

JUST CHROMATIC EXCELLENCE IN ANTI-FUZZY GRAPHS

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Abstract:

Let G be a simple anti-fuzzy graph (AFG). A family $C = \{c_1, \dots, c_k\}$ of anti-fuzzy sets on a set V is called a k -vertex coloring of $G = (V, \sigma, \mu)$ if

(i) $\forall c_i(x) = \sigma(x)$, for all $x \in V$,

(ii) $c_i \wedge c_j = 0$,

(iii) For every strong edge xy of G , $\min\{c_i(\sigma(x)), c_i(\sigma(y))\} = 0$, $(1 \leq i \leq k)$.

The least value of k for which the G has a k -vertex coloring denoted by $\chi(G)$, is called the chromatic number of the anti-fuzzy graph G . Then C is the partition of independent sets of vertices of G in which each set has the same color is called the chromatic partition. An anti-fuzzy graph G is called the just χ -excellent if every vertex of G appears as a singleton in exactly one χ -partitions of G . A just χ -excellent graph of order n is called the tight just χ -excellent graph if G having exactly n , χ -partition. The focal point of this paper is to study the new concept called just chromatic excellence and tight just chromatic excellence in anti-fuzzy graphs. We explain these new concepts through illustrative examples.

Keywords: Antifuzzy graph, Chromatic excellence, Justchromatic excellence, Tight just chromatic excellence.

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1. INTRODUCTION

Graph coloring dates back to 1852, when Francis Guthrie came up with the four color conjecture. Gary Chartrand and Ping Zhang [6] discussed various colorings of graph and its properties in their book entitled Chromatic Graph Theory. A graph coloring is the assignment of a color to each of the vertices or edges or both in such a way that no two adjacent vertices and incident edges share the same color. E. Sambathkumar [18] discussed chromatically fixed, free and totally free vertices of a graph in 1992. Graph coloring has been applied to many real world problems like scheduling, allocation, telecommunications and bioinformatics, etc.

The concept of fuzzy sets and fuzzy relations were introduced by L.A. Zadeh [22] in 1965. A. Rosenfeld [17] considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975. The concept of chromatic number of fuzzy graph was introduced by Munoz et.al. [11] in 2004. C. Eslahchi and B.N. Onagh [2] introduced fuzzy graph coloring of fuzzy graph in 2006. S. Lavanya and R. Sattanathan [10] discussed total fuzzy coloring in 2009. Anjaly Kishore and M.S. Sunitha [1] discussed chromatic number of fuzzy graph in 2013. R. Jahir Hussain and K.S. Kanzul Fathima [7-9] conferred fuzzy coloring of fuzzy graph, using strong arcs and dominator coloring of fuzzy graph in 2015. A. Nagoor Gani and B. Fathima Kani [13] deliberated on fuzzy vertex order coloring in 2016. K.M. Dharmalingam and R. Udaya Suriya [3-5] conferred chromatic excellence, just chromatic excellence and tight just chromatic excellence in fuzzy graph in 2017.

R. Seethalakshmi and R.B. Gnanajothi [19] introduced Anti-fuzzy graph (AFG) in 2016 and discussed various properties in 2017 [20, 21]. R. Muthuraj and A. Sasireka [12] discussed anti-fuzzy graphs in 2017. M.A. Rifayathali et al. [14,16] deliberated anti fuzzy graph coloring and chromatic excellence in anti-fuzzy graph [15] in 2018. Hence in this paper we are introducing a novel concept called just chromatic excellence and tight just chromatic excellence in anti-fuzzy graph and its properties.

2. PRELIMINARIES

2.1. Definition (Zadeh [22])

Let X be a non-empty set. Then a fuzzy set A in X (i.e., a fuzzy subset A of X) is characterized by a function of the form $\mu_A: X \rightarrow [0,1]$, such a function μ_A is called the membership function and for each $x \in X$, $\mu_A(x)$ is the degree of membership of x (membership grade of x) in the fuzzy set A .

In other words, $A = \{(x, \mu_A(x)) / x \in X\}$ where $\mu_A: X \rightarrow [0,1]$.

2.2. Definition (A. Rosenfeld [17])

A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma: V \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$, where for all $u, v \in V$, we have $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$.

2.3. Definition (Eslahchi and Onagh [2])

A family $\Gamma^f = \{\gamma_1, \dots, \gamma_k\}$ of fuzzy sets on V is called a k -fuzzy coloring of $G = (V, \sigma, \mu)$ if

- $\bigvee \Gamma^f = \sigma$,
- $\gamma_i \wedge \gamma_j = 0$,
- For every strong edge xy of G , $\min \{\gamma_i(x), \gamma_i(y)\} = 0$ ($1 \leq i \leq k$).

The least value of k for which the G has a k -fuzzy coloring denoted by $\chi^f(G)$, is called the fuzzy chromatic number of the fuzzy graph G .

2.4. Definition (Dharmalingam and Udaya Suriya [3])

G is fuzzy chromatic excellent if for every vertex of $v \in V(G)$, there exists a fuzzy chromatic partitions Γ^f such that $\{v\} \in \Gamma^f$.

2.5. Definition (Dharmalingam and Udaya Suriya [4])

A fuzzy graph G is just χ^f -excellent if every vertex of G appears as a singleton in exactly one χ^f -partition.

2.6. Definition (Dharmalingam and Udaya Suriya [5])

A just χ^f -excellent graph of order n is the tight just χ^f -excellent graph if G is having exactly n , χ^f -partition.

2.7. Definition (Seethalakshmi and Gnanajothi [19])

An anti-fuzzy graph (AFG) $G = (\sigma, \mu)$ is a pair of functions $\sigma: V \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$, where for all $u, v \in V$, we have $\mu(u, v) \geq \sigma(u) \vee \sigma(v)$.

2.8. Definition (Seethalakshmi and Gnanajothi [19])

Let $G = (\sigma, \mu)$ be an anti-fuzzy graph on (V, E) , the anti-fuzzy degree of a vertex $u \in V$ defined as $d_{af}(u) = \sum_{u \neq v, v \in V} \mu(u, v)$. G is said to be a regular anti-fuzzy graph if each vertex has same anti-fuzzy degree. If $d_{af}(v) = k$ for all $v \in V$, G is said to be a k -regular anti-fuzzy graph.

2.9. Definition (Seethalakshmi and Gnanajothi [20])

Let $G_1 = (\sigma_1, \mu_1)$ on (V_1, E_1) and $G_2 = (\sigma_2, \mu_2)$ on (V_2, E_2) be two anti fuzzy graphs. Then the union of G_1 and G_2 is defined as $G = G_1 \cup G_2 = (\sigma_1 \cup \sigma_2, \mu_1 \cup \mu_2)$ on (V, E) , where $V = V_1 \cup V_2$, $E = E_1 \cup E_2$.

$$(\sigma_1 \cup \sigma_2)(u) = \begin{cases} \sigma_1(u) & \text{if } u \in V_1 - V_2 \\ \sigma_2(u) & \text{if } u \in V_2 - V_1 \\ \min\{\sigma_1(u), \sigma_2(u)\} & \text{if } u \in V_1 \cap V_2 \end{cases}$$

and

$$(\mu_1 \cup \mu_2)(u, v) = \begin{cases} \mu_1(u, v) & \text{if } (u, v) \in E_1 - E_2 \\ \mu_2(u, v) & \text{if } (u, v) \in E_2 - E_1 \\ \min\{\mu_1(u, v), \mu_2(u, v)\} & \text{if } (u, v) \in E_1 \cap E_2. \end{cases}$$

2.10. Definition (Seethalakshmi and Gnanajothi [20])

Let $G_1 = (\sigma_1, \mu_1)$ on (V_1, E_1) and $G_2 = (\sigma_2, \mu_2)$ on (V_2, E_2) be two anti fuzzy graphs. Let $V = V_1 \cup V_2$ and $E = E_1 \cup E_2 \cup E'$ where E' is the set of edges joining of the vertices of V_1 and V_2 . Assume that $V_1 \cap V_2 \neq \emptyset$. Then the join of G_1 and G_2 is $G = (\sigma, \mu) = (\sigma_1 + \sigma_2, \mu_1 + \mu_2)$ on (V, E) is defined as

$$(\sigma_1 + \sigma_2)(u) = (\sigma_1 \cup \sigma_2)(u), \forall u \in V_1 \cup V_2 \quad \text{and} \\ (\mu_1 + \mu_2)(u, v) = \begin{cases} (\mu_1 \cup \mu_2)(u, v) & \text{if } (u, v) \in E_1 \cup E_2 \\ \max\{\sigma_1(u), \sigma_2(v)\} & \text{if } (u, v) \in E'. \end{cases}$$

2.11. Definition (Rifayathali et.al. [15])

The Cartesian product of two anti-fuzzy graphs $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ is defined as an anti-fuzzy graph $G = G_1 \times G_2 : (\sigma_1 \times \sigma_2, \mu_1 \times \mu_2)$ on $G^*: (V, E)$ where $V = V_1 \times V_2$ and $E = \{((\sigma_1, \sigma_2), (\mu_1, \mu_2)) / u_1 = v_1, u_2 v_2 \in E_2 \text{ or } u_2 = v_2, u_1 v_1 \in E_1\}$ with

$$(\sigma_1 \times \sigma_2)(u_1, v_1) = \sigma_1(u_1) \vee \sigma_2(u_2) \text{ for all } (u_1, u_2) \in V_1 \times V_2 \\ (\mu_1 \times \mu_2)((u_1, u_2), (v_1, v_2)) = \begin{cases} \sigma_1(u_1) \vee \mu_2(u_2, v_2), & \text{if } u_1 = v_1 \text{ and } u_2 v_2 \in E_2 \\ \sigma_2(u_2) \vee \mu_1(u_1, v_1), & \text{if } u_2 = v_2 \text{ and } u_1 v_1 \in E_1 \end{cases}$$

2.12. Definition (Rifayathali et.al. [14])

Let G be an anti-fuzzy graph. A family $C = \{c_1, \dots, c_k\}$ of anti-fuzzy sets on a set V is called a k -vertex coloring of $G = (V, \sigma, \mu)$ if

- (i) $\forall c_i(x) = \sigma(x)$, for all $x \in V$
- (ii) $c_i \wedge c_j = 0$
- (iii) For every strong edge xy of G , $\min\{c_i(\sigma(x)), c_i(\sigma(y))\} = 0, (1 \leq i \leq k)$.

The least value of k for which the G has a k -vertex coloring denoted by $\chi(G)$, is called the chromatic number of the anti-fuzzy graph G .

2.13. Definition (Rifayathali et.al. [15])

C is the partition of independent sets of vertices of an anti-fuzzy graph G in which each set has the same color is called the anti-fuzzy chromatic partition.

2.14. Definition (Rifayathali et.al. [15])

A graph G is an anti-fuzzy chromatic excellent if for every vertex of $v \in V(G)$, there exists an anti-fuzzy chromatic partition C such that $\{v\} \in C$.

3. JUST CHROMATIC EXCELLENCE IN ANTI-FUZZY GRAPHS

3.1. Definition

An anti-fuzzy graph G is just χ -excellent if every vertex of G appears as a singleton in exactly one χ -partition.

3.2. Example

Let us consider the anti-fuzzy graph $G = (V, E)$ with vertex set

$V = \{(v_1, 0.3), (v_2, 0.5), (v_3, 0.4), (v_4, 0.8), (v_5, 0.5), (v_6, 0.3), (v_7, 0.2), (v_8, 0.1), (v_9, 0.6), (v_{10}, 0.7)\}$ and edge set E is given below

$$= \{(v_1 v_2, 0.5), (v_2 v_3, 0.6), (v_3 v_4, 0.8), (v_4 v_5, 0.9), (v_5 v_6, 0.6), (v_6 v_7, 0.3), (v_7 v_8, 0.2), \\ (v_9 v_{10}, 0.9), (v_{10} v_1, 0.7), (v_1 v_3, 0.4), (v_3 v_5, 0.5), (v_5 v_7, 0.7), (v_7 v_9, 0.7), (v_9 v_1, 0.6), \\ (v_2 v_4, 0.9), (v_4 v_6, 0.8), (v_6 v_8, 0.4), (v_8 v_{10}, 0.8), (v_{10} v_2, 0.7), (v_8 v_9, 0.6)\}.$$

Then the anti-fuzzy coloring $C = \{c_1, c_2, c_3, c_4\}$

$$c_1(v_i) = \{0.3 \quad i = 1$$

$$c_2(v_i) = \begin{cases} 0.5 & i = 2 \\ 0.5 & i = 5 \\ 0.1 & i = 8 \\ 0 & \text{Otherwise} \end{cases}, c_3(v_i) = \begin{cases} 0.4 & i = 3 \\ 0.3 & i = 6 \\ 0.6 & i = 9 \\ 0 & \text{Otherwise} \end{cases}, c_4(v_i) = \begin{cases} 0.8 & i = 4 \\ 0.2 & i = 7 \\ 0.7 & i = 10 \\ 0 & \text{Otherwise} \end{cases}$$

For the above anti-fuzzy graph $\chi(G) = 4$. Similarly, the anti-fuzzy χ -partitions are

$$C_1 = \{\{v_1\}, \{v_2, v_5, v_8\}, \{v_3, v_6, v_9\}, \{v_4, v_7, v_{10}\}\}, C_2 = \{\{v_2\}, \{v_3, v_6, v_9\}, \{v_4, v_7, v_{10}\}, \{v_5, v_8, v_1\}\},$$

$$C_3 = \{\{v_3\}, \{v_4, v_7, v_{10}\}, \{v_5, v_8, v_1\}, \{v_6, v_9, v_2\}\}, C_4 = \{\{v_4\}, \{v_5, v_8, v_1\}, \{v_6, v_9, v_2\}, \{v_7, v_{10}, v_3\}\},$$

$$C_5 = \{\{v_5\}, \{v_6, v_9, v_2\}, \{v_7, v_{10}, v_3\}, \{v_8, v_1, v_4\}\}, C_6 = \{\{v_6\}, \{v_7, v_{10}, v_3\}, \{v_8, v_1, v_4\}, \{v_9, v_2, v_5\}\},$$

$$C_7 = \{\{v_7\}, \{v_8, v_1, v_4\}, \{v_9, v_2, v_5\}, \{v_{10}, v_3, v_6\}\}, C_8 = \{\{v_8\}, \{v_9, v_2, v_5\}, \{v_{10}, v_3, v_6\}, \{v_1, v_4, v_7\}\},$$

$$C_9 = \{\{v_9\}, \{v_{10}, v_3, v_6\}, \{v_1, v_4, v_7\}, \{v_2, v_5, v_8\}\}, C_{10} = \{\{v_{10}\}, \{v_1, v_4, v_7\}, \{v_2, v_5, v_8\}, \{v_3, v_6, v_9\}\}$$

Therefore every vertex in the above graph appears in a singleton in exactly one χ -partition. Hence above graph is just χ -excellent.

3.3. Remark

- (1) Every just χ -excellent anti-fuzzy graph is χ -excellent graph.
- (2) Let G be any χ -excellent graph. Add a vertex u to every vertex in G such that $\mu(u, v) \geq \sigma(u) \vee \sigma(v)$ for every $v \in V(G)$. Let the resulting graph be H . Then H is χ -excellent but not just χ -excellent.

For every χ -partition of H contains $\{u\}$. Since G is χ -excellent, then for any $v \in V(G)$, there exists a χ -partition C of G such that $\{v\} \in C$. Then $C \cup \{u\}$ is χ -partition of H .

- (3) If G is χ -excellent, then G has exactly one χ -partition (i.e., G is uniquely colorable) if and only if G is complete.

If G is complete, then G is χ -excellent and it has exactly one χ -partition. Conversely, if G is χ -excellent and it has exactly one χ -partition, then every vertex in G must appear as a singleton in that χ -partition. Therefore G is complete.

3.4. Theorem

If G is not complete anti-fuzzy graph and G is χ -excellent, then G has atleast two χ -partitions.

Proof

Let us take C to be a χ -partition of G . Since G is not complete, then there exists atleast non- full degree vertex say u . Let $C_1 = \{\{u\}, V_2, \dots, V_\chi\}$ be a χ -partition of G . Let $v \in V(G)$ such that u and v are not adjacent (i.e. $\mu(u, v) < \sigma(u) \vee \sigma(v)$). Let $v \in V_i$, $2 \leq i \leq \chi$. Then $C_2 = \{\{V_i - \{v\}\}, \{u, v\}, V_3, \dots, V_\chi\}$ is also a χ -partition of G not containing $\{u\}$.

3.5. Theorem

If G is not complete anti-fuzzy graph and G is χ -excellent, then G has at least three χ -partitions.

Proof

We know that any χ -excellent non complete anti-fuzzy graph has at least two χ -partitions (from the Theorem 3.4.). Suppose that G has exactly two χ -partitions C_1 and C_2 . Let $C_1 = \{V_1, V_2, \dots, V_\chi\}$ and $C_2 = \{W_1, W_2, \dots, W_\chi\}$ be the two partitions of G . Since G is χ -excellent and not complete, C_1 has r singletons and C_2 contains atleast $n - r$ singletons. Let C_1 contain $\{u_1\}, \{u_2\}, \dots, \{u_r\}$ and let $\{u_{r+1}\}, \dots, \{u_n\}$ be the elements of C_2 . Then $\langle u_1, u_2, \dots, u_r \rangle$ is complete and also $\langle u_{r+1}, u_{r+2}, \dots, u_n \rangle$ is complete. Therefore in C_1 there will be $\{u_1\}, \{u_2\}, \dots, \{u_r\}, \{u_{r+1}\}, \dots, \{u_n\}$ elements, a contradiction. Hence there are at least three χ -partitions.

3.6. Remark

A similar argument as in the above theorem shows that there are at least four χ -partitions.

3.7. Remark

There exist anti-fuzzy graphs having not full degree vertex and not just χ -excellent but χ -excellent.

3.8. Proposition

If G is just χ -excellent anti-fuzzy graph and $G \neq K_n$ then $\chi = \left\lfloor \frac{n+1}{2} \right\rfloor$. The converse is not true.

3.9. Remark

P_n is not just χ -excellent anti-fuzzy graph but is an induced subgraph of a just χ -excellent anti-fuzzy graph. (If n is odd say $n = 2k + 1$, then P_n is an induced subgraph of cycle C_{2k+3} . If n is even say, $n = 2k$, then P_n is an induced subgraph of cycle C_{2k+1}).

3.10. Remark

Let $G \neq K_n$, be a χ -excellent anti-fuzzy graph with a full degree vertex. Then G is not just χ -excellent.

Proof

Since $G \neq K_n$, then $\chi(G) < n$. Let $\{u\}$ be a full degree vertex of G . Then clearly, G has at least two χ -partitions. Then $\{u\}$ appears in all χ -partitions of G . Therefore G is not just χ -excellent.

3.11. Theorem

If G is a just χ -excellent anti-fuzzy graph and $G \neq K_n$, then any χ -partition of G can contain exactly one singleton.

Proof

Let us assume that there exists a χ -partition C of G containing more than one singleton. Let $C_1 = \{\{u_1\}, \{u_2\}, V_3, \dots, V_\chi\}$ be a partition of G . Since G is just χ -excellent and $G \neq K_n$, no vertex of $V(G)$ is a full degree vertex. Therefore there exists $v_1 \in V(G)$ such that u_1 and v_1 are not adjacent such that $\mu(u_1, v_1) < \sigma(u_1) \vee \sigma(v_1)$. Let $v_1 \in V_i$, $3 \leq i \leq \chi$. Clearly, $|V_i| \geq 2$, for if $V_i = \{v_1\}$, then u_1 and v_1 are adjacent. Let $C_2 = \{\{u_1, v_1\}, \{u_2\}, V_3, \dots, V_i - \{v_1\}, \dots, V_\chi\}$. Then C_2 is a χ -partition containing $\{u_2\}$, which is a contradiction to G is just χ -excellent.

3.12. Corollary

If G is just χ -excellent anti-fuzzy graph and $G \neq K_n$, then $\chi \leq \left\lfloor \frac{n+1}{2} \right\rfloor$.

Proof

Since G is just χ -excellent, then any χ -partition contains exactly one singleton.

Therefore, $n \geq 1 + 2(\chi - 1)$. That is, $n \geq 2\chi - 1$. Hence $\chi \leq \left\lfloor \frac{n+1}{2} \right\rfloor$.

3.13. Remark

- (1) W_6 has chromatic number $4 > \left\lfloor \frac{n+1}{2} \right\rfloor$ and W_6 is χ -excellent. Clearly, W_6 is not just χ -excellent.
- (2) The bound is sharp as seen in C_5 ($\chi(C_5) = 3 = \frac{5+1}{2}$) and C_5 is just χ -excellent.

3.14. Remark

The sum of two just χ -excellent anti-fuzzy graphs need not be just χ -excellent.

For example, Cycle graph C_5 is just χ -excellent but cycle $C_5 + C_5$ is not just χ -excellent.

3.15. Remark

If $G + H$ is a just χ -excellent anti-fuzzy graph, then G and H are just χ -excellent graphs.

Proof

Any chromatic partition of $G + H$ is a union of a chromatic partition of G and H . Then $G + H$ is just χ -excellent so, G and H are just χ -excellent.

3.16. Theorem

If G and H are just χ -excellent anti-fuzzy graphs and one of them is not complete, if the other is K_1 then $G + H$ is not just χ -excellent.

Proof

Let $G = K_1$. Then H is not a complete anti-fuzzy graph. Further, $G + H$ is not complete but it has a full degree vertex. Therefore $G + H$ is not a just χ -excellent graph. Let $G \neq K_1$ and $H \neq K_1$. Since G and H are just χ -excellent, $G, H \neq \bar{K}_n$ for $n \geq 2$. Then any χ -partition of G and H contains atleast two elements. Then for any χ -partition of G with a singleton element, we can associate several χ -partitions of H , giving a χ -partition of $G + H$. Therefore $G + H$ is not just χ -excellent.

3.17. Theorem

Let G and H be two anti-fuzzy graphs. $G + H$ is just χ -excellent if and only if both of them are complete graphs.

Proof

Let us assume that G and H are complete anti-fuzzy graphs. Then $G + H$ is complete anti-fuzzy graph and hence just χ -excellent. Conversely, assume that $G + H$ is just χ -excellent. Therefore, both G and H are just χ -excellent. If G or H is not complete, then using the above Remark 3.15, $G + H$ is not just χ -excellent, a contradiction. Therefore G and H are complete. Hence $G + H$ is complete anti-fuzzy graph.

3.18. Theorem

Let $G \neq K_n$ be just χ -excellent graph. Let $u \in V(G)$. Let $C = \{\{u\}, V_2, \dots, V_\chi\}$ be a χ -partition of G . Then for every vertex in V_i , $2 \leq i \leq \chi$ is adjacent with at least one vertex in V_j , for all $j, j \neq i, 2 \leq j \leq \chi$.

Proof

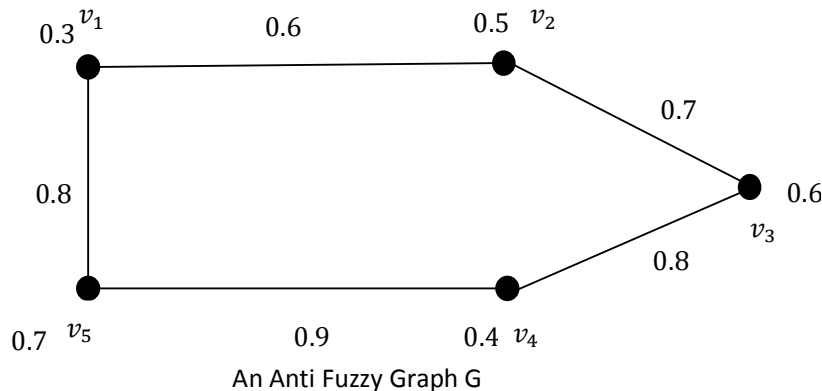
Since G is just χ -excellent, $|V_i| \geq 2$ for all i , $2 \leq i \leq \chi$. Let $v \in V_i$. Suppose v is not adjacent to any vertex of some V_j such that $\mu(u, v) < \sigma(u) \vee \sigma(v)$, for $u \in V_j, j \neq i, 2 \leq j \leq \chi$. Then $C_1 = \{\{u\}, V_2, \dots, V_i - \{v\}, \dots, V_j \cup \{v\}, \dots, V_\chi\}$ is a χ -partition of G (note that $V_i - \{v\} \neq \emptyset$) different from C , a contradiction.

3.19. Definition

A vertex of an anti-fuzzy graph G with respect to a χ -partition C of G is called an anti-fuzzy colorful vertex if it is adjacent to every color class other than the one to which it belongs.

Let $C = \{V_1, V_2, \dots, V_\chi\}$ be a χ -partition of G . Then $u \in V_i$ is said to be anti-fuzzy colorful vertex if u is adjacent to every color class in C -partition but not adjacent to V_i such that $\mu(u, v_i) \geq \sigma(u) \vee \sigma(v_i)$ for some vertex $v_i \in V_1, \dots, V_{i-1}, V_{i+1}, \dots, V_\chi$ and $\mu(u, v_j) < \sigma(u) \vee \sigma(v_j)$ for every $v_j \in V_i$.

3.20. Example



For the above graph $\chi(G) = 3$. Let $C = \{V_1 = \{v_1\}, V_2 = \{v_2, v_4\}, V_3 = \{v_3, v_5\}\}$ be a χ -partition. From this partition $\{v_1\}$ is adjacent to some vertex in V_2 and V_3 , $\{v_2\}$ is adjacent to V_1 and V_3 , $\{v_4\}$ is adjacent to V_3 but not to V_1 , $\{v_3\}$ is adjacent to V_2 but not to V_1 , $\{v_5\}$ is adjacent to V_1 and V_2 . Hence $\{v_1, v_2, v_5\}$ are colorful vertices with respect to the C -partition.

3.21. Corollary

- (1) If G is just χ -excellent then every vertex in $N[u]$, $u \in V(G)$ is an anti-fuzzy colorful vertex in the χ -partition in which $\{u\}$ is an element. Then the number of colorful vertices is $\deg(u) + 1$.
- (2) There exists a χ -partition in which the number of anti-fuzzy colorful vertices are equal to $\Delta(G) + 1$ which is greater than or equal to $\chi(G)$.

3.22. Theorem

Let G be a just χ -excellent anti-fuzzy graph which is not complete. Let $u \in V(G)$ and let $C = \{\{u\}, V_2, \dots, V_\chi\}$ be a χ -partition of G . If $|V_i| \geq 3$ for some $2 \leq i \leq \chi$ then there exists at least some V_j with $|V_j| \geq 3$ containing a vertex not adjacent to u .

Proof

Suppose u is adjacent to every vertex in V_i with $|V_i| \geq 3$ ($2 \leq i \leq \chi$).

Case (1): Let $|V_i| \geq 3$ for all i , $2 \leq i \leq \chi$. Then u is a full degree vertex and it appears as a singleton in every χ -partition of G , which is a contradiction to the fact that G is just χ -excellent and $G \neq K_n$.

Case (2): Let $|V_i| \geq 3$ for all i , $2 \leq i \leq t$ and $|V_{t+1}| = 2$. Let $|V_{t+1}| = \{v_1, v_2\}$. Suppose there exists $V_{t+1}, V_{t+2}, \dots, V_\chi$ such that $|V_{t+j}| = 2$, $2 \leq j \leq \chi - t$ (Note that no V_i , ($2 \leq i \leq \chi$) is a singleton since G is just χ -excellent). Since C is a χ -partition, u is adjacent with at least one vertex in each of $V_{t+1}, V_{t+2}, \dots, V_\chi$. Suppose u is adjacent with v_1 and not adjacent with v_2 in V_{t+1} such that $\mu(u, v_1) \geq \sigma(u) \vee \sigma(v_1)$ and $\mu(u, v_2) < \sigma(u) \vee \sigma(v_2)$ for $v_1, v_2 \in V_{t+1}$. Then u is adjacent with every vertex V_{t+j} , $2 \leq j \leq \chi - 1$ such that $\mu(u, v_i) \geq \sigma(u) \vee \sigma(v_i)$ for every $v_i \in V_{t+j}$, $2 \leq j \leq \chi - 1$.

For, otherwise there exists some vertex $w \in V_{t+j}$ not adjacent with u . Therefore, $C_1 = \{\{u, v_2, w\}, V_2, \dots, V_t, \{v_1\}, \dots, V_{t+j} - \{w\}, \dots, V_\chi\}$ which is a contradiction to the fact that G is just χ -excellent. Hence u is adjacent with every vertex in $V - \{v_1\}$. (Note that if $V_{t+1} = V_\chi$ then also u is adjacent with every vertex in $V - \{v_2\}$). Since G is just χ -excellent there exists a χ -excellent $C_2 = \{\{v_2\}, V'_2, \dots, V'_\chi\}$. Therefore, $u \in V'_i$, a contradiction since u is adjacent with every vertex in $V - \{v_2\}$ such that $\mu(u, v_i) \geq \sigma(u) \vee \sigma(v_i)$ for every vertex $v_i \in V - \{v_2\}$. Hence the theorem.

3.23. Remark

Let G be an anti-fuzzy graph which is just χ -excellent. If there exists a χ -partition in which one of the element is a singleton $\{u\}$ and some other element with cardinality greater than or equal to 3, then there exists a χ -partition in which none of the elements is singleton.

Proof

Let G be a just χ -excellent anti-fuzzy graph satisfying the hypothesis. Then there exists a χ -partition $C = \{\{u\}, V_2, \dots, V_\chi\}$ in which $|V_i| \geq 3$ for some i , $2 \leq i \leq \chi$ and V_i contains a non-neighbour, say, v and u . Then $C_1 = \{\{u, v\}, V_2, \dots, V_i - \{v\}, \dots, V_\chi\}$ is a χ -partition of G in which each class contains at least 2 vertices of G .

3.24. Remark

If G is just χ -excellent and $G \neq K_n$, and $\beta_0(G) = 2$, then the number of χ -partitions of G is exactly 'n'. For, Let $V(G) = \{u_1, u_2, \dots, u_k\}$, then, by the hypothesis there exist χ -partitions $\{\{u_i\}, V_2, \dots, V_k\}$ and $|V_i| = 2$ for all $2 \leq i \leq k$. There $|V(G)| = 2k + 1$. Hence there cannot exist χ -partitions in which one of the element is a singleton.

3.25. Remark

If G is just χ -excellent and $G \neq K_n$, then G has exactly 'n' χ -partitions if and only if in those χ -partitions in which one element is a singleton, the cardinality of any other element of the partition is 2.

3.26. Remark

If G is just χ -excellent anti-fuzzy graph, then $\deg(u) \leq n - 3$ for any vertex $u \in V(G)$.

4. TIGHT JUST CHROMATIC EXCELLENCE IN ANTI-FUZZY GRAPHS

4.1. Definition

A just χ -excellent graph of order n is the tight just χ -excellent graph if G having exactly n , χ -partitions.

4.2. Example

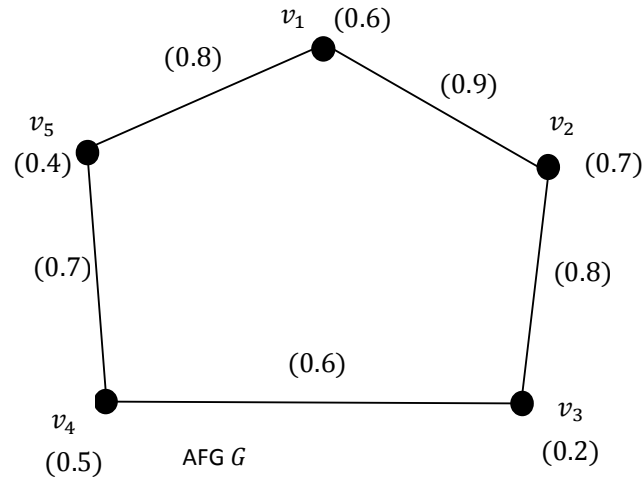


Figure 4.1

The anti-fuzzy coloring $C = \{c_1, c_2, c_3\}$

$$c_1(v_i) = \begin{cases} 0.8 & i = 2 \\ 0 & \text{otherwise} \end{cases}, \quad c_2(v_i) = \begin{cases} 0.6 & i = 1 \\ 0.5 & i = 4 \\ 0 & \text{otherwise} \end{cases}, \quad c_3(v_i) = \begin{cases} 0.4 & i = 3 \\ 0.2 & i = 5 \\ 0 & \text{otherwise} \end{cases}$$

For the above anti-fuzzygraph $\chi(G) = 3$. Similarly, the χ -partitions are

$$C_1 = \{\{v_1\}, \{v_2, v_4\}, \{v_3, v_5\}\}, C_2 = \{\{v_2\}, \{v_1, v_4\}, \{v_3, v_5\}\}, C_3 = \{\{v_3\}, \{v_1, v_4\}, \{v_2, v_5\}\},$$

$$C_4 = \{\{v_4\}, \{v_1, v_3\}, \{v_2, v_5\}\}, C_5 = \{\{v_5\}, \{v_1, v_3\}, \{v_2, v_4\}\}.$$

The graph is just χ -excellent and it has exactly 5, χ -partitions. Hence the graph is tight just χ -excellent.

4.3. Corollary

If G is a just χ -excellent anti-fuzzy graph then G is either tight just χ -excellent or it contains at least $n + 1$, χ -partitions in which no singleton appears.

4.4. Note

Every just χ -excellent graph need not be tight just χ -excellent graph.

4.5. Example

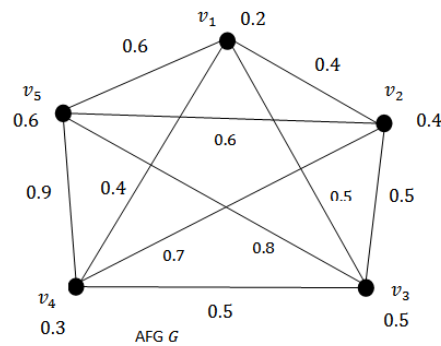


Figure 4.2:

The anti-fuzzy chromatic partition is $\{v_1, v_2, v_3, v_4, v_5\}$. The graph is just χ -excellent but not tight just χ -excellent.

4.6. Remark

If G is non-tight just χ -excellent, then any χ -partition with a singleton class contains at least one class containing more than two elements.

4.7. Theorem

A just χ -excellent graph G is tight just χ -excellent graph if and only if $n = 2\chi - 1$.

Proof

Suppose that G be a just χ -excellent graph with $n = 2\chi - 1$. Since G is just χ -excellent, then there exists a χ -partition containing $\{u\}$ for given any vertex u . The remaining $\chi - 1$ partitions must have at least two elements each. Since in a just χ -excellent graph no χ -partitions can contain two singletons. Therefore the minimum number of elements in any χ -partition are $2(\chi - 1) + 1 = 2\chi - 1 = n = \text{total number of elements}$. Therefore every χ -partition contains singleton and other sets are two element sets. If a χ -partition does not contain singleton, then the total number of elements in the partition are at least $2\chi > n$ which is a contradiction. Hence G is tight just χ -excellent. If G is tight just χ -excellent of order n , then G contains n χ -partitions and every χ -partition must contain singleton set and other $\chi - 1$ partitions are two element sets. Then the number of partitions are $2(\chi - 1) + 1 = 2\chi - 1 = n = \text{total number of elements}$.

4.8. Theorem

If G is just χ -excellent and $G \neq K_n$ then the number of χ -partitions of G are greater than or equal to 5.

Proof

If the graph G is of order 1 or 2, then G is K_1 or K_2 . Then the χ -partitions of G is 1. Let $O(G) = 3$, if $G = K_3$ then the number of χ -partition is 1 and $\chi(G) = 3$. If $\chi(G) = 1$ or 2 then G is not χ -excellent. Let $O(G) \geq 4$. $\chi(G) \geq 3$ then G is χ -excellent.

Case (i) : G is just χ -excellent.

By the theorem 4.7, G is tight just χ -excellent then $O(G) = 2\chi - 1$ which is ≥ 5 .

Therefore the number of χ -partitions = $O(G) \geq 5$.

Case (ii) : G is not tight just χ -excellent, then the number of χ -partitions are $\geq O(G) + 1 \geq 5$.

Conjecture: For every χ -excellent graph G , the number of χ -partitions of G is ≥ 5 .

4.9. Theorem

If G is tight just χ -excellent, then $\chi - 1 \leq \deg(u) \leq 2\chi - 4 = O(G) - 3$ for every $u \in V(G)$.

Proof

For every G that is tight just χ -excellent, we have $|V(G)| = 2\chi - 1$, then clearly, u is not a full degree vertex. Therefore $\deg(u) \leq n - 2 = 2\chi - 3$. If $\deg(u) = n - 2$, then u is not adjacent to exactly one vertex of G say v . Let $C_1 = \{\{v\}, V_1, V_2, \dots, V_\chi\}$ be a χ -partition containing $\{v\}$. Then $u \in V_i$ for some i , $2 \leq i \leq \chi$. Since u is adjacent to every other vertex of G other than v . Then $|V_i| = 1$, that is $V_i = \{u\}$, which is a contradiction to any just χ -excellent graph having a χ -partition containing at most one singleton class. Hence, $\deg(u) \leq n - 3 \leq 2\chi - 4$.

4.10. Theorem

For every positive integer k , there exists a regular Harary anti-fuzzy graph G which is tight just χ -excellent such that $\chi(G) = k + 1$, $|V(G)| = 2k + 1$ and every vertex appears as a singleton in a chromatic partition which is adjacent to every element of $(k - 2)$ doubletons in that partition and adjacent to exactly one element in the remaining two doubleton classes.

Proof

Let us take the graph $H_{2k-2, 2k+1}$. Suppose that S is an independent set with 3 vertices say $\{u_1, u_2, u_3\}$. Then u_1 is not adjacent with only two vertices say v, w where $d(u_1, v) = k$ and $d(u_1, w) = k + 1$. Therefore $u_2 = v$ and $u_3 = w$ but $d(v, w) = 1$. Hence u_2 and u_3 are adjacent, which is a contradiction. Hence $\{u_1, v\}$ is independent. Then $\beta_0(H_{2k-2, 2k+1}) = 2$. Therefore $\frac{n}{\beta_0} \leq \chi$ which implies $\chi \geq k + 1$. Suppose $\Pi = \{\{1\}, \{2, k + 2\}, \{3, k + 3\}, \dots, \{k + 1, 2k + 1\}\}$ be a proper color partition of cardinality $k + 1$ and hence $\chi = k + 1$. Then χ is a partition in which 1 is adjacent with $2, 3, 4, \dots, k, 2k + 1, 2k, \dots, k + 3$. Hence 1 is adjacent with exactly one element 2 and $2k + 1$ in the remaining two doubleton classes $\{2, k + 2\}, \{k + 1, 2k + 1\}$.

4.11. Observation

The anti-fuzzy graph $G = H_{2k-2, 2k+1}$ is $2k-2$ regular, $\beta_0 = 2$ and $\chi \geq k + \frac{1}{2}$. Then the graph admits a $k+1$ color partition, Therefore $\chi(H_{2k-2, 2k+1}) = k+1$ and the degree of every vertex $= 2k-2 = 2\chi-4$. $|V(G)| = 2k+1 = 2\chi-1$. Therefore this graph is tight just χ -excellent and the degree of every vertex is $2\chi-4$.

4.12. Remark

For every positive integer k , there exists a tight just χ -excellent graph, $\chi-1 \leq k \leq 2\chi-4$ and there exists a vertex u with degree k .

4.13. Definition

The anti-fuzzy Kneser graph $K(n, k)$ is the anti-fuzzy graph whose vertices correspond to k -element subsets of a set of n -elements say $V_1, V_2, \dots, V_m$, and where two vertices are adjacent if and only if the corresponding $V_i \cap V_j = \emptyset$ such that $\sigma(V_i) = \sigma(V_1) \vee \sigma(V_2) \vee \dots, \forall v_i \in V_i$ and $\mu(V_i, V_j) = \sigma(V_i) \vee \sigma(V_j)$.

4.14. Proposition

The Kneser graph $K(n, 2)$ is not χ -excellent for $n \geq 3$.

Proof

In a Kneser graph $\chi(K(n, 2)) = n-2$. Then for any $u \in V(K(n, 2))$, $\chi(K(n, 2)) - \{u\} = \chi(K(n, 2))$. Hence $K(n, 2)$ is not χ -critical and hence not χ -excellent.

4.15. Proposition

The Kneser graph $K(n, k)$ where $(k \leq \lfloor \frac{n}{2} \rfloor)$ is not a χ -excellent graph for $n \geq 3$.

Proof

Let $u = \{1, 2, 3, \dots, k\}$. Then $\chi(G-u) = \chi(G) = n-2k+2$. Therefore G is not χ -excellent.

4.16. Theorem

C_{3n+1} is just χ -excellent but not tight just χ -excellent if $n \geq 1$. Further there exists a chromatic partition in which every vertex of the cycle is colorful if and only if $2n+1 \equiv 0 \pmod{3}$.

Proof

Let us take C_{3n} where n is odd. Then the anti-fuzzy chromatic number is 3. Then the χ -partition $C = \{\{u_1, u_4, \dots, u_{3n-2}\}, \{u_2, u_5, \dots, u_{3n-1}\}, \{u_3, u_6, \dots, u_{3n}\}\}$ in which every vertex is anti-fuzzy colorful. Consider C_{3n+1} where n is even. A χ -partition giving $3n-1$ anti-fuzzy colorful vertices is $\{\{u_1, u_4, \dots, u_{3n-2}\}, \{u_2, u_5, \dots, u_{3n-1}\}, \{u_3, u_6, \dots, u_{3n}\}\}$. In above χ -partition except u_1 and u_{3n+1} are colorful. Let $C = \{V_1, V_2, V_3\}$ be a χ -partition of C_{3n+1} , (n -even). For any $v_i, u_i \in V_i$ then u_{i-2} and $u_{i+2} \notin V_i$. Hence $V_1 = \{u_1, u_4, \dots\}$, $V_2 = \{u_3, u_6, \dots\}$, $V_3 = \{u_2, u_5, \dots\}$. Since the total number of vertices is $3n+1$, there exists atleast one V_i such that $|V_i| \geq n+1$. Suppose that $|V_1| \geq n+1$. If $|V_1| = n+1$, then the $(n+1)^{\text{th}}$ term in V_1 is u_{3n+1} which is adjacent to $u_1 \in V_1$, which is a contradiction. Similarly, a contradiction arises if $|V_1| > n+1$. Therefore $|V_1| \leq n$. Similarly $|V_2| \leq n$ and $|V_3| \leq n$ which is a contradiction to $|V| = 3n+1$. If $V_1 = \{u_1, u_4, \dots, u_{3n-2}\}$, $V_2 = \{u_2, u_5, \dots, u_{3n-1}\}$ and $V_3 = \{u_3, u_6, \dots, u_{3n}\}$, then u_{3n+1} cannot be accommodated in V_1 and V_3 , since they contain the adjacent vertices u_1 and u_{3n} respectively. Therefore u_{3n+1} has to be included in V_2 . Here u_{3n} and u_1 will not be anti-fuzzy colorful vertices. Hence the number of anti-fuzzy colorful vertices is at most $3n-1$. Since we have already shown that there exists a χ -partition containing $3n-1$ colorful vertices. Hence the maximum number of anti-fuzzy colorful vertices in any χ -partition of C_{3n+1} (n even) is $3n-1$. Similarly, we can prove that for C_{3n+2} where n is odd, the maximum number of anti-fuzzy colorful vertices in any χ -partition is $3n$.

4.17. Theorem

The maximum number of anti-fuzzy colorful vertices in any anti-fuzzy chromatic partition is $n-2$ for any tight just χ -excellent graph.

Proof

In any tight just χ -excellent graph of order n , every χ -partition contains exactly one singleton class and the maximum degree of a vertex is $2\chi-4$ where $n = 2\chi-1$. The number of anti-fuzzy colorful vertices in any χ -partition is equal to $1 + \deg(v)$ where v appears as a color class in that partition. Therefore the number of anti-

fuzzy colorful vertices in every χ -partition is equal to $1 + 2\chi - 4 = 2\chi - 3 = n - 2$. Hence there is a vertex of degree $n - 3$ in that graph.

4.18. Remark

Every color partition has $n - 2 = (2r + 1)$ colorful vertices in a tight just χ -excellent graph $H_{2r,2r+3}$.

4.19. Remark

The maximum number of anti-fuzzy colorful vertices is less than $n - 2$ for some tight just χ -excellent graphs.

4.20. Theorem

There is no χ -partition containing exactly $(n - 1)$ anti-fuzzy colorful vertices in C_{3n} .

Proof

Let $\{u_1, u_2, \dots, u_{3n}\}$ be the vertices in C_{3n} . Assume that there exists a χ -partition $\mathcal{C} = \{V_1, V_2, V_3\}$ containing exactly $(n - 1)$ anti-fuzzy colorful vertices. Since exactly one vertex u_i is not anti-fuzzy colorful, u_{i-1}, u_{i+1} belong to the same color class of $\mathcal{C}$ say V_1 such that $\mu(u_{i-1}, u_i) < \sigma(u_{i-1}) \vee \sigma(u_i)$ and $\mu(u_i, u_{i+1}) < \sigma(u_i) \vee \sigma(u_{i+1})$. Then every element of V_1 and V_2 is colorful. Let us take $V_2 = \{u_{i1}, u_{i2}, \dots, u_{ir}\}$ such that $\mu(u_{it}, u_{it+1}) < \sigma(u_{it}) \vee \sigma(u_{it+1}), t = 1, 2, \dots, r$, where $(i1 < i2 < \dots < ir)$ and $V_3 = \{u_{j1}, u_{j2}, \dots, u_{js}\}$ such that $\mu(u_{jt}, u_{jt+1}) < \sigma(u_{jt}) \vee \sigma(u_{jt+1}), t = 1, 2, \dots, s$, where $(j1 < j2 < \dots < js)$. In the color classes V_2 and V_3 , ik and $ik + 1$ must have difference 3 and also in jk and $jk + 1$. Therefore in V_2 and V_3 the maximum cardinality of vertices satisfying above property is n . Then no V_i can have cardinality more than n since $\beta_0(C_{3n}) = n$ (see, [16]). If $|V_1| < n$ or, $|V_2| < n$ or, $|V_3| < n$, then one or two of the remaining elements of the partition will have more than n elements, which is a contradiction. Therefore, $|V_1| = n = |V_2| = |V_3|$. Since V_1 and V_3 satisfy the property that the difference between any two suffixes is 3, V_2 also satisfies the same condition, which is again a contradiction. Therefore exactly $n - 1$ anti-fuzzy colorful vertices in a χ -partition is not possible.

4.21. Observation

Every tight just χ -excellent graph is of odd order but a just χ -excellent graph need not be of even order.

5. CONCLUSION

In this paper we have introduced and analyzed the new concept of just chromatic excellence and tight just chromatic excellence in anti-fuzzy graphs by taking into account the anti-fuzzy chromatic partition. Thus this paper paves the way for further studies in chromatic excellence under new parameters.

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