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A NEW MODIFIED OPTIMAL PERFECT MATCHING IN PARTIAL FEASIBLE MATCHING FOR SOLVING FUZZY LINEAR SUM ASSIGNMENT PROBLEMS

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Abstract:
In this paper, the partial feasible assignment cost and the optimal assignment cost for solving fuzzy linear sum assignment problem is computed. Here, c_{ij} is denotes the cost for optimal assigning through the j^{th} job to the i^{th} person. Here $(\widetilde{c_{ij}})$ is ω-trapezoidal fuzzy number and we use the ranking technique.

Keywords: ω- Trapezoidal fuzzy numbers, Fuzzy dual variables, Bipartite graph, Fuzzy Ranking technique.

2010 Mathematics Subject Classifications - 90B80, 09C05, 03E72

1. INTRODUCTION

The assignment problem is a special type of the transportation problem. The assignment supply at all sources is one and the demand at all destinations is one. The assignment problem is in the form of $m \times n$ matrix. Here, we have n jobs assigned to m persons and our objective is to minimize the cost or maximize the profit. Let c_{ij} be the assignment cost that is allocated to the j^{th} job to the i^{th} person. Fuzzy assignment problem is the most powerful tool in fuzzy operations research. Many applications are implemented and new ideas introduced in this criteria. Zadeh [9] first proposed fuzzy sets in 1965. Kaur and Kumar [1] proposed a new approach developed for solving fuzzy transportation problem by using the generalised trapezoidal fuzzy numbers in 2012. Bellman and Zadeh [4] introduced decision making is a fuzzy environment in 1970. Lin and Wen [6] proposed the labelling method for solving the linear fractional programming case in fuzzy assignment problems in 2004 [6]. Thorani and Ravi Shankar [8] proposed new algorithms in classical and linear programming for fuzzy assignment problem with fuzzy cost based on the ranking method in 2013. Jana and Roy [7] proposed the solution of matrix games with generalised trapezoidal Fuzzy Payoffs in 2018. Gupta, Kumar and Sharma[2] proposed applications of Fuzzy Linear Programming with generalized LR flat fuzzy parameters in 2013. Kumar, Singh and Kaur [3] proposed a generalized simplex algorithm to solve fuzzy LPP with ranking of method generalised fuzzy numbers .

The Linear Sum Assignment Problem is most important and special type of linear programming problem in combinatorial optimization. We are given an $m \times n$ matrix and c_{ij} denotes the cost for optimal assigning through the j^{th} job to the i^{th} person and the total sum of the entries minimize the cost or, maximize the profit. In other words, let G be a bipartite graph $G=(U,V;E)$ with a vertex U for each row and a vertex V for each column and the cost or profit associated with edge $[i, j]$ for $i = 1,2,...,n$ and $j = 1,2,...,n$ then the problem is to determine the minimum cost or maximum profit with perfect matching.

2. PRELIMINARIES

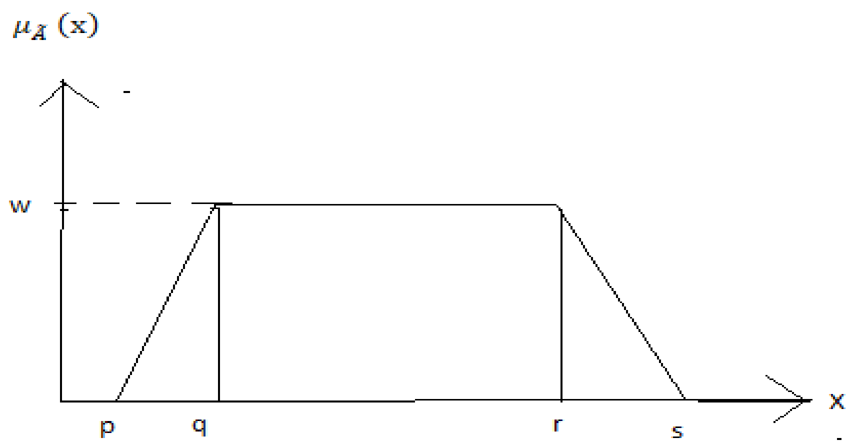
2.1. Definition A fuzzy set Ã defined on the universal set of real numbers R , is said to be a fuzzy number if its membership function has the following properties:

(i) $\mu_{\tilde{A}} : R \rightarrow [0,1]$ is continuous.
(ii) $\mu_{\tilde{A}}(x) = 0 \ \forall x \in (-\infty, p] \cup [s,\infty)$.
(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on $[p,q]$ and strictly decreasing on $[r,s]$.
(iv) $\mu_{\tilde{A}}(x) = \omega \forall x \in [q,r]$, where $\omega \in (0,1]$.

2.2. Definition A fuzzy number $\tilde{A} = (p, q, r, s;\omega)$ is said to be a ω-trapezoidal fuzzy number if its membership function is given by,

$$\mu_{\tilde{A}}(\mathrm{x}) = \begin{cases} \omega\left(\dfrac{x-p}{q-p}\right) & \text{if } p \le x \le q \\ \omega & \text{if } q \le x \le r \\ \omega\left(\dfrac{s-x}{s-r}\right) & \text{if } r \le x \le s \end{cases}$$

where $\omega \epsilon$ (0,1].



2.3. Fuzzy Linear Sum Assignment Problem (FLSAP) Let G be a bipartite graph. $G=(U,V;E)$ having a vertex of U for each row, a vertex of V for each column and ω Trapezoidal fuzzy cost $[\widetilde{c_{ij}}; \omega_{ij}]$ associated with edge (i, j) for $i, j = 1,2,...,n$ then, the problem is to determine a minimum ω- Trapezoidal fuzzy cost which corresponds to an optimal perfect matching in G.

2.4. Fuzzy Partial Assignment (FPA) To determine a feasible fuzzy dual solution and a partial primal solution (where less than "n" rows are assigned) satisfying the complimentary slackness conditions then, if

$$\boldsymbol{\varphi} = \begin{cases} i & \text{if column } j \text{ is assigned to row } i \\ 0 & \text{if column } j \text{ is not assigned to row } i \,. \end{cases}$$

The inverse of row is

$$\boldsymbol{\rho} = \begin{cases} j & \text{if row } i \text{ is assigned to column } j \\ 0 & \text{if row } j \text{ is not assigned to column } j. \end{cases}$$

2.5. Arithmetic operations on ω-Trapezoidal Fuzzy Numbers.

Let $\tilde{A} = (p_1, q_1, r_1, s_1;\omega_1)$ and $\tilde{B} = (p_2, q_2, r_2, s_2;\omega_2)$ be any two ω-trapezoidal fuzzy numbers, the then following operations are,

(i) $\tilde{A} + \tilde{B} = (p_1+p_2, q_1+q_2, r_1+r_2, s_1+s_2;\ \min(\omega_1,\omega_2))$

(ii) $\tilde{A} - \tilde{B} = (p_1 - s_2, q_1 - r_2, r_1 - q_2, s_1 - p_2\ ; \min(\omega_1, \omega_2))$

(iii) $\delta\tilde{A} = \begin{cases} (\delta p_1, \delta q_1, \delta r_1, \delta s_1;\ \omega_1), \delta > 0 \\ (\delta s_1, \delta r_1, \delta q_1, \delta p_1;\ \omega_2), \delta < 0. \end{cases}$

3. MATHEMATICAL FORMULATION OF FUZZY LINEAR SUM ASSIGNMENT PROBLEM

3.1 The Mathematical Formulation of Fuzzy Linear Sum Assignment Problem is given by

Minimize $\tilde{z} = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}$

$\sum_{i=1}^{n} x_{ij} = 1$

$\sum_{j=1}^{n} x_{ij} = 1$

where, x_{ij}= (j^{th} job is assigned to the i^{th} person)

x_{ij} = (j^{th} job is not assigned to the i^{th} person).

3.2. Ranking of ω-Trapezoidal Fuzzy Numbers

The decision maker first takes ω-trapezoidal fuzzy cost and then finds the ranking of the ω-trapezoidal fuzzy cost after the process of optimal decisions. Here, ω-trapezoidal fuzzy cost is the decision variable. The ranking of ω-trapezoidal fuzzy cost to the following comparisons exists:

(i) $\tilde{A} >_R \tilde{B}$ if and only if $R(\tilde{A}) > R(\tilde{B})$.

(ii) $\tilde{A} <_R \tilde{B}$ if and only if $R(\tilde{A}) < R(\tilde{B})$.

(iii) $\tilde{A} =_R \tilde{B}$ if and only if $R(\tilde{A}) = R(\tilde{B})$.

Let $\tilde{A} = (p_1, q_1, r_1, s_1; \omega_1)$ and $\tilde{B} = (p_2, q_2, r_2, s_2; \omega_2)$ be any two ω-trapezoidal fuzzy numbers, the following ranking function is given in [7],

$$R(\tilde{A}) = \frac{\omega_1(2p_1 + q_1 + r_1 + 2s_1)}{6},\ R(\tilde{B}) = \frac{\omega_2(2p_2 + q_2 + r_2 + 2s_2)}{6}.$$

4. A NEW MODIFIED OPTIMAL PERFECT MATCHING IN PARTIAL FEASIBLE MATCHING FOR SOLVING FLSAP

The step by step procedure for a new modified optimal matching in partial feasible matching procedure is as follows:

Step 1: First test whether the number of Persons is equal to the number of Jobs, if it is so, the ω-trapezoidal fuzzy cost assignment

Problem is said to be balanced, then continue step 2. If it is not balanced, a dummy row or a dummy column are introduced.

Step2: Calculate the ranking of ω-trapezoidal fuzzy cost $R[\tilde{C}_{ij}]$ and compute the rank of each cost of ω-trapezoidalfuzzy matrix.

Step3: Calculate the rank of fuzzy dual variables

$R[\tilde{v}_j] = \min\{R[\tilde{c}_{ij}];\ i = 1, \ldots, n$ and $j = 1, \ldots, n$.

$R[\tilde{u}_i] = \min\{R[\tilde{c}_{ij} - \tilde{v}_j];\ i = 1, \ldots, n$ and $j = 1, \ldots, n$.

Step4: Compute a partial feasible solution

If $R[\tilde{c}_{ij} - \tilde{v}_j] = \begin{cases} i \text{ if column } j \text{ is assigned to row } i \\ 0 \text{ if column } j \text{ is not assigned to row } i. \end{cases}$

Step 5: Calculate reduced rank of ω-trapezoidal fuzzy cost $R(\bar{\tilde{c}}_{ij})$

The reduced rank of ω-trapezoidal fuzzy cost $R[\tilde{c}_{ij} - \tilde{u}_i - \tilde{v}_j]$, then form a bipartite graph and find partial feasible matching

performed in j^{th} job to the i^{th} person in G.

Step6: Partial Matching to Perfect Matching

First we take the unassigned row [i^*] and calculate

$g^* = \arg\min\{ R[\tilde{c}_{ij} - \tilde{v}_j];\ j = 1,2,\ldots,n\}$ and $u_{g^*} = c_{ig^*} - v_{g^*}$

$h^* = \arg\min\{ R[\tilde{c}_{ij} - \tilde{v}_j];\ j = 1, 2,\ldots,n,\ g^* \neq h^*\}$ and $u_{h^*} = c_{ih^*} - v_{h^*}$

Step7: If $h^* > g^*$ then calculate $v_{g^{**}} = v_{g^*} - (u_{h^*} - u_{g^*})$.

Step8: Compute the new column and the new assignment introduced

If we consider $T = u_{h^*} - u_{g^*}$ then calculate new pivotal column $v_{g^{**}} = v_{g^*} + T$,

The new assignment is obtained as $z^* = u_{g^*} + T$.

Step9: To find the entering and the leaving assignment

The entering new assignment is z^* and the entering assignment column is known as the key assignment column or pivot
Assignment column. The old assigned element of the key assignment column is leaving assignment and introduced new
assignment z*. Suppose that the key assignment column is already not assigned to any old assignment then introduce the
new assignment z*.

Step10:To find the optimal perfect matching

Each row contains exactly one unmarked zero that is assigned and make an assigning to this single unmarked by encircling it and cross all other zeros in the column of this encircled zero, as these will not be considered for any other future allocations. Repeat in this way until all the rows have been covered.

(ii) Each column contains exactly one unmarked zero that is assigned and make an assigning to this single unmarked zero by
encircling it and cross any other zeros in it. Repeat in this way until all the columns have been covered. Finally perfect
the optimal assignment that is reached.

(iii) Form a bipartite Graph (G) and find optimal perfect matching assigning j^{th} job to the i^{th} person in G.

Step11: STOP.

5. NUMERICAL EXAMPLE

The fuzzy assignment problem associated with four Jobs J_1, J_2, J_3, J_4 and four Persons P_1, P_2, P_3, P_4 respectively. The fuzzy assignment cost to be ω-trapezoidal fuzzy numbers and allocating each row and each column exactly one perfect person.

Table 1:

Persons / jobs	**J1**	**J2**	**J3**	**J4**
P_1	(12,14,16,18;0.7)	(16,18,20,22;0.9)	(18,20,22,24;0.95)	(22,24,26,28;0.99)
P_2	(4,6,8,10;0.3)	(14,16,18,20;0.8)	(12,14,16,18;0.7)	(16,18,20,22;0.9)
P_3	(2,4,6,8;0.2)	(10,12,14,16;0.6)	(12,14,16,18;0.7)	(20,22,24,26;0.97)
P_4	(6,8,10,12;0.4)	(10,12,14,16;0.6)	(4,6,8,10;0.3)	(8,10,12,14;0.5)

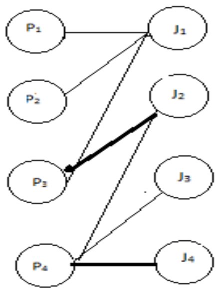
Table 2: Step 2: Calculate $R[\tilde{c}_{ij}]$

10.5	17.1	19.95	24.75
2.1	13.6	10.5	17.1
1	7.8	10.5	22.31
3.6	7.8	2.1	5.5

Table 3: Step 3: Calculate $R[\tilde{c}_{ij} - \tilde{v}_j]$

9.5	9.3	17.85	19.25
1.1	5.8	8.4	11.6
0	**0**	8.4	16.81
2.6	0	0	**0**

Table 4: Calculate $R[\tilde{c}_{ij} - \tilde{u}_i - \tilde{v}_j]$

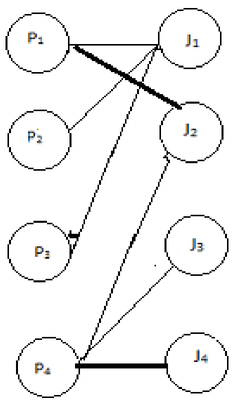


0.2	0	8.55	9.95
0	4.7	7.3	10.5
0	**0**	8.4	16.81
2.6	0	0	**0**

Figure 1: Partial Feasible Matching in Bipartite Graph.
v = (1,7.8,2.1,5.5) ; u = (9.3,1.1,0,0) ; φ = (0,0,2,4) ; ρ= (0,3,0,4).

First we take, $i^* = 1$;u_{g^*}= 9.3 ; u_{h^*}= 9.5 ; $v_{2^{**}} = v_{2^*}$ - (u_{h^*}- u_{g^*}) = 7.6.

Table 5: Calculate new feasible assignment

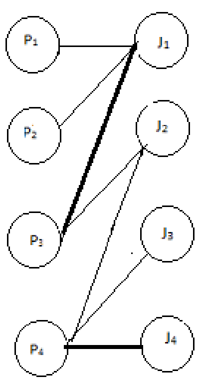


9.5	**9.5**	17.85	19.25
1.1	6	8.4	11.6
0	0.2	8.4	16.81
2.6	0.2	0	**0**

Figure 2: Partial Feasible Matching in Bipartite Graph.
V = (1,7.6,2.1,5.5);φ = (2,0,0,4) ; ρ = (0,1,0,4).

i^*= 3 ; u_{g^*}= 0 ; u_{h^*}= 0.2 ; $v_{2^{**}} = v_{2^*}$ - (u_{h^*}- u_{g^*}) = 0.8.

Table 6: Calculate new feasible assignment



9.7	9.5	17.85	19.25
1.3	6	8.4	11.6
0.2	0.2	8.4	16.81
2.8	0.2	0	**0**

Figure 3: Partial Feasible Matching in Bipartite Graph.
V = (0.8,7.6,2.1,5.5); φ = (0,0,1,4) ; ρ = (3,0,0,4).

i^* = 2 ; u_{g^*}= 1.3 ; u_{h^*}= 6 ; $v_{2^{**}} = v_{2^*}$ - (u_{h^*} - u_{g^*}) = -3.9.

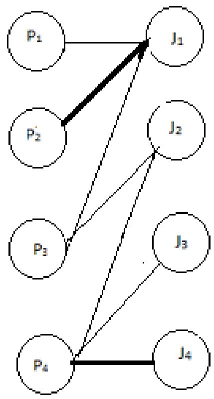


Table 7: Calculate new feasible assignment

14.4	9.5	17.85	19.25
6	6	8.4	11.6
4.9	0.2	8.4	16.81
7.5	0.2	0	**0**

Figure 4: Partial Feasible Matching in Bipartite Graph

V = (-3.9,7.6,2.1,5.5); φ = (0,1,0,4) ; ρ = (2,0,0,4).

$i^* = 3$; $u_{g^*}= 0.2$; $u_{h^*}= 4.9$; $v_{2^{**}} = v_{2^*} - (u_{h^*} - u_{g^*}) =2.9$.

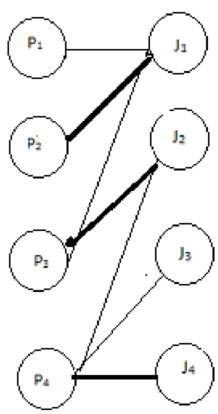


Table 8: Calculate new feasible assignment

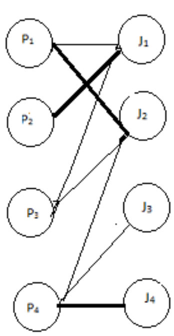
14.4	14.2	17.85	19.25
6	10.7	8.4	11.6
4.9	**4.9**	8.4	16.81
7.5	4.9	0	**0**

Figure 5: Partial Feasible Matching in Bipartite Graph

V = (-3.9,2.9,2.1,5.5); φ = (0,1,2,4) ; ρ = (2,3,0,4).

$i^* = 1$; $u_{g^*}=14.2$;$u_{h^*}= 14.4$; $v_{2^{**}} = v_{2^*} - (u_{h^*} - u_{g^*}) =2.7$.

Table 9: Calculate new feasible assignment



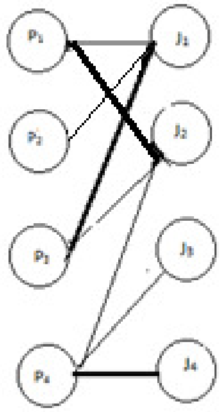
14.4	**14.4**	17.85	19.25
6	10.9	8.4	11.6
4.9	5.1	8.4	16.81
7.5	5.1	0	**0**

Figure 6: Partial Feasible Matching in Bipartite Graph

V = (-3.9,2.7,2.1,5.5); φ = (2,1,0,4) ; ρ = (2,1,0,4).

$i^* = 3$; $u_{g^*}= 4.9$; $u_{h^*}=5.1$; $v_{2^{**}} = v_{2^*} - (u_{h^*} - u_{g^*}) = -4.1$.

Table 10: Calculate new feasible assignment



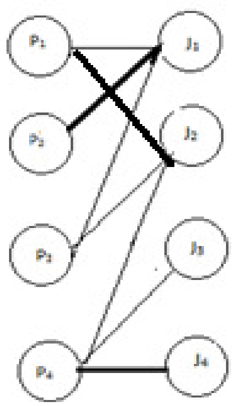
14.6	**14.4**	17.85	19.25
6.2	10.9	8.4	11.6
5.1	5.1	8.4	16.81
7.7	5.1	0	**0**

Figure 7: Partial Feasible Matching in Bipartite Graph

V = (-4.1,7.6,2.1,5.5); φ = (2,0,1,4) ; ρ = (3,1,0,4).

$i^* = 2$; $u_{g^*} = 6.2$; $u_{h^*} = 8.4$; $v_{2^{**}} = v_{2^*} - (u_{h^*} - u_{g^*}) = -6.3$

Table 11: Calculate new feasible assignment



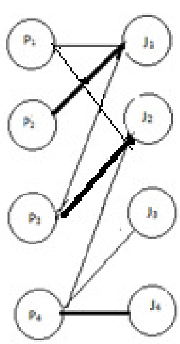
16.8	**14.4**	17.85	19.25
8.4	10.9	8.4	11.6
7.3	5.1	8.4	16.81
9.9	5.1	0	**0**

Figure 8: Partial Feasible Matching in Bipartite Graph

V = (-6.3,7.6,2.1,5.5); φ = (2,1,0,4) ; ρ = (2,1,0,4).

$i^* = 3$; $u_{g^*} = 5.1$; $u_{h^*} = 7.3$; $v_{2^{**}} = v_{2^*} - (u_{h^*} - u_{g^*}) = 5.4$

Table 12: Calculate new feasible assignment



16.8	16.6	17.85	19.25
8.4	13.1	8.4	11.6
7.3	**7.3**	8.4	16.81
9.9	7.3	0	**0**

Figure 9: Partial Feasible Matching in Bipartite Graph

V = (-6.3,5.4,2.1,5.5); φ = (0,1,2,4) ; ρ = (2,3,0,4).

$i^* = 1$; $u_{g^*} = 16.6$; $u_{h^*} = 16.8$; $v_{2^{**}} = v_{2^*} - (u_{h^*} - u_{g^*}) = 5.2$

Table 13: Calculate new feasible assignment

16.8	**16.8**	17.85	19.25
8.4	13.3	8.4	11.6
7.3	7.5	8.4	16.81
9.9	7.5	0	**0**

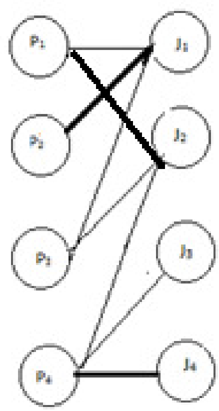


Figure 10: Partial Feasible Matching in Bipartite Graph

V = (-6.3,5.2,2.1,5.5); φ = (2,1,0,4) ; ρ = (2,1,0,4).

$i^* = 3$; $u_{g^*} = 7.3$; $u_{h^*} = 7.5$; $v_{2^{**}} = v_{2^*} - (u_{h^*} - u_{g^*}) = -6.5$

Table 14: Calculate new feasible assignment

17	**16.8**	17.85	19.25
8.6	13.3	8.4	11.6
7.5	7.5	8.4	16.81
10.1	7.5	0	**0**

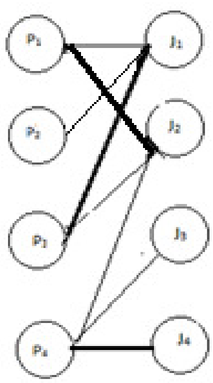


Figure 11: Partial Feasible Matching in Bipartite Graph

V = (-6.5,5.2,2.1,5.5); φ = (2,0,1,4) ; ρ = (3,1,0,4).

$i^* = 2$; $u_{g^*} = 8.4$; $u_{h^*} = 8.6$; $v_{2^{**}} = v_{2^*} - (u_{h^*} - u_{g^*}) = 1.9$

Table 15: Calculate new optimal assignment

17	***16.8***	18.05	19.25
8.6	13.3	***8.6***	11.6
7.5	7.5	8.6	16.81
10.1	7.5	0.2	**0**

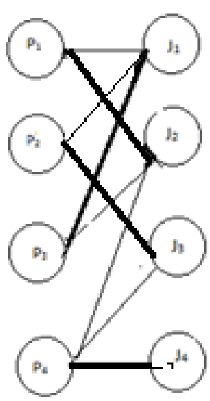


Figure 12: Optimal Perfect Matching in Bipartite Graph

V = (-6.5,5.2,1.9,5.5); φ = (2,3,0,4) ; ρ = (3,1,2,4).

The ω-Trapezoidal Fuzzy Optimal Assignment Table

Persons / jobs	$\mathbf{J_1}$	$\mathbf{J_2}$	$\mathbf{J_3}$	$\mathbf{J_4}$
$\mathbf{P_1}$	(12,14,16,18;0.7)	***(16,18,20,22;0.9)***	(18,20,22,24;0.6)	(22,24,26,28;0.7)
$\mathbf{P_2}$	(4,6,8,10;0.3)	(14,16,18,20;0.8)	***(12,14,16,18;0.7)***	(16,18,20,22;0.9)
$\mathbf{P_3}$	***(2,4,6,8;0.2)***	(10,12,14,16;0.6)	(12,14,16,18;0.7)	(20,22,24,26;0.8)
$\mathbf{P_4}$	(6,8,10,12;0.4)	(10,12,14,16;0.6)	(4,6,8,10;0.3)	***(8,10,12,14;0.5)***

The Optimal Perfect Matching Schedule is $P_1 \to J_2, P_2 \to J_3, P_3 \to J_1, P_4 \to J_4$.
The Optimal Perfect Matching in ω-trapezoidal fuzzy assignment cost is
(2, 4, 6, 8; 0.2) + (16,18,20,22;0.9) + (12,14,16,18;0.7) + (8,10,12,14;0.5) = *(38,46,54,62 ; 0.2)*
The Rank of Optimal Perfect Matching in fuzzy assignment cost is $R[\widetilde{C}_{ij}] = 10$.

5. CONCLUSION

The partial feasible matching in bipartite graph and optimal perfect matching in bipartite graph by using ω-trapezoidal fuzzy assignment cost are discussed. The optimal assigning through the j^{th} job to the i^{th} person and the total ω-trapezoidal fuzzy assignment cost is the Minimum.

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