

SOLVING A SYSTEM OF NONLINEAR FRACTIONAL DIFFERENTIAL EQUATIONS USING MAHGOUB ADOMIAN DECOMPOSITION METHOD

S. Katrin Lydia*

Author Affiliation:

Ph. D. Research Scholar, Department of Mathematics, Bishop Heber College, Trichy, Tamil Nadu 620017, India.

***Corresponding Author:**

S. Katrin Lydia, Ph. D. Research Scholar, Department of Mathematics, Bishop Heber College, Trichy, Tamil Nadu 620017, India

E-mail:katrinlydia1987@gmail.com

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Abstract:

In this article, MahgoubAdomian Decomposition method is developed to obtain an approximate solution of the nonlinear system of Fractional Differential Equations. This method combines Mahgoub transform and Adomian decomposition method. Here, the nonlinear terms have been handled with the help of Adomian polynomials. The fractional derivatives are described in the Caputo sense. Some examples are given to demonstrate that our newly method is very efficient and accurate.

Keywords: Mahgoub Transform, Adomian Decomposition method, Fractional Differential Equations, Caputo derivative.

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1. INTRODUCTION

Fractional differential equations (FDEs) are generalization of integer order differential equations to arbitrary non-integer orders. Nowadays, FDEs have attracted many scientists and engineers because they have been applied in various fields such as mechanics, signal processing, image processing, bioengineering, control engineering, viscoelasticity and polymer networks [1]. Many mathematicians have begun to show much more attention in finding the numerical solution of linear and nonlinear Fractional Differential Equations. Some of the methods are Homotopy Analysis Method [2], Differential Transform Method [3], and Adomian Decomposition Method [4]. Several new integral transform methods have been proposed by many researchers to find the analytical solution of linear FDEs. Some of them are Sumudu [5], Elzaki [6], Laplace [7], Mahgoub [8] and Natural [9]. For solving nonlinear system of FDEs, the Adomian decomposition method was combined, with Sumudu transform method [10], with Elzaki transform method [11], with Laplace transform method [12], with Mahgoub transform method [13] and with Natural transform method [14].

In this paper, the Mahgoub Adomian Decomposition method (MADM) has been developed for finding the approximate solution of nonlinear system of FDEs with Caputo derivatives. This paper is organized as follows: Section 2 consists of basic definitions of fractional calculus and Mahgoub transform of fractional derivatives.

Section 3 constructs the MADM for finding the approximate solutions for nonlinear system of fractional differential equations. Section 4 gives some examples of FDEs to show the efficiency of this method.

2. PRELIMINARIES AND NOTATIONS

In this section, the fundamental definitions of fractional calculus, Mahgoub transform and Mahgoub transform of fractional derivatives are given which are used in this paper.

Definition 2.1: A real function $f(t)$, $t > 0$ is said to be in the space C_μ , $\mu \in \mathbb{R}$ if there exists a real number $p > \mu$ such that $f(t) = t^p f_1(t)$ where $f_1(t) \in C[0, \infty)$ and it is said to be in the space C_μ^m if and only if $f^{(m)} \in C_\mu$, $m \in \mathbb{N}$.

Definition 2.2: The Riemann-Liouville fractional integral operator of order $\alpha \geq 0$, of a function $f \in C_\mu$, $\mu \geq -1$, is defined as

$$J^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-x)^{\alpha-1} f(x) dx, \alpha > 0, t > 0,$$

$$J^0 f(t) = f(t) \quad (2.1)$$

Definition 2.3: The fractional derivative of $f(t)$ in the Caputo sense is defined as

$${}^c D^{-\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-x)^{m-\alpha-1} f^{(m)}(x) dx. \quad (2.2)$$

form $-1 < \alpha \leq m$, $m \in \mathbb{N}$, $t > 0$ and $f \in C_{-1}^m$

Definition 2.4: Mahgoub transform is defined on the set of continuous functions and exponential order. We consider functions in the set A defined by

$$A = \left\{ f(t): |f(t)| < P e^{\frac{|t|}{\epsilon_i}} \text{ if } t \in (-1)^i \times [0, \infty), \quad i = 1, 2; \epsilon_i > 0 \right\}$$

where ϵ_1, ϵ_2 may be finite or infinite and the constant P must be finite.

Let $f \in A$, then the Mahgoub transform is defined as

$$M[f(t)] = H(u) = u \int_0^\infty f(t) e^{-ut} dt, \quad t \geq 0, \epsilon_1 \leq u \leq \epsilon_2 \quad (2.3)$$

Mahgoub transform of simple functions are given below:

$$\begin{aligned} (i) M[1] &= 1 \\ (ii) M[t] &= \frac{1}{u} \\ (iii) M[t^2] &= \frac{2}{u^2} \\ (iv) M[t^n] &= \frac{n!}{u^n} = \frac{\Gamma(n+1)}{u^n} \end{aligned}$$

The Mahgoub Transform for derivatives are:

$$\begin{aligned} (i) M[f'(t)] &= uH(u) - uf(0) \\ (ii) M[f''(t)] &= u^2 H(u) - u^2 f(0) - uf'(0) \\ (iii) M[f^n(t)] &= u^n H(u) - \sum_{k=0}^{n-1} u^{n-k} f^{(k)}(0) \end{aligned}$$

Theorem 2.5:

Let $f \in A$ and the Mahgoub transform $M[f(t)] = H(u)$ then,

$$\begin{aligned} \text{(i)} \quad & M[tf(t)] = -\frac{d}{du}H(u) + \frac{1}{u}H(u) = \left(-\frac{d}{du} + \frac{1}{u}\right)H(u) \\ \text{(ii)} \quad & M[t^2f(t)] = \frac{d^2}{du^2}H(u) - \frac{2}{u}\frac{d}{du}H(u) + \frac{2}{u^2}H(u) = \left(-\frac{d}{du} + \frac{1}{u}\right)^2 H(u) \\ \text{(iii)} \quad & M[t^n f(t)] = \left(-\frac{d}{du} + \frac{1}{u}\right)^n H(u) = \sum_{k=0}^n (-1)^{n-k} a_k^n \frac{1}{u^k} \frac{d^{n-k}}{du^{n-k}} H(u) \end{aligned}$$

where $a_k^n = na_{k-1}^{n-1}$, $a_0^n = 1$, $a_n^n = n!$

Theorem 2.6: [13]

Let $m \in \mathbb{N}$ and $\alpha > 0$ be such that $m-1 < \alpha \leq m$ and $H(u)$ be the Mahgoub transform of the function $f(t)$, then the Mahgoub transform of Caputo fractional derivative of $f(t)$ of order α is given by

$$M[{}_0^c D_t^\alpha f(t)] = u^\alpha H(u) - \sum_{k=0}^{m-1} u^{\alpha-k} f^{(k)}(0), \quad (2.4)$$

3. CONSTRUCTION OF MAHGOUB ADOMIAN DECOMPOSITION METHOD FOR FDEs

Considering the general nonlinear system of fractional differential equations of the form

$$\begin{aligned} {}^c D^\alpha x(t) + Rx(t) + Fx(t) &= g(t) \\ {}^c D^\beta y(t) + Ry(t) + Fy(t) &= h(t) \end{aligned} \quad \text{where } 0 < \alpha, \beta \leq 1 \quad (3.1)$$

subject to the initial conditions $x(0) = g(t)$ and $y(0) = h(t)$. (3.2)

where ${}^c D^\alpha x(t)$, ${}^c D^\beta y(t)$ are the Caputo fractional derivatives of the functions $x(t), y(t)$ respectively. R is the linear differential operator, F is the general nonlinear differential operator and $g(t), h(t)$ are the source terms. Applying the Mahgoub Transform and using Theorem 2.6 in (3.1) we get,

$$\begin{aligned} M(x(t)) &= \frac{1}{u^\alpha} \sum_{k=0}^{m-1} u^{\alpha-k} x^{(k)}(0) + \frac{1}{u^\alpha} M(g(t)) - \frac{1}{u^\alpha} M[Rx(t) + Fx(t)] \\ M(y(t)) &= \frac{1}{u^\beta} \sum_{k=0}^{m-1} u^{\beta-k} y^{(k)}(0) + \frac{1}{u^\beta} M(h(t)) - \frac{1}{u^\beta} M[Ry(t) + Fy(t)] \end{aligned} \quad (3.3)$$

Using initial conditions then (3.3) becomes

$$\begin{aligned} M(x(t)) &= g(t) + \frac{1}{u^\alpha} M(g(t)) - \frac{1}{u^\alpha} M[Rx(t) + Fx(t)] \\ M(y(t)) &= h(t) + \frac{1}{u^\beta} M(h(t)) - \frac{1}{u^\beta} M[Ry(t) + Fy(t)] \end{aligned} \quad (3.4)$$

Applying the inverse Mahgoub transform in (3.4) we obtain

$$\begin{aligned} x(t) &= G(t) - M^{-1} \left[\frac{1}{u^\alpha} M[Rx(t) + Fx(t)] \right], \\ y(t) &= H(t) - M^{-1} \left[\frac{1}{u^\beta} M[Ry(t) + Fy(t)] \right]. \end{aligned} \quad (3.5)$$

Noting that $G(t)$ and $H(t)$ are arising from the nonhomogeneous term and given initial conditions. Now we assume an infinite series solutions form

$$x(t) = \sum_{n=0}^{\infty} x_n(t), \quad y(t) = \sum_{n=0}^{\infty} y_n(t) \quad (3.6)$$

Using (3.6) rewriting (3.5) as follows:

$$\begin{aligned}\sum_{n=0}^{\infty} x_n(t) &= G(t) - M^{-1} \left[\frac{1}{u^{\alpha}} M[R \sum_{n=0}^{\infty} x_n(t) + F \sum_{n=0}^{\infty} A_n] \right] \\ \sum_{n=0}^{\infty} y_n(t) &= H(t) - M^{-1} \left[\frac{1}{u^{\beta}} M[R \sum_{n=0}^{\infty} y_n(t) + F \sum_{n=0}^{\infty} B_n] \right]\end{aligned}\quad (3.7)$$

where, A_n and B_n are Adomian polynomials representing the nonlinear terms $Fx(t)$ and $Fy(t)$, respectively. For the nonlinear functions the first few Adomian polynomials are given by

$$\begin{aligned}A_0 &= F(x_0), & B_0 &= F(y_0) \\ A_1 &= F^{(1)}(x_0)x_1, & B_1 &= F^{(1)}(y_0)y_1 \\ A_2 &= F^{(1)}(x_0)x_2 + \frac{1}{2!}F^{(2)}(x_0)x_1^2, & B_2 &= F^{(1)}(y_0)y_2 + \frac{1}{2!}F^{(2)}(y_0)y_1^2 \\ \vdots & & \vdots & \end{aligned}$$

By comparing both sides of (3.7) we get

$$\begin{aligned}x_0(t) &= G(t), y_0(t) = H(t), \\ x_1(t) &= -M^{-1} \left[\frac{1}{u^{\alpha}} M[Rx_0(t) + A_0] \right], & y_1(t) &= -M^{-1} \left[\frac{1}{u^{\beta}} M[Ry_0(t) + B_0] \right] \\ x_2(t) &= -M^{-1} \left[\frac{1}{u^{\alpha}} M[Rx_1(t) + A_1] \right], & y_2(t) &= -M^{-1} \left[\frac{1}{u^{\beta}} M[Ry_1(t) + B_1] \right] \\ \vdots & & \vdots & \end{aligned}$$

Continuing in this manner to get the general recursive relation,

$$\begin{aligned}x_{n+1}(t) &= -M^{-1} \left[\frac{1}{u^{\alpha}} M[Rx_n(t) + A_n] \right], n \geq 1 \\ y_{n+1}(t) &= -M^{-1} \left[\frac{1}{u^{\beta}} M[Ry_n(t) + B_n] \right], n \geq 1.\end{aligned}$$

4. ILLUSTRATIVE EXAMPLES

In this section, we present the illustrative examples of the system of fractional differential equations by implementing the proposed method in this article. The results for these examples demonstrate that the proposed methods are accurate, effective and convenient.

Example 4.1 Consider the system of nonlinear fractional differential equations of the form

$$\begin{aligned}D^{\alpha}x(t) &= \frac{1}{2}x(t) \\ D^{\beta}y(t) &= y(t) + x^2(t), & 0 < \alpha, \beta \leq 1\end{aligned}\quad (4.1)$$

Subject to the initial conditions, $x(0) = 1$; $y(0) = 0$ (4.2)

The exact solutions for the (4.1) are $x(t) = e^{\frac{t}{2}}$ and $y(t) = te^t$ for $\alpha = \beta = 1$.

Applying the Mahgoub transform on both sides in (4.1), we get

$$\begin{aligned}M(D^{\alpha}x(t)) &= \frac{1}{2}M(x(t)) \\ M(D^{\beta}y(t)) &= M(y(t)) + M(x^2(t))\end{aligned}\quad (4.3)$$

Applying Theorem 2.6 and using the initial conditions (4.2) in (4.3) and we get

$$\begin{aligned}M(x(t)) &= 1 + \frac{1}{2u^\alpha} M(x(t)) \\M(y(t)) &= \frac{1}{u^\beta} M(y(t)) + \frac{1}{u^\beta} M(x^2(t))\end{aligned}\quad (4.4)$$

In the view of (3.7), we have

$$\begin{aligned}M\left(\sum_{n=0}^{\infty} x_n(t)\right) &= 1 + \frac{1}{2u^\alpha} M\left(\sum_{n=0}^{\infty} x_n(t)\right) \\M(\sum_{n=0}^{\infty} y_n(t)) &= \frac{1}{u^\beta} M(y(t)) + \frac{1}{u^\beta} M(\sum_{n=0}^{\infty} A_n)\end{aligned}\quad (4.5)$$

where $x^2(t) = \sum_{n=0}^{\infty} A_n$.

Thus, $A_0 = x_0^2$,

$$\begin{aligned}A_1 &= 2x_0x_1, \\A_2 &= 2x_0x_2 + x_1^2, \\A_3 &= 2x_0x_3 + 2x_1x_2, \\A_4 &= 2x_0x_4 + 2x_1x_3 + x_2^2.\end{aligned}$$

We know that $x(0) = x_0 = 1$ and $y(0) = y_0 = 0$. Taking the inverse Mahgoub transform and using the above recursive relation, the first few terms of the Mahgoub Adomian Decomposition series are derived as follows:

$$\begin{aligned}x_1 &= \frac{1}{2} \frac{t^\alpha}{\Gamma(\alpha+1)} \\y_1 &= \frac{t^\beta}{\Gamma(\beta+1)} \\x_2 &= \frac{1}{4} \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} \\y_2 &= \frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)} + \frac{t^{2\beta}}{\Gamma(2\beta+1)} \\x_3 &= \frac{1}{8} \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} \\y_3 &= \frac{t^{3\beta}}{\Gamma(3\beta+1)} + \frac{t^{\alpha+2\beta}}{\Gamma(\alpha+2\beta+1)} + \frac{\Gamma(2\alpha+1)}{4\Gamma^2(\alpha+1)} \frac{t^{2\alpha+\beta}}{\Gamma(2\alpha+\beta+1)} + \frac{t^{2\alpha+\beta}}{2\Gamma(2\alpha+\beta+1)}\end{aligned}$$

and so on.

Finally, the approximate solutions are given by

$$\begin{aligned}x(t) &= 1 + \frac{1}{2} \frac{t^\alpha}{\Gamma(\alpha+1)} + \frac{1}{4} \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{1}{8} \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \frac{1}{16} \frac{t^{4\alpha}}{\Gamma(4\alpha+1)} + \frac{1}{32} \frac{t^{5\alpha}}{\Gamma(5\alpha+1)} + \dots \\y(t) &= \frac{t^\beta}{\Gamma(\beta+1)} + \frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)} + \frac{t^{2\beta}}{\Gamma(2\beta+1)} + \frac{t^{3\beta}}{\Gamma(3\beta+1)} + \frac{t^{\alpha+2\beta}}{\Gamma(\alpha+2\beta+1)} + \frac{\Gamma(2\alpha+1)}{4\Gamma^2(\alpha+1)} \frac{t^{2\alpha+\beta}}{\Gamma(2\alpha+\beta+1)} \\&\quad + \frac{t^{2\alpha+\beta}}{2\Gamma(2\alpha+\beta+1)} + \dots\end{aligned}$$

when $\alpha = \beta = 1$, we get the approximate solution of exact solutions of (4.1).

Table 1 shows the exact solution and the approximate solution obtained by our method corresponding to distinct values of t . The Approximate solution is very much close to the exact solution. Fig. 1 shows the approximate solution of Eqn. (4.1) obtained for values of (a) when $\alpha = \beta = 1$ and (b) when $\alpha = 0.7, \beta = 0.6$.

Table1: Numerical Solution of (4.1)

t	$\alpha = \beta = 1$				$\alpha = 0.7; \beta = 0.6$	
	$x(t)$		$y(t)$		MADM	
	Exact	MADM	Exact	MADM	$x(t)$	$y(t)$
0.1	1.051271	1.051271	0.110517	0.110517	1.118280	0.405462
0.2	1.105171	1.105171	0.244281	0.244277	1.201599	0.759576
0.3	1.161834	1.161834	0.405000	0.404932	1.279237	1.157226
0.4	1.221403	1.221402	0.596730	0.596620	1.354986	1.611382
0.5	1.284025	1.284025	0.824361	0.824011	1.430414	2.13028
0.6	1.349859	1.349857	1.093271	1.092367	1.506375	2.721058
0.7	1.419068	1.419064	1.409627	1.407589	1.583415	3.390614
0.8	1.491825	1.491818	1.780433	1.776284	1.661916	4.145901
0.9	1.568312	1.568299	2.213643	2.205830	1.742170	4.994049
1.0	1.648721	1.648697	2.718282	2.704438	1.824415	5.942419

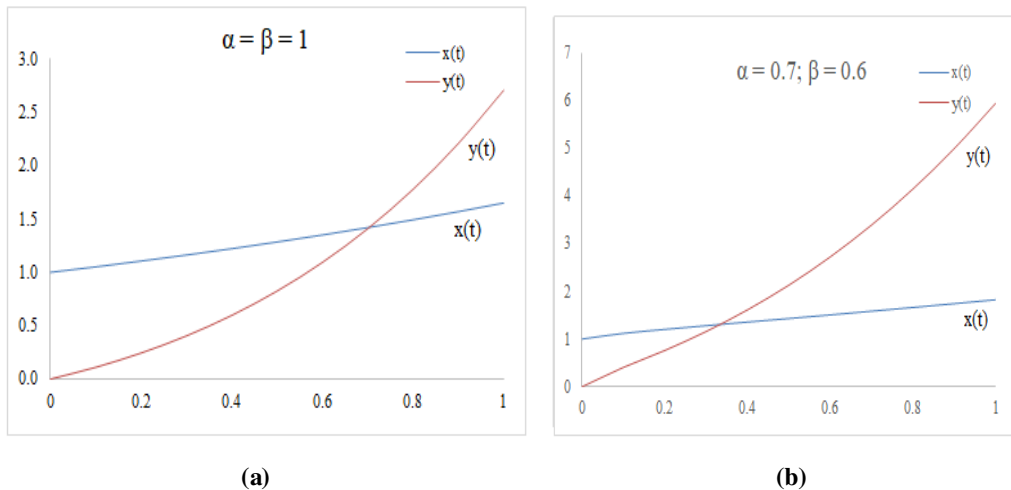


Figure 1: The Plots of system (4.1) (a) when $\alpha = \beta = 1$ and (b) when $\alpha = 0.7, \beta = 0.6$.

Example 4.2

Consider the system of nonlinear fractional differential equations of the form

$$D^\alpha x(t) = 2y^2(t),$$

$$D^\beta y(t) = tx,$$

$$D^\gamma z(t) = y(t)z(t), \quad 0 < \alpha, \beta, \gamma \leq 1 \quad (4.6)$$

$$\text{subject to the initial conditions } x(0) = 0; y(0) = 1; z(0) = 1. \quad (4.7)$$

Applying the Mahgoub transform on both sides in (4.6), we get

$$M(D^\alpha x(t)) = 2M(y^2(t)),$$

$$\begin{aligned}
M(D^\beta y(t)) &= M(tx), \\
M(D^\gamma z(t)) &= M(y(t)z(t))
\end{aligned} \tag{4.8}$$

Using the above theorem and the initial conditions (4.7) in (4.8) we get

$$\begin{aligned}
M(x(t)) &= \frac{2}{u^\alpha} M(y^2(t)) \\
M(y(t)) &= 1 - \frac{1}{u^\beta} \frac{d}{du} M(x(t)) + \frac{1}{u^{\beta+1}} M(x(t)) \\
M(z(t)) &= 1 + \frac{1}{u^\gamma} M(y(t)z(t))
\end{aligned}$$

In the view of (3.7), we have

$$\begin{aligned}
M\left(\sum_{n=0}^{\infty} x_n(t)\right) &= \frac{2}{u^\alpha} M\left(\sum_{n=0}^{\infty} A_n\right) \\
M(\sum_{n=0}^{\infty} y_n(t)) &= 1 - \frac{1}{u^\beta} \frac{d}{du} M(\sum_{n=0}^{\infty} x_n(t)) + \frac{1}{u^{\beta+1}} M(\sum_{n=0}^{\infty} x_n(t)) \\
M(\sum_{n=0}^{\infty} z_n(t)) &= 1 + \frac{1}{u^\gamma} M(\sum_{n=0}^{\infty} B_n)
\end{aligned}$$

where $y^2(t) = \sum_{n=0}^{\infty} A_n$ and $y(t)z(t) = \sum_{n=0}^{\infty} B_n$.

$$\begin{aligned}
\text{Thus, } A_0 &= y_0^2, & B_0 &= y_0 z_0 \\
A_1 &= 2y_0 y_1, & B_1 &= y_0 z_1 + y_1 z_0 \\
A_2 &= 2y_0 y_2 + y_1^2, & B_2 &= y_0 z_2 + y_1 z_1 + y_2 z_0 \\
A_3 &= 2y_0 y_3 + 2y_1 y_2, & B_3 &= y_0 z_3 + y_1 z_2 + y_2 z_1 + y_3 z_0
\end{aligned}$$

We know that $x(0) = x_0 = 0$, $y(0) = y_0 = 1$ and $z(0) = z_0 = 1$. Taking the inverse Mahgoub transforms and using the above recursive relation, the first few terms of the Mahgoub Adomian Decomposition series are derived as follows:

$$\begin{aligned}
x_1 &= \frac{2t^\alpha}{\Gamma(\alpha+1)} & y_1 &= 0 & z_1 &= \frac{t^\gamma}{\Gamma(\gamma+1)} \\
x_2 &= 0 & y_2 &= \frac{2\alpha t^{\alpha+\beta+1}}{\Gamma(\alpha+\beta+2)} + \frac{2t^{\alpha+\beta+1}}{\Gamma(\alpha+\beta+2)} & z_2 &= \frac{t^{2\gamma}}{\Gamma(2\gamma+1)} \\
x_3 &= \frac{8\alpha t^{2\alpha+\beta+1}}{\Gamma(2\alpha+\beta+2)} + \frac{8t^{2\alpha+\beta+1}}{\Gamma(2\alpha+\beta+2)} & y_3 &= 0 & z_3 &= \frac{t^{3\gamma}}{\Gamma(3\gamma+1)} + \frac{2\alpha t^{\alpha+\beta+\gamma+1}}{\Gamma(\alpha+\beta+\gamma+2)} + \frac{2t^{\alpha+\beta+\gamma+1}}{\Gamma(\alpha+\beta+\gamma+2)} \\
&\vdots & & \vdots & & \vdots
\end{aligned}$$

Finally, the approximate solutions are given by

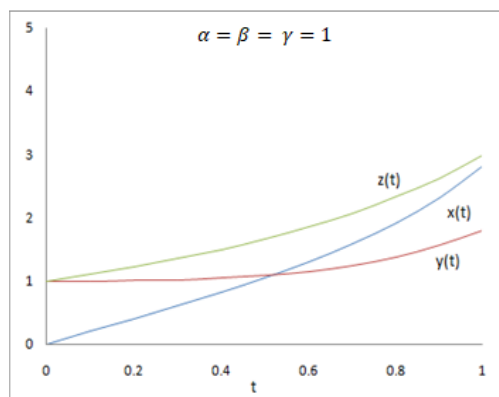
$$\begin{aligned}
x(t) &= \frac{2t^\alpha}{\Gamma(\alpha+1)} + \frac{8\alpha t^{2\alpha+\beta+1}}{\Gamma(2\alpha+\beta+2)} + \frac{8t^{2\alpha+\beta+1}}{\Gamma(2\alpha+\beta+2)} + \dots \\
y(t) &= 1 + \frac{2\alpha t^{\alpha+\beta+1}}{\Gamma(\alpha+\beta+2)} + \frac{2t^{\alpha+\beta+1}}{\Gamma(\alpha+\beta+2)} + \frac{8\alpha(2\alpha+\beta+1)t^{2\alpha+2\beta+2}}{\Gamma(2\alpha+2\beta+3)} + \frac{8(2\alpha+\beta+1)t^{2\alpha+2\beta+2}}{\Gamma(2\alpha+2\beta+3)} \\
&\quad + \frac{8\alpha t^{2\alpha+2\beta+2}}{\Gamma(2\alpha+2\beta+3)} + \dots
\end{aligned}$$

$$z(t) = 1 + \frac{t^\gamma}{\Gamma(\gamma + 1)} + \frac{t^{2\gamma}}{\Gamma(2\gamma + 1)} + \frac{t^{3\gamma}}{\Gamma(3\gamma + 1)} + \frac{2\alpha t^{\alpha+\beta+\gamma+1}}{\Gamma(\alpha + \beta + \gamma + 2)} + \frac{2t^{\alpha+\beta+\gamma+1}}{\Gamma(\alpha + \beta + \gamma + 2)} + \dots$$

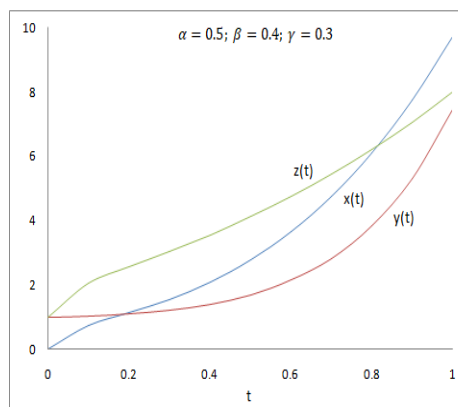
Table 2 shows the approximate solution obtained by our method corresponding to distinct values of t . Fig. 2 shows the approximate solution of the (4.6) obtained for values of (a) when $\alpha = \beta = \gamma = 1$ and (b) when $\alpha = 0.5; \beta = 0.4; \gamma = 0.3$.

Table 2: Numerical Solution of Eqn. (4.6)

t	$\alpha = \beta = \gamma = 1$			$\alpha = 0.5; \beta = 0.4; \gamma = 0.3$		
	$x(t)$	$y(t)$	$z(t)$	$x(t)$	$y(t)$	$z(t)$
0.1	0.200068	1.000668	1.105189	0.733499	1.021424	2.039882
0.2	0.401110	1.005363	1.221699	1.122638	1.085711	2.550349
0.3	0.605727	1.018233	1.351439	1.554260	1.203457	3.043500
0.4	0.818452	1.043674	1.497070	2.093702	1.394863	3.561862
0.5	1.045926	1.086497	1.662153	2.784124	1.692368	4.125691
0.6	1.297094	1.152138	1.851294	3.661798	2.145317	4.748429
0.7	1.583422	1.246948	2.070302	4.760328	2.825786	5.440594
0.8	1.919143	1.378627	2.326347	6.112360	3.835380	6.211210
0.9	2.321531	1.556883	2.628141	7.750433	5.312895	7.068456
1	2.811217	1.794415	2.986111	9.707481	7.442787	8.019998



(a)



(b)

Figure 2: The Plots of the system (4.6), (a) when $\alpha = \beta = \gamma = 1$ & (b) when $\alpha = 0.5; \beta = 0.4; \gamma = 0.3$.

5. CONCLUSION

In this paper, the Mahgoub transform method combined with the Adomian decomposition method is successfully applied to solve the system of nonlinear fractional differential equations. Thus, the results show that this method is a powerful mathematical tool for solving the systems of nonlinear fractional differential equations.

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