$int(A))).

The family of all (δ_i, δ_j)F-γ spo (respectively (δ_i, δ_j)F-γ spc) sets of X is denoted by (δ_i, δ_j)F-γ SPO(X) and (respectively (δ_i, δ_j)F-γ SPC(X)).

Example 3.2: Let (X, δ_1, δ_2) be a Fbts with X = {a, b, c}, $\delta_1 = \{\tilde{0}, \tilde{1}, A\}$, $\delta_2 = \{\tilde{0}, \tilde{1}, B\}$and fuzzy sets A=$\{a_{0.5}, b_{0.6}, c_{0.3}\}$, $B = \{a_{0.3}, b_{0.8}, c_{0.6}\}$. Here (δ_1, δ_2) F-γ o sets = $\{\tilde{0}, \tilde{1}, A, B\}$. The sets A and B (resp. A' andB') are (δ_1, δ_2) F-γ-spo (resp. (δ_1, δ_2) F-γ-spc).

Theorem 3.3: Let A be a fuzzy subset of a Fbts (X, δ_i, δ_j). Then the following are equivalent

(a) A is (δ_i, δ_j) F-γ-semi preopen set.
(b) A′ is (δ_i, δ_j) F-γ-semi preclosed set.
(c) There exists a (δ_i, δ_j) F-γ po set U in X, such that U $\leq$ A $\leq \delta_j$cl(U).

(d) There exists a (δ_i, δ_j) F-γ pc set V in X, such that $\delta_j$int(V) $\leq$ A′$\leq$ V.

Proof: (a) $\Leftrightarrow$ (b) Follows from Definition 3.1 and Lemma 2.6[2].

(a) $\Rightarrow$ (c) A is (δ_i, δ_j) F-γ-spo implies that A $\leq \delta_j$cl (δ_i int ($\delta_j\gamma$cl(A))). Let U = δ_i int ($\delta_j\gamma$cl(A)). Then U is a (δ_i, δ_j) F-γ po set. Then A $\leq \delta_j$cl (U) and by [17] (Theorem 3.6) there exists a δ_i F o set H in X such that U $\leq$ H = A $\leq\delta_j$ cl(U).

(c) $\Rightarrow$ (a) Now suppose that there exists a (δ_i, δ_j) F-γ po set U in X, such that U $\leq$ A $\leq \delta_j$cl(U). Then U $\leq \delta_i$ int ($\delta_j\gamma$cl(U)). As U $\leq$ A, $\delta_j\gamma$cl(U) $\leq \delta_j\gamma$cl(A). Then δ_i int ($\delta_j\gamma$cl(U)) $\leq \delta_i$ int ($\delta_j\gamma$cl(A)). Consider A $\leq \delta_j$cl(U). Then A $\leq \delta_j$cl (δ_i int ($\delta_j\gamma$cl(U))) $\leq \delta_j$cl (δ_i int ($\delta_j\gamma$cl(A))) which implies A is (δ_i, δ_j) F-γ-spo.

(b) $\Rightarrow$ (d) A′ is (δ_i, δ_j) F-γ-spc. By definition 3.1, A′$\geq \delta_j$int (δ_i cl ($\delta_j\gamma$int(A′))). Let V = $\delta_i$cl($\delta_j\gamma$int(A′)) and so V is a (δ_i, δ_j) F-γ pc set. Then A′ $\geq \delta_j$int(V) and by [17] (Theorem (3.7) there exists a δ_i F c set F = A′ in X such that δ_j γint(V) $\leq$ F = A′$\leq$V.

(d) $\Rightarrow$ (b) Suppose that there exists a (δ_i, δ_j) F-γ pc set V in X, such that δ_j int(V) $\leq$ A′$\leq$ V. Then V $\geq \delta_i$ cl ($\delta_j\gamma$int(V)). As A′ $\geq \delta_j$int(V) $\geq \delta_j$ int (δ_i cl ($\delta_j\gamma$int(V))). Thus A′ is (δ_i, δ_j) F-γ-spc.

Remark 3.4: The concepts of (δ_i, δ_j) F-γ-semi preopen (resp. (δ_i, δ_j) F-γ-semi preclosed) and (δ_j, δ_i) F-γ-semi preopen (resp. (δ_j, δ_i) F-γ-semi preclosed) set are independent. The following example illustrates this.

Example 3.5: Let (X, δ_1, δ_2) be a F-bts where X ={a, b, c}, δ_1={$\tilde{0}$, $\tilde{1}$,A, E} and δ_2= {$\tilde{0}$, $\tilde{1}$, B, C} with fuzzy sets A={$a_{0.5}$, $b_{0.7}$, $c_{0.4}$}, E={$a_{0.5}$, $b_{0.8}$, $c_{0.5}$}, B ={$a_{0.6}$, $b_{0.7}$, $c_{0.4}$}, C ={$a_{0.4}$, $b_{0.3}$, $c_{0.5}$}. Here (δ_1, δ_2) F-γ-o sets = {$\tilde{0}$,$\tilde{1}$, A, E, B}and (δ_1, δ_2) F-γ-spo sets = {$\tilde{0}$,$\tilde{1}$, A, B, E}. Also (δ_2, δ_1) F-γ-o sets = {$\tilde{0}$,$\tilde{1}$, A, E, B, C} and (δ_2, δ_1) F-γ-spo sets = {$\tilde{0}$,$\tilde{1}$, A, E, B, C}.
Let D = {$a_{0.5}$, $b_{0.1}$, $c_{0.3}$}. Consider $\delta_2$cl (δ_1 int ($\delta_2\gamma$cl(D))) = δ_2 cl(A) = C′. Thus, D is (δ_1, δ_2) Fγspo but $\delta_1$cl (δ_2 int ($\delta_1\gamma$cl(D)))=0 and D >0 so D is not (δ_2, δ_1) Fγspo.
Now $\delta_1$cl (δ_2 int ($\delta_1\gamma$cl(C)))=A′ and C $\leq$ A′ but $\delta_2$cl (δ_1 int ($\delta_2\gamma$cl(C))) = 0. Thus, C is not (δ_1, δ_2) Fγspo and C is (δ_2, δ_1) Fγspo.

Theorem 3.6: In a F-bts (X, δ_i, δ_j), if a fuzzy set A is (δ_i, δ_j) F-γ-spo ((δ_i, δ_j) F-γ-spc) and (δ_j, δ_i)F-γ-sc ((δ_j, δ_i) F-γ-so), then A is (δ_i, δ_j) F-γ-so ((δ_i, δ_j) F-γ-sc).

Proof: Let A be a (δ_i, δ_j) F-γ-spo and (δ_j, δ_i) F-γ-sc set.A is (δ_j, δ_i) F-γ-sc implies that δ_i int ($\delta_j\gamma$cl (A)) $\leq$ A. That is δ_i int (δ_i int ($\delta_j\gamma$cl (A))) $\leq \delta_i$ int (A) $\leq \delta_i$ int ($\delta_j\gamma$cl (A)) as A $\leq \delta_j$ cl (A) $\leq \delta_j\gamma$cl (A). Thus, δ_i int ($\delta_j\gamma$cl (A)) = δ_i int (A).

Using this, in the definition of (δ_i, δ_j) F-γ-spo, we have A $\leq\delta_j$cl(δ_i int($\delta_j\gamma$cl(A))) $\leq\delta_j$cl(δ_i int(A)). Hence A is (δ_i, δ_j) F-γ-so.

Theorem 3.7: In a F-bts (X, δ_i, δ_j), if a fuzzy set A is (δ_i, δ_j) F-γ-spo ((δ_i, δ_j) F-γ-spc) and (δ_j, δ_i)F-γ-c ((δ_j, δ_i) F-γ-o), then A is (δ_i, δ_j) F-γ-o ((δ_i, δ_j) F-γ-c).

Proof: Follows from Theorem 3.6 and from [16].

Theorem 3.8: In a F-bts (X, δ_i, δ_j), a fuzzy set A of X is (δ_i, δ_j) F-γ-spo if and only if there exists a δ_j F-γ-c H set such that A $\leq \delta_j$cl (δ_i int(H)).

Proof: Suppose A is (δ_i, δ_j) F-γ-spo, then A $\leq \delta_j$cl (δ_i int ($\delta_j\gamma$cl(A)). Take H = $\delta_j\gamma$cl(A). Thus, we have H is a δ_j F-γ-c set and A $\leq \delta_j$cl (δ_i int(H)).

Conversely suppose there exists a δ_j F-γ-c set H such that A $\leq \delta_j$cl (δ_i int(H)). Put H = $\delta_j\gamma$cl(A). Then A is (δ_i, δ_j) F-γ-spo.

Theorem 3.9: In a F-bts (X, δ_i, δ_j),a fuzzy set B of X is (δ_i, δ_j) F-γ-spc if and only if there exists a δ_j F-γ-o set G such that $\delta_j$int (δ_i cl(G))$\leq$ B.

Proof: Follows from Definition 3.1.

Proposition 3.10: The union of two (δ_i, δ_j)F-γ spo sets is a (δ_i, δ_j)F-γ spo set in a F-bts (X, δ_i, δ_j).

Theorem 3.11: An arbitrary union of (δ_i, δ_j)Fγ spo sets is a (δ_i, δ_j)Fγ spo set in a F-bts (X,δ_i,δ_j).

Proof: Let $\{A_\alpha\}_{\alpha\in A}$ be a collection of (δ_i, δ_j) F-γ spo sets in a F-bts (X, δ_i, δ_j). Then for each $\alpha\in A$, A_α is (δ_i, δ_j)F-γ spo. Then $\vee_{\alpha\in\Delta}$ $A_\alpha \leq \vee_{\alpha\in\Delta}\delta_j$cl ($\delta_i$ int ($\delta_j\gamma$cl (A))) = $\delta_j$cl [$\vee_{\alpha\in\Delta}\delta_i$ int ($\delta_j\gamma$cl (A))] $\leq\delta_j$cl (δ_i int ($\vee_{\alpha\in\Delta}\delta_j\gamma$cl (A)))= $\delta_j$cl (δ_i int ($\delta_j\gamma$cl ($\vee_{\alpha\in\Delta}$ A_α))). Thus, {$\vee_{\alpha\in\Delta}$ A_α} is (δ_i, δ_j) F-γ po.

Remark 3.12: Intersection of two (δ_i, δ_j)F-γ spo sets need not be a (δ_i, δ_j) F-γ spo set. Also, the intersection of a (δ_i, δ_j) Fγspo set and aδ_i Fo set is not necessarily (δ_i, δ_j) Fγspo and the intersection of a (δ_i, δ_j) Fγspo and a (δ_i, δ_j) Fsso is not necessarily (δ_i, δ_j) Fγspo which is shown in the following example. It should be noted that in ordinary topological setting the intersection of a semi-preopen set and an open set is semi-preopen [1].

Example 3.13: Let (X, δ_1, δ_2) be a F-bts with X = {a, b, c}, $\delta_1 = \{\tilde{0},\tilde{1},A\}$ and $\delta_2 = \{\tilde{0},\tilde{1},B\}$ where $A = \{a_{0.7}, b_{0.5}, c_{0.4}\}$and $B = \{a_{0.8}, b_{0.3}, c_{0.2}\}$. Here ($\delta_1$, δ_2) Fγspo sets = $\{\tilde{0},\tilde{1},A,B\}$. Let $C = \{a_{0.2}, b_{0.8}, c_{0.5}\}$ then $A \wedge C = \{a_{0.2}, b_{0.5}, c_{0.4}\}$. C is ($\delta_1$, δ_2) Fγspo but A ∧ C is not (δ_1, δ_2) Fγspo since $\delta_2$cl (δ_1 int ($\delta_2\gamma$cl(A))) = 0 and A ∧ C $\nleq$ 0.

The intersection of a (δ_i, δ_j) Fγspo set and aδ_i Fo set is not necessarily (δ_i, δ_j) Fγspo. Here A is δ_1 Fo and C is (δ_1, δ_2) Fγspo but A ∧ C is not (δ_1, δ_2) Fγspo.

Consider δ_1 int (δ_2 cl (δ_1 int (A))) =1 and A ≤ 1. Thus, A is (δ_1, δ_2) Fsso whereas δ_1 int (δ_2 cl (δ_1 int (A ∧ C))) = 0 and so A ∧ C is not (δ_1, δ_2) Fγspo.

The intersection of a (δ_i, δ_j) F-γspo set and a (δ_i, δ_j) Fsso set is not necessarily (δ_i, δ_j) Fγspo. Here A is (δ_1, δ_2) Fsso and C is (δ_1, δ_2) Fγspo but A ∧ C is not (δ_1, δ_2) Fsso.

Proposition 3.14: Intersection of two (δ_i, δ_j)Fγspc sets is a (δ_i, δ_j)Fγspc set in a F-bts (X, δ_i, δ_j).

Theorem 3.15: Anyarbitrary intersection of (δ_i, δ_j) F-γspc sets is a (δ_i, δ_j) F-γspc set in a F-bts (X,δ_i,δ_j).

Proof Consider $\{A_\alpha\}_{\alpha\in\Delta}$as a collection of ($\delta_i$, δ_j) F-γ-spc sets in (X, δ_i, δ_j). For each α∈ Δ, A_α is a (δ_i, δ_j) F-γ-spc. Then for each α∈ Δ, A_α' is (δ_i, δ_j) F-γ-spo which implies $\vee_{\alpha\in\Delta}A_\alpha$' is ($\delta_i$, δ_j) F-γ-spo. By Theorem 3.3,($\vee_{\alpha\in\Delta}A_\alpha$')' is ($\delta_i$, δ_j) F-γ-spc. $\wedge_{\alpha\in\Delta}$(A_α')' is (δ_i, δ_j) F-γ-spc. Thus, $\wedge_{\alpha\in\Delta}A_\alpha$ is (δ_i, δ_j) F-γ-spc.

Remark 3.16: The union of two (δ_i, δ_j) Fγspc sets need not be (δ_i, δ_j) Fγspc which is illustrated below.

Example 3.17: Let (X, δ_1, δ_2) be a F-bts withX = {a, b ,c}, $\delta_1 = \{\tilde{0},\tilde{1},A\}$ and $\delta_2 = \{\tilde{0},\tilde{1},B\}$ where A and B are fuzzy sets defined in X as$A = \{a_{0.5}, b_{0.8}, c_{0.4}\}$ and $B = \{a_{0.8}, b_{0.1}, c_{0.6}\}$. ($\delta_1$, δ_2) Fγpc sets= $\{\tilde{0},\tilde{1},A',B'\}$. Let $C = \{a_{0.2}, b_{0.7}, c_{0.8}\}$ then $C' = \{a_{0.8}, b_{0.3}, c_{0.2}\}$ and A′ ∨ C′ = {$a_{0.8}$, $b_{0.3}$, $c_{0.6}$}. Here A′ ∨ C′ is not (δ_1,δ_2) Fγpc. Thus, union of two (δ_i, δ_j) Fγpc sets need not be (δ_i, δ_j) Fγpc.

Theorem 3.18: Every δ_iF o set is (δ_i, δ_j) F-γ spo and every δ_iF γ o is (δ_i, δ_j) F-γ spo in a F-bts(X,δ_i,δ_j).

Proof: Let A be δ_iF o. Then δ_i int ($\delta_j\gamma$cl(A)) = $\delta_j\gamma$cl(A) ≥ A which implies A ≤δ_i int ($\delta_j\gamma$cl (A)). Then $\delta_j$cl (A) ≤$\delta_j$cl (δ_i int ($\delta_j\gamma$cl (A))). That is A ≤$\delta_j$cl (δ_i int ($\delta_j\gamma$cl (A))). Thus, A is (δ_i, δ_j) F-γ spo.

Now let A be δ_iF γ o. Then $\delta_i\gamma$int (A) = δ_i int (A) = A. Then, as before A is (δ_i, δ_j) F-γ spo.

Theorem 3.19: Every (δ_i, δ_j) F-γ o set is (δ_i, δ_j) F-γ spo in a F-bts (X, δ_i, δ_j).
Proof Let A be (δ_i, δ_j)F-γ o. By the definition 2.12**,** A ≤δ_i int (δ_j cl (A))∨δ_j cl(δ_i int (A)). That is A ≤δ_i int (δ_j cl(A)) ∨ δ_j cl(A). Then A ≤δ_i int ($\delta_j$cl(A)) which implies that A ≤δ_j cl(δ_i int ($\delta_j\gamma$cl(A))). Thus A is (δ_i, δ_j) F-γ spo.

Theorem 3.20: Every (δ_i, δ_j) Fssoset is (δ_i, δ_j) F - γ spo in a F-bts (X, δ_i, δ_j).
Proof Let A be (δ_i, δ_j) Fsso. Then A ≤δ_i int (δ_j cl (δ_i int (A))). That isA ≤δ_j cl (δ_i int (A)) which gives that A ≤$\delta_j$cl(δ_i int ($\delta_j\gamma$cl(A))). Then A is (δ_i, δ_j) F-γ spo.

Theorem 3.21: Every (δ_i, δ_j)Fso set is (δ_i, δ_j)F-γ spo and every (δ_i, δ_j)F-γ sois (δ_i, δ_j)F-γ spo in a F-bts(X,δ_i,δ_j).

Proof: Let A be (δ_i, δ_j) Fso. Then A ≤δ_j cl (δ_i int (A)) which implies that A ≤$\delta_j$cl(δ_i int ($\delta_j\gamma$cl(A))). Then A is (δ_i, δ_j) F-γ spo.

Similarly, if A isa (δ_i, δ_j) F-γ so, then A is (δ_i, δ_j) F-γ spo.

Theorem 3.22: Every (δ_i, δ_j) F po set is (δ_i, δ_j) F-γ spo in a F-bts(X, δ_i, δ_j).

Proof: Follows from the Definition 2.10.

Remark 3.23: The converse of the above theorems need not be true which is illustrated below.

Example 3.24: Let (X, δ_1, δ_2) be a F-bts where X = {a, b, c}, $\delta_1 = \{\tilde{0}, \tilde{1}, A\}$ and $\delta_2 = \{\tilde{0}, \tilde{1}, B\}$with fuzzy sets $A = \{a_{0.6}, b_{0.2}, c_{0.3}\}$and $B = \{a_{0.4}, b_{0.4}, c_{0.5}\}$. Here (δ_1, δ_2) Fγoset = $\{\tilde{0}, \tilde{1}, A\}$.

We have $\delta_2$cl (δ_1 int ($\delta_2\gamma$cl(A))) = 1 = $\delta_2$cl (δ_1 int ($\delta_2\gamma$cl(B))), so (δ_1, δ_2) Fγspo sets = $\{\tilde{0}, \tilde{1}, A, B\}$. Here B (resp. B') is (δ_1, δ_2) Fγspo (resp. (δ_1, δ_2) F γ spc) but not δ_1 Fo (resp. δ_1 F c) as well as not δ_1 Fγo

(resp. δ_1Fγ c).Also, B (resp. B') is not (δ_1, δ_2) Fγo (resp. (δ_1, δ_2) F γ c).

Consider δ_2 cl (δ_1 int γ(B)) = 0 and B > 0. Thus, B (resp. B') is (δ_1, δ_2) Fγspo (resp. (δ_1, δ_2)Fγspc) but not (δ_1, δ_2)Fγso (resp. (δ_1, δ_2)Fγsc) and not (δ_1, δ_2)Fso (resp. (δ_1, δ_2) Fsc).

Now consider $\delta_1$int(δ_2 cl (B))=A but B>A. Thus, B (resp. B') is not (δ_1,δ_2)Fpo(resp. (δ_1,δ_2) Fpc).

Here δ_1 int (δ_2 cl (δ_1 intB))=0 and B>0. Thus, B (resp. B') is not (δ_1, δ_2)Fsso(resp. (δ_1, δ_2)Fssc).

Theorem 3.25: Every (δ_i, δ_j) F γpo set is (δ_i, δ_j) F γ spo in a F-bts(X, δ_i, δ_j).

Proof: Let A be (δ_i, δ_j) F-γpo. From the Definition 2.17 we have A $\leq \delta_i$ int ($\delta_j\gamma$cl(A)). That is δ_j cl(A)$\leq \delta_j$ cl(δ_i int ($\delta_j\gamma$cl(A))) which implies A $\leq \delta_j$ cl(δ_i int ($\delta_j\gamma$cl(A))).

Remark 3.26: The converse of the above theorem is not true which is illustrated the following example.

Example 3.27: Let X={a, b, c}, $\delta_1 = \{\tilde{0}, \tilde{1}, A\}$ and $\delta_2 = \{\tilde{0}, \tilde{1}, B\}$then (X, δ_1, δ_2) is a F-bts with fuzzy sets $A = \{a_{0.4}, b_{0.3}, c_{0.2}\}$ and $B = \{a_{0.5}, b_{0.3}, c_{0.4}\}$. Here (δ_1, δ_2) Fγoset = $\{\tilde{0}, \tilde{1}, A, B\}$.

Here, $\delta_2$cl (δ_1 int ($\delta_2\gamma$cl(A))) = B′ = $\delta_2$cl (δ_1 int ($\delta_2\gamma$cl(B))), A ≤ B′ and B ≤B′, so (δ_1, δ_2) Fγspo set= $\{\tilde{0}, \tilde{1}, A, B\}$.

Consider δ_1 int ($\delta_2\gamma$cl(B)) = A and B > A. Thus, B (resp. B') is (δ_1, δ_2) Fγspo (resp. (δ_1, δ_2) F γ spc) but not (δ_1, δ_2) Fγpo (resp. (δ_1, δ_2) Fγpc).

Theorem 3.28: Every (δ_i, δ_j) Fspo set is (δ_i, δ_j) F γ spo in a F-bts(X, δ_i, δ_j).

Proof: Follows from the Definition 2.11.

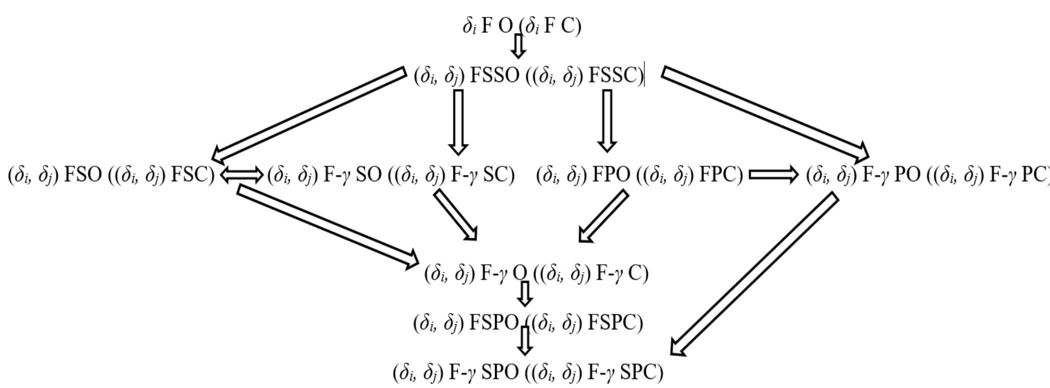
Remark 3.29: Converse of the above theorem does not hold as given in the following example.

Example 3.30: Let X = {a, b, c}, $\delta_1 = \{\tilde{0}, \tilde{1}, A\}$ and $\delta_2 = \{\tilde{0}, \tilde{1}, B\}$then (X, δ_1, δ_2) is a F-bts with fuzzy sets $A = \{a_{0.8}, b_{0.4}, c_{0.6}\}$and $B = \{a_{0.3}, b_{0.2}, c_{0.4}\}$. Here (δ_1, δ_2) Fγoset= $\{\tilde{0}, \tilde{1}, A\}$.

Here, $\delta_2$cl (δ_1 int ($\delta_2\gamma$cl(A))) = 1 = $\delta_2$cl (δ_1 int ($\delta_2\gamma$cl(B))). Therefore (δ_1, δ_2) Fγsposet= $\{\tilde{0}, \tilde{1}, A, B\}$.

Consider $\delta_2$cl (δ_1 int ($\delta_2$cl(B)))=0 and B > 0. Thus B (resp. B') is (δ_1 , δ_2) Fγspo (resp. (δ_1, δ_2) F γ spc) but not (δ_1, δ_2) Fspo (resp. (δ_1, δ_2) Fspc).

Remark 3.31: Thus, we have that(δ_i, δ_j) F γ spo sets are weaker than the concepts of (δ_i, δ_j) Fsso, (δ_i, δ_j) Fso, (δ_i, δ_j) Fpo, (δ_i, δ_j) F γ o, (δ_i, δ_j) F γ so,(δ_i, δ_j) F γ po and (δ_i, δ_j) Fspo set. This is shown in the figure.



Theorem 3.32: Let (X, δ_i, δ_j) be a F-bts and A be a fuzzy set of X. Then A is (δ_i, δ_j) Fγspo if and only if for each fuzzy point $x_\beta \in$ A there exists a (δ_i, δ_j) Fγspo set H such that $x_\beta \in$ H $\leq$ A.

Proof : Suppose A is a (δ_i, δ_j) Fγspo set. By Theorem 3.3, there exists a (δ_i, δ_j) Fγpo set H such that H $\leq$ A. Now take H = A for every $x_\beta \in$ A. Then H is (δ_i, δ_j) Fγspo.

Conversely, suppose for every fuzzy point $x_\beta \in$ A, there exists a (δ_i, δ_j) Fγspo set H_β such that $x_\beta \in H_\beta \leq$ A. Then $\{H_\beta\}$ is a collection of (δ_i, δ_j) Fγspo sets such that, for every $x_\beta \in$ A, $x_\beta \in H_{\beta i} \leq$ A, $\beta_i \in \Delta$ and further $\cup_{\beta i \in \Delta} H_{\beta i}$ = A. Now, $H_{\beta i}$ is (δ_i, δ_j) Fγspo which implies that $\cup_{\beta i \in \Delta} H_{\beta i}$ = A is (δ_i, δ_j) Fγspo.

4.(δ_i, δ_j) FUZZY-γ-SEMI PREINTERIOR AND (δ_i, δ_j) FUZZY-γ-SEMI PRECLOSURE

Definition 4.1: . Let A be a fuzzy set of a F-bts (X, δ_i, δ_j). Then the (δ_i, δ_j) γ- semi preclosure ($(\delta_i, \delta_j)\gamma$-spcl, for short) and (δ_i, δ_j) γ-semi preinterior ($(\delta_i, \delta_j)\gamma$-spint, for short) of A are defined as
(i) (δ_i, δ_j) γ - sp cl (A) = $\wedge$ { B : B is (δ_i, δ_j) F-γ-sp closed and A $\leq$ B}.(ii) (δ_i, δ_j) γ - sp int (A) = $\vee$ { B : B is (δ_i, δ_j) F-γ-sp open and B $\leq$ A}.

Proposition 4.2: Let A be a fuzzy set of a F-bts (X, δ_i, δ_j). Then

(a) (δ_i, δ_j) γ-sp cl (A′) = ((δ_i, δ_j) γ-sp int (A))′.
(b) (δ_i, δ_j) γ-sp int (A′) = ((δ_i, δ_j) γ-sp cl (A))′.

Proof: Follows from the Definition 4.1.

Definition 4.3: Let (X, δ_i, δ_j) be a fuzzy bitopological space and x_β is a fuzzy point of X. A fuzzy set A of X is called

(a) (δ_i, δ_j) F-γ semi pre neighbourhood (briefly, (δ_i, δ_j) F-γ-semi pre nbhd) of $x_\beta$if there exists a (δ_i, δ_j) F-γ-spo set O such that $x_\beta \in$ O $\leq$ A.
(b) (δ_i, δ_j) F-γ semi preq neighbourhood (briefly, (δ_i, δ_j) F-γ-semi pre q nbhd) of $x_\beta$if there exists a (δ_i, δ_j) F-γ-spo set O such that x_βq O $\leq$ A.

A. Properties of (δ_i, δ_j) F-γ-Semi preinterior and (δ_i, δ_j) F-γ-Semi preclosure Operators

Theorem 4.4: Let (X, δ_i, δ_j) be a F-bts.Then for any fuzzy sets A and B of X,

(a) (δ_i, δ_j) γ-sp int($\tilde{0}$) = $\tilde{0}$ and (δ_i, δ_j) γ-sp int($\tilde{1}$) = $\tilde{1}$.
(b) δ_i int(A) $\leq$ (δ_i, δ_j) γ-sp int(A) $\leq$ A.
(c) A is (δ_i, δ_j) F-γ spo if and only if A = (δ_i, δ_j) γ-sp int(A).
(d) $(\delta_i, \delta_j)\gamma$-sp int(A) is (δ_i, δ_j)F-γ-spo set and $(\delta_i, \delta_j)\gamma$-sp int ($(\delta_i, \delta_j)\gamma$-sp int(A)) = $(\delta_i, \delta_j)\gamma$-sp int(A).
(e) If A $\leq$ B, then (δ_i, δ_j) γ-sp int (A) $\leq$ (δ_i, δ_j) γ-sp int (B).

Proof: (a), (b) and (c) follow from the Definition 4.1.(d) From Definition 4.1 it follows that(δ_i, δ_j) γ-sp int (A) is (δ_i, δ_j)F-γ-spo. From (c) other result holds.

(e) Let A $\leq$ B. From (b) we have, (δ_i, δ_j) γ-sp int (A) $\leq$ A $\leq$ B. By (d), (δ_i, δ_j) γ-sp int (A) $\leq$ (δ_i, δ_j) γ-sp int (B).

Proposition 4.5: Let (X, δ_i, δ_j) be a F-bts and A and B be any two fuzzy sets of X. Then

(a) $(\delta_i, \delta_j)\gamma$-sp int (A $\wedge$ B) = (δ_i, δ_j) γ-sp int (A) $\wedge$ (δ_i, δ_j) γ-sp int (B).
(b) (δ_i, δ_j) γ-sp int (A $\vee$ B) $\geq$ (δ_i, δ_j) γ-sp int (A) $\vee$ (δ_i, δ_j) γ-sp int (B).

Proof: (a) By Theorem 4.4, (δ_i, δ_j) γ-sp int (A $\wedge$ B) $\leq$ (δ_i, δ_j) γ-sp int (A), (δ_i, δ_j) γ-sp int (A $\wedge$ B) $\leq$ (δ_i, δ_j) γ sp int (B). Then, (δ_i, δ_j) γ-sp int(A $\wedge$ B) $\leq$ (δ_i, δ_j) γ-sp int(A) $\wedge$ (δ_i, δ_j) γ-sp int(B).
Let C $\in$ [$(\delta_i, \delta_j)\gamma$-sp int (A) $\wedge$ (δ_i, δ_j)γ-sp int (B)]. Then C is a (δ_i, δ_j) F-γ spo set and C $\leq$ A $\wedge$ B.
Then C $\leq$ (δ_i, δ_j) γ-sp int (A $\wedge$ B). Thus, [(δ_i, δ_j) γ-sp int(A) $\wedge$ (δ_i, δ_j) γ-sp int (B)] $\leq$ $(\delta_i, \delta_j)\gamma$ -sp int(A$\wedge$B). Hence, (δ_i, δ_j) γ-sp int (A $\wedge$ B) = (δ_i, δ_j) γ-sp int (A) $\wedge$ (δ_i, δ_j) γ-sp int (B).
(b) By Theorem 4.4, (δ_i, δ_j) γ-sp int (A$\vee$ B) $\geq$ (δ_i, δ_j) γ-sp int (A) $\vee$ (δ_i, δ_j) γ-sp int (B).

Remark 4.6: Equality does not hold in Proposition 4.5, which is shown by the example below.

Example 4.7: Let (X, δ_1, δ_2) be a F-bts with X={a, b, c}, δ_1 ={$\tilde{0}$, $\tilde{1}$,A}, δ_2 ={$\tilde{0}$, $\tilde{1}$, B, C} and fuzzy sets A={$a_{0.5}$, $b_{0.7}$, $c_{0.4}$}, B ={$a_{0.4}$, $b_{0.2}$, $c_{0.5}$}, C ={$a_{0.4}$, $b_{0.3}$, $c_{0.5}$}. Then (δ_1, δ_2) Fγo set = {$\tilde{0}$, $\tilde{1}$, A, B} and (δ_1, δ_2) F-γ-spo set = {$\tilde{0}$, $\tilde{1}$, A, B, C}. Let D ={$a_{0.5}$, $b_{0.8}$, $c_{0.3}$} then C $\vee$ D = {$a_{0.5}$, $b_{0.8}$, $c_{0.5}$}. Here D and C $\vee$ D are not (δ_1, δ_2) F-γ-spo.

Now (δ_1, δ_2) γ-sp int (C ∨ D) = A ∨ B ∨ C ={$a_{0.5}$, $b_{0.7}$, $c_{0.5}$}, (δ_1, δ_2) γ-sp int(C) = C and (δ_1, δ_2) γ-sp int(D) = 0. Thus, (δ_1, δ_2) γ-sp int (C) ∨ (δ_1, δ_2) γ-sp int (D) = C. Hence (δ_1, δ_2) γ-sp int (C ∨ D) ≥ (δ_1, δ_2) γ-sp int(C) ∨ (δ_1, δ_2) γ-sp int(D).

Theorem 4.8: Let *(X, δ_i, δ_j)* be a F-bts. Then for fuzzy sets A and B of X, the following holds:

(a) (δ_i, δ_j) γ-sp cl($\tilde{0}$) = $\tilde{0}$ and (δ_i, δ_j) γ-sp cl($\tilde{1}$) = $\tilde{1}$.
(b) A ≤ (δ_i, δ_j) γ-sp cl(A) ≤ δ_i cl(A).
(c) A is (δ_i, δ_j) F-γ spc if and only if A = (δ_i, δ_j) γ-sp cl(A).
(d) (δ_i, δ_j) γ-sp cl(A) is (δ_i, δ_j) F-γ-spc set and (δ_i, δ_j) γ-sp cl((δ_i, δ_j) γ-sp cl(A)) = (δ_i, δ_j) γ-sp cl(A).
(e) If A ≤ B, then (δ_i, δ_j) γ-sp cl(A) ≤ (δ_i, δ_j) γ-sp cl(B)

Proof: Follows from the Definition 4.1.

Proposition 4.9: Let *(X, δ_i, δ_j)* be a F-bts and A and B be any two fuzzy sets of X. Then

(a) (δ_i, δ_j) γ-sp cl(A ∨ B) = (δ_i, δ_j) γ-sp cl(A) ∨ (δ_i, δ_j) γ-sp cl(B).
(b) (δ_i, δ_j) γ-sp cl(A ∧ B) ≤ (δ_i, δ_j) γ-sp cl(A) ∧ (δ_i, δ_j) γ-sp cl(B).

Proof: Follows by taking the complement from the relations of Proposition 4.5.

Remark 4.10: Equality need not hold in Proposition 4.9 which is illustrated as follows.

Example 4.11: Let (X, δ_1, δ_2) be a F-bts with X ={a, b, c}, δ_1={$\tilde{0}$, $\tilde{1}$, A}, δ_2= {$\tilde{0}$, $\tilde{1}$, B, C} with fuzzy sets A={$a_{0.4}$, $b_{0.6}$, $c_{0.3}$}, B ={$a_{0.3}$, $b_{0.4}$, $c_{0.2}$}, C ={$a_{0.4}$, $b_{0.4}$, $c_{0.5}$}. Here (δ_1, δ_2)F-γspc sets = {$\tilde{0}$, $\tilde{1}$, A′, B′, C′}.

Let D ={$a_{0.7}$, $b_{0.6}$, $c_{0.2}$} and so C′ ∧ D′ = {$a_{0.3}$, $b_{0.4}$, $c_{0.5}$}. Then (δ_1, δ_2) γ-sp cl (C′ ∧ D′) = A′ ∧ C′ = {$a_{0.6}$, $b_{0.4}$, $c_{0.5}$}, (δ_1, δ_2) γ-sp cl(C′) = C′, (δ_1, δ_2) γ-sp cl(D′) = B′ and C′∧B′ = C′. Thus, (δ_i, δ_j) γ-sp cl(C′ ∧ D′) ≤ (δ_i, δ_j) γ-sp cl(C′) ∧ (δ_i, δ_j) γ-sp cl(D′).

5. CONCLUSION

In this paper, the notions of (δ_i, δ_j) F-γ-semi-preopen and (δ_i, δ_j) F-γ-semi-preclosed sets in fuzzy bitopological spaces are introduced and their properties are discussed along with examples and counter examples and also their relationship with other sets are studied.

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