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FIXED POINT THEOREMS FOR INTEGRAL TYPE CONTRACTION IN FUZZY METRIC SPACES USING ALTERING DISTANCE FUNCTION

A. Nagoor Gani[1] , M. Mohamed Althaf[2,*]

Authors Affiliation:
[1]P.G. & Research Department of Mathematics, Jamal Mohamed College (Autonomous), Tiruchirappalli, Tamil Nadu 620020, India.
E-mail: ganijmc@yahoo.co.in
[2] P.G. & Research Department of Mathematics, Jamal Mohamed College (Autonomous), Tiruchirappalli, Tamil Nadu 620020, India.
E-mail: althaf2maths@gmail.com

***Corresponding Author:**
M. Mohamed Althaf, P.G. & Research Department of Mathematics, Jamal Mohamed College (Autonomous), Tiruchirappalli, Tamil Nadu 620020, India.

E-mail: althaf2maths@gmail.com

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Abstract:
This paper aims to ascertain the presence of fixed point for mapping delineated on a complete fuzzy metric space gratifying a contractive condition of integral type contraction through altering distance function.

Keywords: fuzzy metric, Integral type contraction, altering distance function.

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1. INTRODUCTION

The theory of fuzzy set was invented by Zadeh in 1965[15]. Many authors extensively applied this theory in different fields such as Control theory, Engineering Sciences, Neural networks, etc. In 1975 the notion of fuzzy metric space was initiated by Kramosil and Michalek [9]. Later on, George and Veeramani [4] gave the modified concept of fuzzy metric spaces due to Kramosil and Michalek [9] and analyzed a Hausdorff topology of fuzzy metric spaces. Recently, Gregori et al. [7] gave many interesting examples of fuzzy metric in the sense of George and Veeramani [9] and have also applied these fuzzy metrics to color image processing. In 1988 Grabiec [6] proved an analog of the popular Banach contraction theorem and Edelstein fixed point theorems in fuzzy metric spaces. Grabiec [6] established the above theory in the Cauchy sequence of fuzzy metric spaces. M.S. Khan et al. [8] propounded a new notion of Banach fixed point theorem in metric spaces by presenting a control function that is called an altering distance function in 1984. Recently, Shen et al.[13] introduced the notion of control function in fuzzy metric space. Branciari [3] proved fixed point for Banach contraction mappings of integral type contraction on metric spaces and obtained fixed point result for a self-mapping T.

In this paper, we show the existence and uniqueness of fixed point theorems for integral type contraction in strong fuzzy metric spaces by using the altering distance function.

2. PRELIMINARIES

Definition 2.1:
A fuzzy set $\widetilde{A}$ is defined by $\widetilde{A} = \left\{\left(x, \mu_A(x)\right) : x \in A, \mu_A(x) \in [0,1]\right\}$. In the pair $(x, \mu_A(x))$, the first element x belongs to the classical set A, the second element $\mu_A(x)$ belongs to the interval $[0,1]$ and is called the membership function.

Definition 2.2:
A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is a continuous *t*-norm if it satisfies the following conditions:

(1) $*$ is associative and commutative;
(2) $*$ is continuous;
(3) $a * 1 = a$ for all $a \in [0,1]$;
(4) $a * b \leq c * d$, whenever, $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0,1]$.

Examples 2.3:

i. Lukasievicz *t*-norm: $a * b = \max\{a + b - 1, 0\}$
ii. Product *t*-norm: $a * b = a.b$
iii. Minimum *t*-norm: $a * b = min(a, b)$

Definition 2.4:
A fuzzy metric space is an ordered triple $(X, M,*)$ such that X is a nonempty set, $*$ is a continuous *t*-norm and M is a fuzzy set on $X \times X \times (0, \infty) \to [0,1]$ satisfies the following conditions:
For all $x, y, z \in X$ and $s, t > 0$

(KM1) $M(x, y, 0) = 0$;
(KM2) $M(x, y, t) = 1$ if and only if $x = y, t > 0$;
(KM3) $M(x, y, t) = M(y, x, t)$;
(KM4) $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$;
(KM5) $M(x, y, .): [0, \infty) \to [0,1]$ is left-continuous.
Then *M* is called a fuzzy metric on *X*.

Definition 2.5:
A fuzzy metric space is an ordered triple such that *X* is a nonempty set, $*$ is a continuous *t*-norm and M is a fuzzy set on $X \times X \times (0, \infty) \to [0,1]$ and satisfies the following conditions:
For all $x, y, z \in X$ and $s, t > 0$

(GV1) $M(x, y, t) > 0, \forall\, t > 0$;
(GV2) $M(x, y, t) = 1$ if and only if $x = y, t > 0$;
(GV3) $M(x, y, t) = M(y, x, t)$;
(GV4) $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$;
(GV5) $M(x, y, .): (0, \infty) \to [0,1]$ is continuous.
Then *M* is called a fuzzy metric on *X*.

Definition 2.6:
Let $(X, M,*)$ be a fuzzy metric space. M is said to be strong if it satisfies the following axiom instead of the definition (2.5) in (GV4) that is
(GV4′) $M(x, z, t) \geq M(x, y, t) * M(y, z, t), x, y, z \in X, t > 0$,
then $(X, M,*)$ is called a strong fuzzy metric space.

Definition 2.7:
Let $(X, M,*)$ be a fuzzy metric space, for $t > 0$, an open ball $B(x, r, t)$ with a centre $x \in X$ and a radius $0 < r < 1$ is defined by

$$B(x, r, t) = \{y \in X : \mathrm{M}(\mathrm{x}, \mathrm{y}, \mathrm{t}) > 1 - r\}.$$

A subset $A \subset X$ is called open if for each$x \in A$, there exist $t > 0$ and $0 < r < 1$ such that $B(x, r, t) \subset A$. Let τ denote the family of all open subsets of X. Then τ is a topology on X, called the topology induced by the fuzzy metric M.

Definition 2.8:

Let $(X, M,*)$ be a fuzzy metric space

- **(i)** A sequence $\{x_n\}$ in X is said to be convergent to a point x in $(X, M,*)$ if $\lim_{n\to\infty} M(x, y, t) = 1$ for all $t > 0$.
- **(ii)** A sequence $\{x_n\}$ in X is called a Cauchy sequence in $(X, M,*)$ if for each $0 <\in< 1$ and $t > 0$, there exists $n_0 \in N$ such that $M(x_n, x_m, t) > 1 - \in$ for each $n, m \geq n_0$.
- **(iii)** A fuzzy metric space in which every Cauchy sequence is convergent itself is said to be complete.
- **(iv)** A fuzzy metric space in which every sequence has a convergent subsequence is said to be compact.

Lemma 2.9:

Let $(X, M,*)$ be a fuzzy metric space then for all $u, v \in X$, $M(u, v, \ .)$ is a non-decreasing function.

Proof:

If $M(u, v, t) > M(u, v, s)$ for some $0 < t < s$.
Then $M(u, v, t) * M(v, v, s - t) \leq M(u, v, s) < M(u, v, t)$,
thus $M(u, v, t) < M(u, v, t) < M(u, v, t)$, (since $M(v, v, s - t) = 1$)
which is a contradiction.
Therefore the fuzzy metric M is a non-decreasing function.

Definition 2.10:

A function $\varphi: [0, 1] \to [0, 1]$ is a called control function (an altering distance function) if it satisfies the following properties:

(AD1) φ is strictly decreasing and continuous;
(AD2) $\varphi(\lambda) \geq 0, \forall\, \lambda \neq 1$ and $\varphi(\lambda) = 0$ if and only if $\lambda = 1$.
It is obvious that $\lim_{\lambda\to 1^-} \varphi(\lambda) = \varphi(1) = 0$.
Here Φ denotes the class of functions.

Definition 2.11:

The set of mapping $\gamma: [0,1] \to [0,1]$ satisfying the following conditions

i) γ is continuous,

ii) If $\int_0^{\psi(\delta)} \gamma(x)dx \leq \int_0^{\psi(\delta)} \gamma(x)dx - \int_0^{\varphi(\delta)} \gamma(x)dx$,

iii) If $\int_0^{\psi(\delta)} \gamma(x)dx = 0$ then $\delta = 0$.

where $\varphi \in \Phi$ and ψ is a continuous function.

3. MAIN RESULTS

Theorem 3.1:

Let $(X, M,*)$ be a complete fuzzy metric space. Let φ be an altering distance function and $\psi: [0,1] \to [0,1]$ be a continuous function. Let $\gamma: [0, \infty] \to [0,1]$ be an integral function and T be a self mapping on X such that

$$\int_0^{\psi(M(Tu,Tv,t))} \gamma(x)dx \leq \int_0^{\psi\left(M(u,v,t)\right)} \gamma(x)dx - \int_0^{\varphi\left(M(u,v,t)\right)} \gamma(x)dx \qquad (1)$$

for every $u, v \in X$ and $t \in (0, \infty)$. Then T is a unique fixed point on X.

Proof:

Take $u_0 \in X$ is an arbitrary element and define the sequence $\{u_n\} \subset X$ as $u_n := Tu_{n-1}$, For $n = 1,2,3, \ldots$

Assume that, $u_n \neq u_{n+1} = Tu_n$, for all $n \geq 1$.

Take $u = u_{n-1}$ and $v = u_n$ in (1) we have,

$$\int_0^{\psi(M(u_n,u_{n+1},t))} \gamma(x)dx = \int_0^{\psi(M(Tu_{n-1},Tu_n,t))} \gamma(x)dx$$
$$\leq \int_0^{\psi(M(u_{n-1},u_n,t))} \gamma(x)dx - \int_0^{\varphi(M(u_{n-1},u_n,t))} \gamma(x)dx$$

If $\psi(M(u_{n-1}, u_n, t)) \geq \varphi(M(u_n, u_{n+1}, t))$, then we have

$$\int_0^{\psi(M(u_n,u_{n+1},t))} \gamma(x)dx \leq \int_0^{\psi(M(u_{n-1},u_n,t))} \gamma(x)dx - \int_0^{\varphi(M(u_{n-1},u_n,t))} \gamma(x)dx$$

$\Rightarrow \int_0^{\psi(M(u_n,u_{n+1},t))} \gamma(x)dx = 0$ and $\int_0^{M(u_n,u_{n+1},t)} \gamma(x)dx = 0$.

This gives $u_n = u_{n+1} = Tu_n$, which is a contradiction to our assumption.

This implies that, $\int_0^{\psi(M(u_{n-1},u_n,t))} \gamma(x)dx = \int_0^{M(u_{n-1},u_n,t)} \gamma(x)dx$,

$$\int_0^{\psi(M(u_n,u_{n+1},t))} \gamma(x)dx \leq \int_0^{\psi(M(u_{n-1},u_n,t))} \gamma(x)dx - \int_0^{\varphi(M(u_{n-1},u_n,t))} \gamma(x)dx$$
$$\leq \int_0^{\psi(M(u_{n-1},u_n,t))} \gamma(x)dx$$

Because ψ is a non-decreasing function, then

$$\int_0^{\psi(M(u_n,u_{n+1},t))} \gamma(x)dx \leq \int_0^{\psi(M(u_n,u_{n-1},t))} \gamma(x)dx \quad \text{for all } n \geq 1.$$

Therefore the sequence $\left\{\int_0^{\psi(M(u_n,u_{n+1},t))} \gamma(x)dx > 0\right\}$ is decreasing and converges to $s > 0$. Taking the limits as $n \to \infty$ we obtain

$$\int_0^{\psi(s)} \gamma(x)dx \leq \int_0^{\psi(s)} \gamma(x)dx - \int_0^{\varphi(s)} \gamma(x)dx$$

$\Rightarrow \int_0^{\varphi(s)} \gamma(x)dx = 0$ and hence $s = 0$.

$\lim_{n\to\infty} \int_0^{M(u_n,u_{n+1},t)} \gamma(x)dx = 1$.

Claim: Let T have a periodic point, there exists a $p \in N$ and a point $z \in X$ such that $z = T^p z$.

Assume the contrary, that is, T has no periodic point. Then, all the elements of the sequence $\{u_n\}$ are distinct, $i.e., u_n \neq u_m$ for all $n \neq m$.

Suppose that $\{u_n\}$ is not a Cauchy sequence. Find subsequences $\{u_{n(i)}\}$ and $\{u_{m(i)}\}$ of $\{u_n\}$ with $n(i) > m(i) > i$ such that

$$\int_0^{M(u_{m(i)},u_{n(i)},t)} \gamma(x)dx \geq 1$$

where $n(i)$ is the smallest integer satisfying,

$$\int_0^{M(u_{m(i)},u_{n(i)-1},t)} \gamma(x)dx \leq 1$$

On applying the triangular inequality (FM3) we get

$$1 \leq \int_0^{M(u_{m(i)},u_{n(i)},t)} \gamma(x)dx \leq \int_0^{M(u_{m(i)},u_{n(i)-1},t)} \gamma(x)dx + \int_0^{M(u_{n(i)-1},u_{n(i)},t)} \gamma(x)dx$$
$$\leq 1 + \int_0^{M(u_{n(i)-1},u_{n(i)},t)} \gamma(x)dx$$

Taking the limit as $i \to \infty$, we get $\lim_{i\to\infty} \int_0^{M\left(u_{n(i)},u_{m(i)},t\right)} \gamma(x)dx = 1$

From (FM3) we have,

$\int_0^{M\left(u_{n(i)},u_{m(i)},t\right)} \gamma(x)dx \le \int_0^{M\left(u_{n(i)},u_{n(i)-1},t\right)} \gamma(x)dx + \int_0^{M\left(u_{n(i)-1},u_{m(i)},t\right)} \gamma(x)dx.$

Then $\quad 1 \le \lim_{i\to\infty} \int_0^{M\left(u_{n(i)-1},u_{m(i)-1},t\right)} \gamma(x)dx \le 1.$

Therefore, $\lim_{i\to\infty} \int_0^{M\left(u_{n(i)-1},u_{m(i)-1},t\right)} \gamma(x)dx = 1.$

Now we substitute $u = u_{n(i)-1}$ and $v = u_{m(i)-1}$ in (1), which yields

$$\int_0^{\psi\left(M\left(Tu_{n(i)-1},Tu_{m(i)-1},t\right)\right)} \gamma(x)dx = \int_0^{\psi\left(M\left(Tu_{n(i)},Tu_{m(i)},t\right)\right)} \gamma(x)dx$$
$$\le \int_0^{\psi\left(M\left(u_{n(i)-1},u_{m(i)-1},t\right)\right)} \gamma(x)dx - \int_0^{\varphi\left(M\left(u_{n(i)-1},u_{m(i)-1},t\right)\right)} \gamma(x)dx$$

Clearly, as $i \to \infty$ we have $M\left(u_{n(i)-1}, u_{m(i)-1}\right) \to 1$ and $M\left(u_{n(i)-1}, u_{m(i)-1}\right) \to 1$.

$$\int_0^{\psi(1)} \gamma(x)dx \le \int_0^{\psi(1)} \gamma(x)dx - \int_0^{\varphi(1)} \gamma(x)dx$$

which implies that $\varphi(1) = 0$. But this contradicts our assumption that $\{u_n\}$ is not Cauchy sequence.

Thus, $\{u_n\}$ must be Cauchy Sequence, since $(X, M,*)$ is complete, then $\{u_n\}$ converges to a limit, say $u \in X$. Let $u = u_n$ and $v = u$ in (1). This gives

$$\int_0^{\psi(M(Tu_n,Tu,t)} \gamma(x)dx \le \int_0^{\psi(M(u_n,u,t))} \gamma(x)dx - \int_0^{\varphi(M(u_n,u,t))} \gamma(x)dx$$

Note that $M(u_n, u, t) \to 1$ as $n \to \infty$. If $M(u_n, u, t) = M(u_n, u_{n+1,} t)$, then we have $M(u_n, u, t) \to 1$ as $n \to \infty$, then the sequence $\{u_n\}$ converges to u. Since ψ is a continuous function, we have

$$0 \le \lim_{n\to\infty} \int_0^{\psi(M(Tu_n,Tu,t)} \gamma(t)dt \le \lim_{n\to\infty} \left(\int_0^{\psi(M(u_n,u,t))} \gamma(t)dt - \int_0^{\varphi(M(u_n,u,t))} \gamma(t)dt\right) = 1$$

Hence $\lim_{n\to\infty} M(Tu_n, Tu, t) = 1$.

If $u_{n+1} = Tu_n \to Tu$. Since X is Hausdorff, thus $u = Tu$.

If $M(u_n, u, t) = M(u, Tu, t)$ passing to the limit as $n \to \infty$, we get $\varphi(M(u, Tu, t) = 0$, hence $M(u, Tu, t) = 1$, that is $u = Tu$.

This is a contradiction to our assumption that T has no periodic point.

Therefore, T has a periodic point, that is, $z = T^p z$ for some $z \in X$and $p \in N$.

Uniqueness:

If $p = 1$, then $z = Tz$, so z is a fixed point of T.

We claim that the fixed point of T is $T^{p-1}z$, for $p > 1$.

Assume that $T^{p-1}z \ne T(T^{p-1}z)$. Then $M(T^{p-1}z, T^p u, t) > 0$ and $\varphi(M(T^{p-1}z, T^p u, t) > 0$.

Taking $u = T^{n-1}z$ and $v = T^p z$ in (1), we have

$$\int_0^{\psi(M(z,Tz,t)} \gamma(t)dt = \int_0^{\psi(M\left(T^p z,T^{p+1}z,t\right))} \gamma(t)dt$$
$$\le \int_0^{\psi\left(M\left(T^{p-1}z,T^p z,t\right)\right)} \gamma(t)dt - \int_0^{\varphi(M(u_n,u,t))} \gamma(t)dt$$

which shows that, $\int_0^{M\left(T^{p-1}z,T^p z,t\right)} \gamma(t)dt = \int_0^{M\left(T^p z,T^{p+1}z,t\right)} \gamma(t)dt$

or, $\quad \int_0^{\psi\left(M(z,Tz,t)\right)} \gamma(t)dt = \int_0^{\psi\left(M\left(T^p z,T^{p+1}z,t\right)\right)} \gamma(t)dt$

$$\leq \int_0^{\psi(M(T^p z, T^{p+1} z, t))} \gamma(t)dt - \int_0^{\varphi(M(T^p z, T^{p+1} z, t))} \gamma(t)dt.$$

Thus we get $\varphi(M(T^p z, T^{p+1} z, t)) = 0$ and hence $M(T^p z, T^{p+1} z, t) = M(z, Tz, t) = 1$, which is not possible since $p > 1$. If, $M(T^{p-1} z, T^p z, t) = M(T^{p-1} z, T^p z, t)$, then

$$\int_0^{\psi(M(z,Tz,t))} \gamma(t)dt = \int_0^{\psi\left(M(T^p z, T^{p+1} z, t)\right)} \gamma(t)dt \leq \int_0^{\psi(M(T^{p-1} z, T^p z, t))} \gamma(t)dt - \int_0^{\varphi\left(M(T^{p-1} z, T^p z, t)\right)} \gamma(t)dt$$

and taking ψ as non decreasing, $M(z, Tz, t) < M((T^{p-1} z, T^p z, t)$.

Now we write $u = T^{p-2} z$ and $v = T^{p-1} z$ in (1) and we get

$$\psi\left(M(T^{p-1} z, T^p z, t)\right) \leq \psi\left(M(T^{p-2} z, T^{p_1} z, t)\right) - \varphi(M(T^{p-2} z, T^{p-1} z, t))$$

$$\psi\left(M(T^{p-1} z, T^p z, t)\right) \leq \psi\left(M(T^{p-1} z, T^p z, t)\right) - \varphi\left(M(T^{p-1} z, T^p z, t)\right)$$

which is possible only if $\varphi\left(M(T^{p-1} z, T^p z, t)\right) = 0$ and hence, $M(T^{p-1} z, T^p z, t) = 1$.

However, we have assumed that $M(T^{p-1} z, T^p z, t) > 0$.

Thus $M(T^{p-2} z, T^{p-1} z, t) = M(T^{p-2} z, T^{p-1} z, t)$, so that

$$\int_0^{\psi\left(M(T^{p-1} z, T^p z, t)\right)} \gamma(t)dt \leq \int_0^{\psi\left(M(T^{p-2} z, T^{p-1} z, t)\right)} \gamma(t)dt - \int_0^{\varphi\left(M(T^{p-2} z, T^{p-1} z, t)\right)} \gamma(t)dt$$

$$\leq \psi\left(M(T^{p-2} z, T^{p-1} z, t)\right).$$

which implies $M(T^{p-2} z, T^{p-1} z, t) \geq M(T^{p-1} z, T^p z, t)$

Since $\psi$is non-decreasing this leads to

$$0 < \int_0^{M(z,Tz,t)} \gamma(t)dt = \int_0^{M(T^p z, T^{p+1} z, t)} \gamma(t)dt$$

$$\leq \int_0^{M(T^{p-1} z, T^p z, t)} \gamma(t)dt$$

$$\leq \int_0^{M(T^{p-2} z, T^{p-1} z, t)} \gamma(t)dt.$$

By the inequality

$$0 < \int_0^{M(z,Tz,t)} \gamma(t)dt = \int_0^{M(T^p z, T^{p+1} z, t)} \gamma(t)dt$$

$$\leq \int_0^{M(T^{p-1} z, T^p z, t)} \gamma(t)dt$$

$$\leq \int_0^{M(T^{p-2} z, T^{p-1} z, t)} \gamma(t)dt$$

$$\leq \cdots$$

$$\leq \int_0^{M(z,Tz,t)} \gamma(t)dt.$$

which gives that $M(z, Tz, t) < M(z, Tz, t)$.

This is the contradiction to our assumption $M(T^{p-1} z, T^p z, t) > 0$, that is, $M(T^{p-1} z, T^p z, t) = 0$ and $T^{p-1} z$ is the fixed point of T.

We assume that T has two distinct fixed points, say z and w.

Then taking $u = z$ and $v = w$ in (1), we have

$$\psi\left(M(z, w, t)\right) = \psi\left(M(Tz, Tw, t)\right) \leq \psi\left(M(z, w, t)\right) - \varphi(M(z, w, t)$$

Thus we have

$$\int_0^{\psi(M(z,w,t))} \gamma(t)dt \leq \int_0^{\psi(M(z,w,t))} \gamma(t)dt - \int_0^{\varphi(M(z,w,t))} \gamma(t)dt.$$

Implying $\varphi(M(z,w,t)) = 0$, and hence $\mathrm{M}(z,w,t) = 1$,

Hence $z = w$ which completes the proof of the uniqueness.

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