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THE TRIPLE AND QUADRUPLE COMPLETE PARTITIONS OF INTEGERS

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Abstract:
Complete partitions are introduced by S.K. Park [6] and the representation of a positive integer in terms of a sum of smaller numbers with certain conditions has been developed by Mac Mahon [5] in perfect partitions. This paper presents the concepts of complete partition, double complete partition and an attempt is made for the triple and quadruple complete partitions of integers.

Keywords: Partitions, Complete Partitions, Double complete partitions.

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1. INTRODUCTION

The theory of partitions is an area of additive number theory, a subject concerning the representation of integers as sums of other integers [1, 2]. The partition function , denoted by $p(n)$ (see, [3,4,5]) is defined as the number of ways that the positive integer n can be written as a sum of positive integers, as in $n = a_1 + a_2 + ... + a_r$. Mac Mahon [5] studied perfect partitions of n which are partitions of n such that every integer m with $1 \le m \le n$ is uniquely represented in one and only one way. A partition of a positive integer n is a finite non-decreasing sequence $\mu = (\mu_1, \mu_2, ..., \mu_k)$ such that $\sum_{i=1}^{k} \mu_i = n$ and $\mu_i > 0$ for all $i = 1, 2, ..., k$. The μ_i are called the parts of the partition and k is called the length of the partition. We sometimes write $\mu = (1^{m_1} 2^{m_2} ....)$, which means there are exactly m_i parts equal to i in the partition μ. A complete partition [6] of an integer n is a partition $\mu=(\mu_1,\mu_2,....,\mu_k)$ of n, with $\mu_1=1$, such that each integer $i, 1\le i\le n$, can be represented as a sum of elements of $\mu_1,\mu_2,...,\mu_k$. In other words, each i can be expressed as $\sum_{j=1}^{k}\beta_j\mu_j$, where β_j is either 0 or 1.

A double complete partition [7] of an integer n is a partition $\mu=(\mu_1^{m_1}\mu_2^{m_2}...\mu_l^{m_l})$ of n such that each integer m, with $2\le m\le n-2$ can be represented by at least two different ways as a sum $\sum_{i=1}^{l}\beta_i\mu_i$ with $\beta_i\in\{0,1,2,....m_l\}$.

Now, we define the triple and quadruple complete partitions of integers.

2. TRIPLE AND QUADRUPLE COMPLETE PARTITIONS

Definition 2.1: For any integer $n\ge 8$, the *triple complete partition* of an integer n is a partition $\mu=(\mu_1^{m_1}\mu_2^{m_2}...\mu_l^{m_l})$ of n such that each integer m, with $3\le m\le n-3$ can be represented *at least in three different ways as a sum* $\sum_{i=1}^{l}\beta_i\mu_i$ with $\beta_i\in\{0,1,2,....m_l\}$.

Definition 2.2: For any integer $n\ge 11$, the *quadruple complete partition* of an integer n is a partition $\mu=(\mu_1^{m_1}\mu_2^{m_2}...\mu_l^{m_l})$ of n such that each integer m, with $4\le m\le n-4$ can be represented *at least in four different ways as a sum* $\sum_{i=1}^{l}\beta_i\mu_i$ with $\beta_i\in\{0,1,2,....m_l\}$.

Theorem 2.3: If a partition $\mu=(\mu_1^{m_1}\mu_2^{m_2}...\mu_l^{m_l})$ of a positive integer $n\ge 8$ is a triple complete partition then $\mu_{i+1}\le\sum_{j=1}^{i}m_j\,\mu_j-2$ with $i\ge 2$ and μ should have at least three 1's, one 2 and one 3 (or) one 1, three 2's and one 3 as its parts.

Proof : For any integer n, its triple complete partition can be obtained by taking the value as $n\ge 8$, and the parts of the integer n should be equivalent to $(\mu_1^{m_1}\mu_2^{m_2}...\mu_l^{m_l})$. We can prove this theorem by considering the parts of the integer as $n=\mu_1{}^{m_1}\mu_2{}^{m_2}\mu_3{}^{m_3}$. That is, $n=1^{m_1}2^{m_2}3^{m_3}$ with $m_1\ge 3, m_2, m_3\ge 1$ and $\mu_3\le m_1+m_2$ is a triple complete partition of the integer $n=m_1\mu_1+m_2\mu_2+m_3\mu_3$. If it is a triple complete partition, then for every integer r, $3\le r\le\sum_{j=1}^{i}m_j\mu_j-3$ can be written in three different ways using the parts $1,2$ and 3. Therefore, $m_1\mu_1$, $m_2\mu_1+m_3\mu_3$ and $m_3\mu_3$ are the three representations of n with μ satisfies the condition $\mu_{i+1}\le\sum_{j=1}^{i}m_j\mu_j-2$. Now we check the condition $\mu_{i+1}\le\sum_{j=1}^{i}m_j\mu_j-2$ for n. Let us assume that

$n=\mu_1^{m_1}\mu_2\mu_3$ and $n=\mu_1\mu_2{}^{m_2}\mu_3$ be a triple complete partitions of n with $m_1=m_2=m_3=3$ and

$$\mu_1=1,\mu_2=2,\mu_3=3 \qquad (1)$$

If we take $n = \mu_1^{m_1}\ \mu_2\ \mu_3$ is a triple complete partition then it should satisfy the condition

$\mu_{i+1} \le \sum_{j=1}^{i} m_j\ \mu_j - 2$ with $i \ge 2$. Here $n = 8$ by equation (1) and $3 \le 3$. Therefore, μ satisfies the condition. If we take $n = \mu_1 \mu_2^{m_2} \mu_3$ then by equation (1) $n = 10$ and $3 \le 5$. If it is true for $n = \mu_1^{m_1}\mu_2^{m_2}\mu_3^{m_3}$ then it is also true for $n = (\mu_1^{m_1}\mu_2^{m_2}...\mu_l^{m_l})$. Hence μ satisfies the condition

$$\mu_{i+1} \le \sum_{j=1}^{i} m_j \mu_j - 2.$$

Corollary 2.4 : Let $\mu = (\mu_1\mu_2...\mu_l)$ be a triple complete partition of a positive integer n. Then

$$\mu_{i+1} \le \sum_{j=1}^{i} 2^{j-1}\mu_j.$$

Proof : For a triple complete partition, $n \ge 8$ and μ_1, μ_2 and μ_3 should be equivalent to $1, 2$ and 3 respectively.

$$\mu_{i+1} \le \mu_1 + \mu_2 + ... + \mu_j \le 2\mu_1 + 2\mu_2 + ... + 2\mu_j \le 2^{j-1}\mu_1 + 2^{j-1}\mu_2 + ... + 2^{j-1}\mu_j \le \sum_{j=1}^{i} 2^{j-1}\mu_j.$$

3. CONCLUSION

From the concept of complete partition an attempt has been made here for the triple and quadruple complete partitions of integers. This may be extended upto $k-$tuple complete partitions of integers.

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