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SUPER VERTEX GRACEFUL GRAPHS UNDER CERTAIN OPERATIONS

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Abstract
In this paper, an analysis is made on Jelly fish, merge graphs, Shells and $K_{1,n}@2P_m$ under super vertex graceful labelling using different operations.

Keywords: Jelly fish, merge graphs, Shells and $K_{1,n}@2P_m$, graceful graphs, super vertex graceful graphs.

2010 Mathematics Subject Classification: 05C78.

1. INTRODUCTION

Graph labeling is one of the important areas of research in graph theory in which the problems are concerned with the interaction between suitably defined vertex and edge labeling. The first bonafide labeling problem is known as band width problem which is concerned with minimizing the difference between the labels of adjacent vertices. The graph labeling has applications in Network Addressing, Circuit Layout and Design of Error-correcting codes subject to minimizing the maximum error. A systematic study of various applications of graph theory is carried out in Bloom and Golomb [2]. Super vertex graceful labeling was introduced by Sin Min Lee, Elo Leung and Ho Kuen Ng [8] and they made an analysis on unicyclic graphs under the mapping. N. Murugesan and R. Uma [6,7] have studied super vertex graceful mapping on paths, square graphs, firecracker graphs, spider graphs etc. In this paper, an analysis is made on Jelly fish, merge graphs, Shells and $K_{1,n}@2P_m$.

2. PRELIMINARIES

2.1 Super vertex graceful labeling

A graph G with p vertices and q edges , vertex set $V(G)$ and edge set $E(G)$, is said to be super vertex graceful (SVG), if there exists a function pair (f, f^+) where f is a bijection from $V(G)$ onto P, f^+ is a bijection from $E(G)$ onto Q, such that $f^+(u,v) = f(u) + f(v)$ for any $(u,v) \in V(G)$, where P and Q are finite set of integers defined as follows:

$$P = \begin{cases} \pm1, \pm2, \dots, \pm\frac{p}{2} \text{ if } p \text{ is even} \\ 0, \pm1, \pm2, \dots, \pm\frac{p-1}{2} \text{ if } p \text{ is odd} \end{cases} \quad \text{and} \quad Q = \begin{cases} \pm1, \pm2, \dots, \pm\frac{q}{2} \text{ if } q \text{ is even} \\ 0, \pm1, \pm2, \dots, \pm\frac{q-1}{2} \text{ if } q \text{ is odd.} \end{cases}$$

2.1.1 Examples

In the graph given in fig 2.1, both the size and order is 5. Therefore $P = Q = \{ -2,-1,0,1,2\}$. Here f^{+} and f are defined such that f^{+} (-1,1)= 0; f^{+} (-1,2) = 1; f^{+} (0,2) = 2; f^{+} (1,-2)= -1; f^{+} (-2,0) = -2.Then G is SVG. It is interesting to note that C_5 is SVG. Bloom and Golomb [2], discussed that C_5 is not graceful.

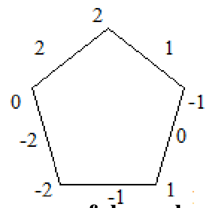

Figure 2.1: A super vertex graceful graph with order and size 5.

2.2 Jelly Fish

The Jelly fish graph $J(m,n)$ is obtained by joining a 4-cycle whose vertices are v_1, v_2, v_3, v_4 with vertices v_1 and v_3 defined by an edge and appending m pendent edges to v_2 and n pendent edges to v_4. V.Lakshmi Alias Gomathi, A. Nellaimurugan and A. Nagarajan [10] have proved that the jelly fish is elegant graph. Also, S. Sriram, R. Govindarajan [12], have shown that the jelly fish satisfies 1- near cordial labeling.

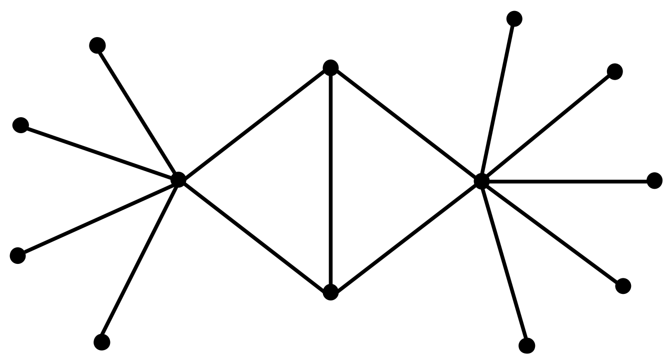

Figure 2.2: Jelly fish graph J(4, 5).

2.3 Merge graph:

A merge graph $G_1 * G_2$ can be formed from two graphs G_1 and G_2 by merging a node of G_1 with a node of G_2 . Merge of $K_{1,n}$ with $K_{2,n}$ is depicted in the figure. Merging of two graphs satisfy super edge – magic labelling, if one of the graph is star and the other is a tree with 5 vertices or 3 vertices and this was proved by A. Solaoraju and R. Raziabegam [11].

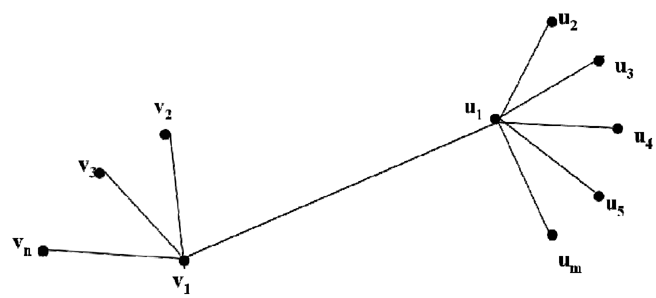

Figure 2.3: Merge graph of $S_n * S_m$.

2.4 Shell:

A shell graph is a cycle C_n with $(n-3)$ chords sharing a common end points called the apex. Shell graphs are denoted as $[C(n, n-3)]$.

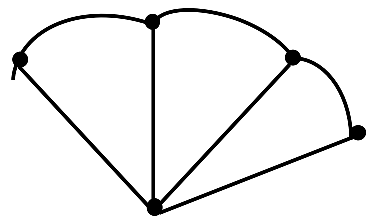

Figure 2.4: The Shell graph [C(4,4-4)].

2.5 $K_{1,n}@2P_m$: The graph $K_{1,n}@2P_m$ means that 2 copies of the path of length m is attached with each pendent vertex of $K_{1,n}$.

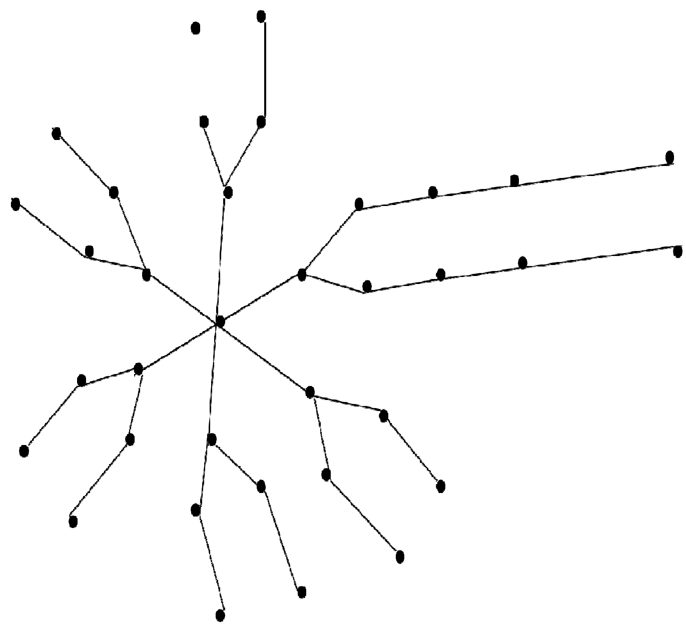

Figure 2.5: $K_{1,n}@2P_m$

3. MAIN RESULTS

3.1 Theorem:
Jelly fish $J(m,n)$ is super vertex graceful if $m = n = 2k\ , k > 1$.

Proof:
Let $G = \{V, E, f\}$ be a Jelly fish $J(m,m)$ of order and size $p = 2m+4$ and $q = 2m+4-1$. Let $V = \{v_1, v_2, v_3, v_4, x_i, y_i\}$ be the set of vertices where v_i's are vertices of the cycle, x_i's and y_i's are the pendant vertices adjacent to v_1 and v_3 respectively. Then $E = \{E_1, E_2, E_3\}$ where, $E_1 = \{e_{ij} = (v_i, v_j),\ E_2 = \{e_{1j} = (v_1, x_i)\}$ and $E_3 = \{e_{3j} = (v_3, y_i)\}$ as shown in the figure below. The function $f: V \to P$ induces super vertex graceful map $f^+: V \times V \to Q$ if "f" , P, Q are defined as given below: $P = \{\pm 1, \pm 2, \pm 3, \dots, \pm m+2\}, Q =$

$\{0, \pm 1, \pm 2, \pm 3, \dots, \pm m+2\}$; $f(v_1) = \frac{p}{2}, f(v_2) = \frac{p}{2} - 1, f(v_3) = -f(v_1), f(v_4) = -f(v_2); f(x_n) = \frac{p}{2}, f(x_i) = i+1, i = 1, 2, \dots, \frac{p}{2} - 2;\ f(y_i) = -f(x_i),\ i = 1, 2, \dots, \frac{p}{2} - 2$.

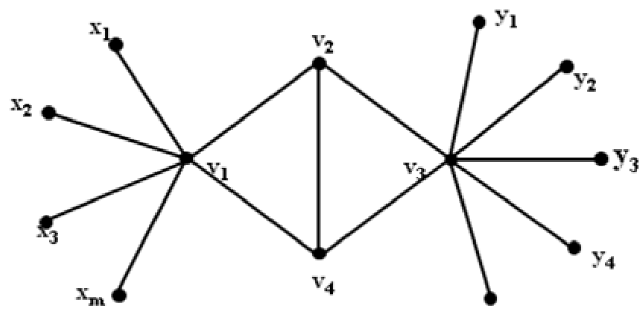

Figure 3.1: Jelly fish graph $J(m, m)$.

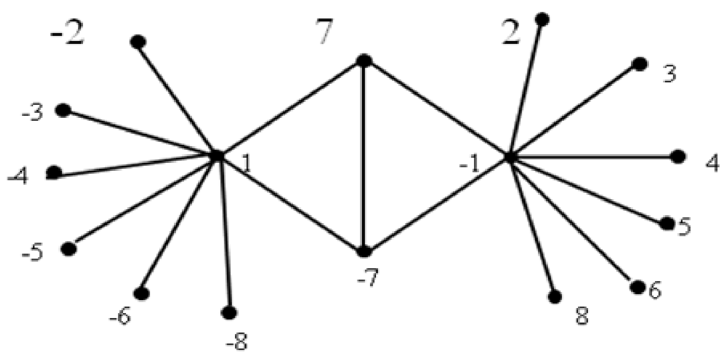

Figure 3.2: Super vertex graceful Jelly fish $J(6, 6)$.

3.2 Example: The graph given in the figure 3.2 is of order 16 and size 16. Then $P = \{\pm 1, \pm 2, \dots, \pm 8\}$, $Q = \{\pm 1, \pm 2, \dots, \pm 8\}$, $f(v_1) = 1, f(v_2) = 7, f(v_3) = -1, f(v_4) = -7$, $f(x_1) = 2, \dots, f(y_6) = -8$ satisfies the functions given in the above theorem leading to $J_{6,6.}$

3.3 Theorem:
The Merge of $S_n * S_{n-1}$ is Super vertex graceful.

Proof: Let $G = \{V, E, f\}$ is a graph obtained by merging the apex vertex of star graph S_{n-1} with one of the pendant vertex of star graph S_n. Then the order of G is '$2n$' and size is '$2n$-1', where the order of S_n is 'n' with vertex set $\{v_1, v_2, v_{3,\dots,} v_n\}$ and the order of S_{n-1} is 'n' with vertex set $\{u_1, u_2, u_{3,\dots,} u_{n-1}\}$. The apex vertices of S_n and S_{n-1} are v_1 and u_1 respectively. The edge set of S_n is $e_{ij} = (v_i, v_j)$ and $S_n - 1$ is $e_{ij} = (u_i, u_j)$. The vertex u_1 of S_{n-1} is merged with the vertex v_2 of S_n as shown in the figure and the edge (u_1, v_2) is the edge $e_{12} = (v_1, v_2)$. By the definition of the super vertex graceful labeling, 'f' is a mapping from $V \times V \to P$ where P is the label of vertices defined in $P = \{\pm 1, \pm 2, \dots, \pm n\}$. The mapping "$f$" defined below admits the induced labeling $f^+ : V \times V \to Q$ as super vertex mapping, where $Q = \left\{\pm 1, \pm 2, \dots, \pm \frac{2n-1}{2}\right\}; f(v_1) = 1; f(u_1) = -1; f(v_i) = -i$ and $f(u_i) = -f(v_i)$ for $i = 2, 3, \dots, n-1$. Hence the graph is super vertex graceful graph.

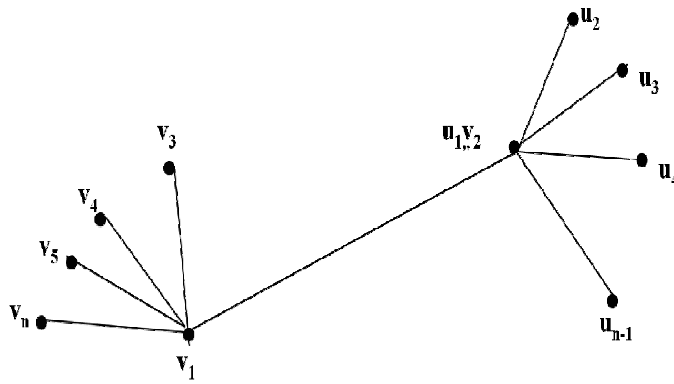

Figure 3.3: Merge graph of $S_n * S_{n-1}$

3.4 Example:

The graph given in the figure is the merge of S_5 and S_4. The order of the graph is 10 and size is 9. Then by the definition of super vertex labeling $P = \{\pm1, \pm2, \pm3, \pm4, \pm5\}$ and $Q = \{0, \pm1, \pm2, \pm3, \pm4\}$. Then $f(v_1) = 1, f(v_1) = 1, f(u_1) = 1, f(v_2) = -2, f(u_2) = -2, \dots, f(v_5) = 5, f(u_5) = -5$. Thus, $f^+(v_1, u_1) = 0$ and similarly f^+ induces the edge labels in the set Q. Hence the graph is super vertex graceful.

3.5 Theorem:

The graph $K_{1,n}@2P_m$ is super vertex graceful for all values of m only if $n = 2$.

Proof:

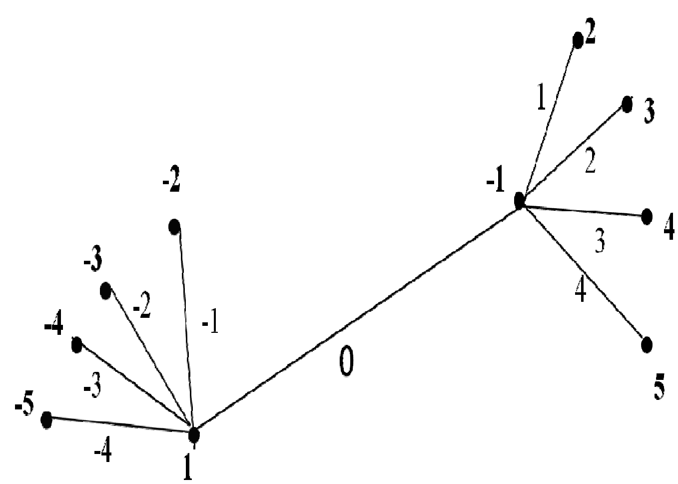

Figure3.4: Super vertex graceful $S_5 * S_{5-1}$.

Let $G = K_{1,n}@2P_m$ be a graph. By the definition of $K_{1,n}@2P_m$ where, the vertex set $V = \{y, x_1, x_2, \dots x_n, \; u_{11}, u_{12}, \dots u_{1m}, \; u_{21}, \dots u_{nm}, \; v_{11}, v_{12}, \dots v_{1m}, v_{21}, \dots v_{nm}\}$. Let $'x'$ be the apex vertex of $K_{1,n}$ and $x_1, x_2, \dots, x_n$ be the adjacent vertices of x. u_{ij} be the j^{th} vertex of the path incident with the vertex x_j and similarly v_{ij} be the j^{th} vertex of the another copy of the path incident with the vertex x_j. By the definition of super vertex graceful definition the vertex label P and edge set label Q are defined as $P = 0, \pm 1, \pm 2, \ldots, \frac{n}{2}$ and $Q = \pm 1, \pm 2, \ldots, \frac{n}{2}$. Now, let us define the map $f: V \times V \to P$ as follows:

$f(y) = 0; f(x_1) = \frac{n}{2}; f(x_2) = -\frac{n}{2},\ f(u_{1,1}) = 1, f(v_{1,1}) = m + 1$

$f(u_{1,2i}) = -(f(x_1) + (2i - 2)), i = 1, 2, \ldots, m,\ f(u_{1,2i+1}) = f(u_{1,2i-1}) + 1,\ i = 1, 2, \ldots, m$

$f(v_{1,2i+1}) = f(v_{1,2i-1}) + 1, i = 1, 2, \ldots, m,\ f(v_{1,2i}) = f(v_{1,2i-1}) - 1, i = 1, 2, \ldots, m.$

Then the map $f^+: V \times V \to E$ defined by $f(e_{ij}) = f(y_i, y_j) = f(y_i) + f(y_j)$ is a bijective map, resulting G to be a Super vertex graceful graph.

3.6 Example:

Let us consider $G = K_{1,2}@2P_4$ as an example as shown in the figure below. By the definition, the order $p = 2(2 \times 4) + 2 + 1 = 19$ and $q = 18$. Then by the definition of Super vertex graceful map $f: V \times V \to P$. Let y be the apex vertex and $\{x_1, x_2\}$ be adjacent to the apex vertex and $\{u_{11}, u_{12}, u_{13}, v_{11}, v_{12}, v_{13}, \ldots\}$ are the vertices of the path that is incident with the vertex x_1 and similarly the vertices $\{u_{21}, v_{21}\}$. Then $f(y) = 0; f(x_1) = -9$, $f(x_2) = 9$. The graph given below depicts the labels.

3.7 Remark:

The shell graph $[C(n, n-3)]$ is super vertex graceful only when $n = 5$ and $n = 7$ where n represents the number of vertices of the cycle C_n.

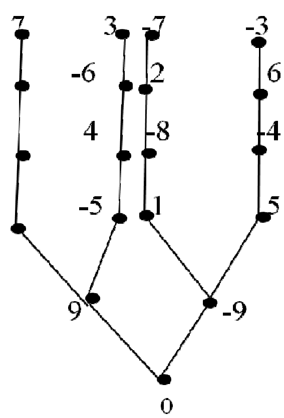

Figure 3.5: Super vertex graceful $K_{1,2}@2P_4$.

The labeling of $[C(n, n-3)]$ for $n = 5$ and $n = 7$ is given in the following figures. For all other values of n, the shell graphs are not super vertex graceful because the apex vertex is adjacent to all the vertices and if degree of vertex is $n - 5$ and the size is even, there does not exist a super vertex graceful map.

3.8 Example:

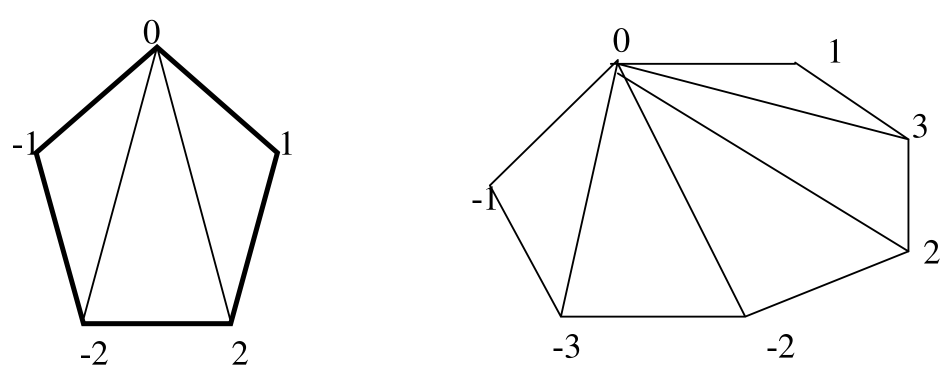

Figure 3.6: Super vertex graceful $[C(5, 5-3)]$, $[C(7, 7-3)]$.

4. DISCUSSION

The labeling of graphs depends on the degree of vertices. This has been analyzed in various research articles. In future, the necessary and sufficient condition for a graph to be super vertex graceful can be obtained.

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