Original Article

Bulletin of Pure and Applied Sciences.
Vol. 38E (Math & Stat.), No.1, 2019. P.441-449

Available online at
www.bpasjournals.com

Print version ISSN 0970 6577
Online version ISSN 2320 3226
DOI: 10.5958/2320-3226.2019.00048.1

FUZZY LINEAR PROGRAMMING PROBLEM USING POLYNOMIAL PENALTY METHOD

A. Nagoor Gani[1,*], R. Yogarani[2]

Authors Affiliation:
[1]P.G. and Research Department of Mathematics, Jamal Mohamed College (Autonomous), Tiruchirappalli, Tamil Nadu 620020, India.
[2]Department of Mathematics, Arul Anandar College, Madurai, Tamil Nadu 625514, India.

[*]**Corresponding Author:**
A. Nagoorgani, P.G. and Research Department of Mathematics, Jamal Mohamed College (Autonomous), Tiruchirappalli, Tamil Nadu 620020, India.

E-mail: ganijmc@yahoo.co.in

Received on 18.02.2019 Revised on 29.05.2019 Accepted on 08.06.2019

Abstract:
In this paper we focus on a polynomial order even penalty function for solving a fuzzy linear programming problem and develop an algorithm which gives a better rate of convergence to achieve the optimal solution to the problem in hand. Some numerical examples are included to exhibit the efficiency of the new algorithmic procedure developed by us.

Keywords: Fuzzy Linear Programming, triangular fuzzy number, polynomial penalty method, polynomial order even.

2010 Mathematics Subject Classification: 03E72, 90C70.

1. INTRODUCTION

The Penalty method is an alternative method to solve the fuzzy linear programming problem. In fact the Penalty method is a procedure for approximating constrained optimization problems by unconstrained problems by adding to the objective function a term that prescribes a high cost for violation of the constraints. Wright [17], Zboon et al. [19] and Parwadi et al. [13] give high cost to infeasible points in their works. Associated with this method is a parameter σ or δ that determines the penalty, hence, consequently the degree to which the unconstrained problem approximates the original constrained problem.

2. PRELIMINARIES

The fuzzy set theory has been useful to many disciplines such as control theory and management sciences, mathematical modeling. Real-world decision making problems are often uncertain (or) vague in some ways. In 1965, Zadeh introduced the concept of fuzzy set theory while the concept of fuzzy linear programming at the general level was first proposed by Tanaka and Asai [15].

2.1 Fuzzy Sets

A fuzzy set A is defined by $\tilde{A} = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0,1]\}$. A pair $(x, \mu_A(x))$, the first element x belongs to the classical set A, and the second element $\mu_A(x)$ belongs to the interval [0, 1], called the *membership function*.
A fuzzy set can also be denoted by, $\tilde{A} = \{\mu_A(x)/x : x \in A, \mu_A(x) \in [0,1]\}$. Here the symbol '/' does not represent the division sign. It indicates that the top number $\mu_A(x)$ is the membership value of the element x on the bottom.

2.2 FUZZY NUMBERS

Definition

Dubois. D and Prade. H [21] introduced the notion of fuzzy numbers. A fuzzy subset $\tilde{A}$ of the real line R with membership function $\mu_{\tilde{A}}: R \to [0,1]$ is called a fuzzy number if

i $\tilde{\boldsymbol{A}}$ is normal, (i.e.) there exists an element x_0 such that $\boldsymbol{\mu_{\tilde{A}}(x_0) = 1}$.

ii. $\tilde{\boldsymbol{A}}$ is fuzzy convex, (i.e.)

$$\mu_{\tilde{A}}[\lambda x_1 + (1-\lambda)x_2] \geq \mu_{\tilde{A}}(x_1) \wedge \mu_{\tilde{A}}(x_2), x_1, x_2 \in R, \forall \lambda \in [0,1]$$

iii. $\mu_{\tilde{A}}$ is upper continuous, and

iv. supp $\tilde{A}$is bounded, where supp $\tilde{A} = \{x \in R: \boldsymbol{\mu_{\tilde{A}}}(x) > 0\}$.

2.2.1 Generalized Fuzzy Number

Any fuzzy subset of the real line R, whose membership function satisfies the following conditions, is a generalized fuzzy number

i. $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to [0, 1],
ii. $\mu_{\tilde{A}}(x) = 0, -\infty < x \leq a_1$,
iii. $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$,
iv. $\mu_{\tilde{A}}(x) = 1, a_2 \leq x \leq a_3$,
v. $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$,
vi. $\mu_{\tilde{A}}(x) = 0, a_4 \leq x < \infty$,

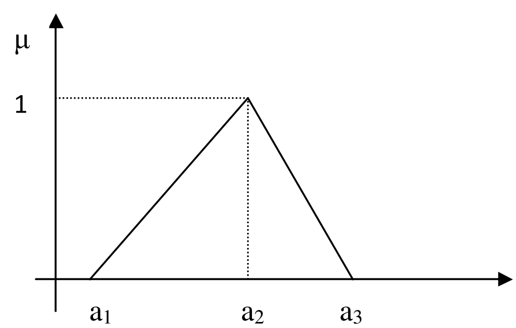
where a_1, a_2, a_3, a_4 are real numbers.

There are different fuzzy numbers. In the present research, the researchers have confined themselves only to the triangular fuzzy number and the trapezoidal fuzzy number.

2.3 Triangular Fuzzy Number

The fuzzy set $\tilde{\boldsymbol{A}} = (\boldsymbol{a_1}, \boldsymbol{a_2}, \boldsymbol{a_3})$ where $\boldsymbol{a_1} \leq \boldsymbol{a_2} \leq \boldsymbol{a_3}$ and defined on R, is called the triangular fuzzy number if membership function of $\tilde{\boldsymbol{A}}$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$



Triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$

2.4 Operations on Fuzzy Numbers

Though different methods are available for the operation of fuzzy numbers, the function principle and the extension principle are used for the operation of fuzzy numbers in the present paper.

2.4.1. Function Principle

The function principle was introduced by Hsieh and Chen [10] to treat fuzzy arithmetical operations. This principle is used for the operation of addition, multiplication, subtraction ,and division of fuzzy numbers.

Suppose $\tilde{\boldsymbol{A}} = (a_1, a_2, a_3)$ and $\widetilde{\boldsymbol{B}} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers. Then

i. The addition of $\tilde{A}$ and $\tilde{B}$ is
$\tilde{A}+\tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$ where $a_1, a_2, a_3, b_1, b_2, b_3$ are real numbers.

ii. The product of $\tilde{A}$ and $\tilde{B}$ is
$\tilde{A} \times \tilde{B} = (c_1, c_2, c_3)$ where $T = \{a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3\}$
$c_1 = \min T$, $c_2 = a_2 b_2$, $c_3 = \max T$
If $a_1, a_2, a_3, b_1, b_2, b_3$ are all non zero positive real numbers, then
$\tilde{A} \times \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3)$.

iii. $-\tilde{B} = (-b_3, -b_2, -b_1)$ then the subtraction of $\tilde{B}$ from $\tilde{A}$ is
$\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$ where $a_1, a_2, a_3, b_1, b_2, b_3$ are real numbers.

iv. $\frac{1}{\tilde{B}} = \tilde{B}^{-1} = 1/b_3, 1/b_2, 1/b_1$ where b_1, b_2, b_3 are all non zero real numbers then
$$\tilde{A}/\tilde{B} = (a_1/b_3, a_2/b_2, a_3/b_1)$$

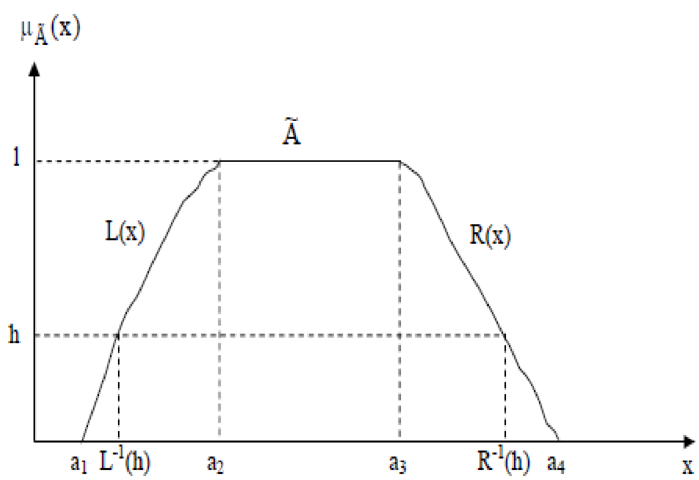
v. Let $\alpha \in R$, then $\alpha\tilde{A}$ $= (\alpha a_1, \alpha a_2, \alpha a_3)$ if $\alpha \geq 0$,
$= (\alpha a_3, \alpha a_2, \alpha a_1)$ if $\alpha < 0$.

2.5. Defuzzification

Defuzzification is the process of transforming fuzzy values to crisp values. Defuzzification methods have been widely studied for some years and are applied to fuzzy systems. The major idea behind these methods was to obtain a typical value from a given set according to some specified characters. Defuzzification method provides a correspondence from the set of all fuzzy sets into the set of all real numbers.

2.5.1. Graded Mean Integration Representation Method

Hsieh and Chen [10] introduced the Graded Mean Integration Representation Method based on the integral value of graded mean *h*-level of generalized fuzzy number for defuzzifying the generalized fuzzy number.



The graded mean *h*-level value of generalized fuzzy number $\tilde{A}$

Let $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$ be a generalized fuzzy number with left reference function L and right reference function R. Let L^{-1} and R^{-1} be the inverse functions of L and R respectively and the graded mean *h*-level value of generalized fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$ is $\frac{h}{2}[L^{-1}(h) + R^{-1}(h)]$ then the graded mean integration representation of $\tilde{A}$ is $p(\tilde{A})$ where

$$p(\tilde{A}) = \frac{\frac{1}{2}\int_0^1 h[L^{-1}(h)+R^{-1}(h)]dh}{\int_0^1 h dh} \quad \text{with } 0 < h \leq 1.$$

By the above formula, the graded mean integration representation of triangular fuzzy number

$\tilde{A} = (a_1, a_2, a_3)$ is given by

$$p(\tilde{A}) = \frac{a_1+4a_2+a_3}{6}.$$

2.5.2. Median Rule

One way of transforming a fuzzy set into a real number is characterized by choosing the median that divides the area under the membership function into two equal parts (median rule). If $\tilde{A} = (a_1, a_2, a_3)$ is a triangular fuzzy number, then by the median rule the defuzzified value of $\tilde{A}$ is given by

$$p(\tilde{A}) = \frac{a_1 + 2a_2 + a_3}{4}.$$

2.6 Fuzzy Linear Programming Problem

Consider the following fuzzy linear programming problem:
Maximum (or Minimum) $z = \tilde{c}x$
Constraints of the form
$Ax(\leq, =, \geq)\tilde{b}_j$, j=1,2,…,m
and the nonnegative conditions of the fuzzy variables $\tilde{x} \geq (0,0,0)$ where $\tilde{c}^T = (\tilde{c}_1,\ldots,\tilde{c}_n)$ is an n-dimensional constant vector, $A \in R^{m\times n}$, $\tilde{x} = (\tilde{x}_i)$, i=1,2,..,n and $\tilde{b}_j$ are non-negative fuzzy variable vectors such that $\tilde{x}_i$ and $\tilde{b}_j \in F(R)$ for all $1 \leq i \leq n$, $1 \leq j \leq m$, is called a fuzzy linear programming (FLP) problem.

2.7. Feasible solution: We say that vector $\tilde{x} \in F(R)^n$ is a feasible solution if and only if $\tilde{x}$ satisfies the constraints of the problem.

2.8. Optimal Solution: A feasible solution $\tilde{x}^*$ is an optimal solution if for all feasible solution $\tilde{x}$, we have $\tilde{c}\tilde{x}^* \geq \tilde{c}\tilde{x}$.

2.9. Fuzzy basic feasible solution: Here we describe fuzzy basic solution (FBFS) for the FLP problems which established by Nezam et al. [22] for the FVLP problem .Consider the system $A\tilde{x} = \tilde{b}$ and $\tilde{x} \geq \tilde{0}$. Let $A=[a_{ij}]_{m\times n}$. Assume that rank(A)=m. Partition A as $[B \;\; N]$ where B is nonsingular. It is obvious that rank (B)=m. Let y_j be the solution to $B_y = a_j$. It is apparent that the basic solution $\tilde{x}_B = \left(\tilde{x}_{B_1}, \tilde{x}_{B_2}, \ldots, \tilde{x}_{B_m}\right)^T = B^{-1}\tilde{b}$, $\tilde{x}_N = 0$ is a solution of $A\tilde{x} = \tilde{b}$. We call $\tilde{x}$, accordingly partitioned as $(\tilde{x}_B^T \tilde{x}_N^T)^T$, a fuzzy basic solution corresponding to the basis B. If $\tilde{x}_B \geq \tilde{0}$, then the fuzzy basic solution is feasible and the corresponding fuzzy objective value is $\tilde{z} = \tilde{c}_B\tilde{x}_B$, where $\tilde{c}_B = (\tilde{c}_{B_1}, \ldots, \tilde{c}_{B_m})$. Now corresponding to every fuzzy non basic variable $\tilde{x}_j, 1 \leq j \leq n, j \neq B_i$, and i=1, … , m, define $z_j = c_B y_j = c_B B^{-1} a_j$). If $\tilde{x}_B > \tilde{0}$, then $\tilde{x}$ is called non degenerate fuzzy basic feasible solution.

3. MAIN RESULTS

Consider the problem Minimize $z = \tilde{c}x$
Subject to $Mx_j = \tilde{N}_j, j = 1,2.$
where $M \in R^{m\times n}, \tilde{c}, x \in R^n, \tilde{N} \in R^m, \tilde{c} = (c_1, c_2, c_3), \tilde{N} = \left(n_1, n_2, n_3\right)$ (1)
Without loss of generality, assume that M has full rank m. The notation ~ denotes the fuzzy quantity described a triangular fuzzy number.
Assume that the problem (1) has at least one feasible solution.
For any scalar, $\delta > 0$ we define the polynomial penalty function $\tilde{p}(x, \delta)$ for the fuzzy linear programming problem.

3.1 Polynomial Penalty Method

Consider the polynomial penalty function $\tilde{p}(x, \delta)$ for the fuzzy linear programming problem (1).

$$\tilde{p}\left(x, \delta\right): R^n \to R \text{ by } \tilde{p}(x, \delta) = \tilde{c}x + \delta \sum_{j=1}^{m} (M_j x - \tilde{N}_j)^{\beta}, \quad (2)$$

where $\beta > 0$ is an even number. Here M_j and N_j denote the rows of the matrices M and N respectively. The positive even number β is chosen to ensure that the function (2) is convex. Hence $\tilde{p}(x, \delta)$ has a global minimum, where δ is the penalty parameter and $\beta > 0$ is an even number because the positive even number β

is selected so that the function (2) remains convex. The Polynomial Penalty Method uses the ordinary Lagrangian function to change the constraints $M_j x - \widetilde{N}_j$, (j=1, 2,…,m) and replace them by $(M_j x - \widetilde{N}_j)^{\beta}$.
The Polynomial Penalty Method for solving a sequence of the form
Minimize $p(x, \delta^k)$
subject to $x \leq 0$,
where δ^K is a rational penalty parameter sequence, satisfying $0 < \delta^k < \delta^{k+1}$ for all k, $\delta^k \to \infty$. Let $\{\delta^k\}$ be an increasing sequence of positive penalty parameters such that $\delta^k \to \infty$ as $k \to \infty$.

(a) $\{\tilde{c}\, x^k\}$ is non decreasing.

(b) $\{\sum_{i=1}^{m}(M_j x^k - \widetilde{N}_j)\}^{\beta}$ is non-inccreasing.

(c) $\tilde{P}(x^k, \delta^k)$ is non-decreasing.

3.2 Algorithm:

Given that $Mx_j = \widetilde{N}_j, x \geq 0, \delta^1 > 0$, the number of iteration I.

1. Write min $z = \tilde{c}x, Mx_j \leq \widetilde{N}_j, x_j \geq 0, j = 1,2$.
2. Convert the Lagrangian polynomial penalty function of the form
$$\tilde{p}(x) = [Mx_j - \widetilde{N}_j]^{\beta}, j = 1,2$$
where β is an even number, to
$$\tilde{p}(x, \delta) = \tilde{c}x + \delta[Mx_j - \widetilde{N}_j]^{\beta}.$$
3. The problem can be changed into the standard form of unconstrained problems
$$\text{Min}\, \tilde{p}(x, \delta) = \tilde{c}x + \delta[Mx_j - \widetilde{N}_j]^{\beta}.$$
4. Applying the first order necessary condition for optimality, taking the limit $\delta \to \infty$ we get the optimal value of the given fuzzy linear programming problem.
5. Compute $p(x^k, \delta^k) = \min_{x \geq 0} p(x, \delta^k)$, then minimize x^k & $\delta = 10, k = 1,2, \ldots, k = I$ then stop. Otherwise go to the step 5.

4. NUMERICAL EXAMPLE

Problem 4.1:

Consider the problem
Minimize $z = (3,4,5)x_1 + (2,3,4)x_2$
$2x_1 + 3x_2 \geq (5,6,7)$
$4x_1 + x_2 \geq (3,4,5),\ x_j \geq 0, j = 1,2.$

Solution:

Let Minimize $z = (3,4,5)x_1 + (2,3,4)x_2$
$-2x_1 - 3x_2 \leq -(5,6,7)$
$-4x_1 - x_2 \leq -(3,4,5), x_j \geq 0, j = 1,2.$
$\tilde{p}(x) = [-2x_1 - 3x_2 + (5,6,7)]^2 + [-4x_1 - x_2 + (3,4,5)]^2$,
$\tilde{p}(x, \delta) = (3,4,5)x_1 + (2,3,4)x_2 + \delta[-2x_1 - 3x_2 + (5,6,7)^2] + \delta[-4x_1 - x_2 + (3,4,5)]^2$,
The problem can be changed into the standard form of unconstrained problems
Min $\tilde{p}(x, \delta) = (3,4,5)x_1 + (2,3,4)x_2 + \delta[-2x_1 - 3x_2 + (5,6,7)^2] + \delta[-4x_1 - x_2 + (3,4,5)]^2$,
Applying the first order necessary condition for optimality, the derivatives of $\tilde{p}(x, \delta)$with respect to x_1 and x_2 respectively give as follows

$$\frac{\partial \tilde{p}(x,\delta)}{x_1} = 40\delta x_1 + 20\delta x_2 - \delta(44,56,68) + (-5,-4,-3), \quad (3)$$

$$\frac{\partial \tilde{p}(x,\delta)}{x_2} = 20\delta x_1 + 20\delta x_2 - \delta(36,44,52) + (2,3,4) \quad (4)$$

On solving (3) and (4) we get,

$$x_1 = \frac{(8,12,16)}{20} + \frac{(-3,-1,1)}{20\delta}, \quad x_2 = \frac{(28,32,36)}{20} - \frac{(-7,-1,3)}{20\delta}.$$

Taking the limit $\delta \to \infty$ we get ,

$$x_1 = \lim_{\delta \to \infty} \frac{(8,12,16)}{20} + \frac{(-3,-1,1)}{20\delta} = \frac{(8,12,16)}{20},$$

$x_2 = \lim_{\delta\to\infty} \frac{(28,32,36)}{20} - \frac{(-7,-1,3)}{20\delta} = \frac{(28,32,36)}{20}$.
When we look at the few iterations of the optimal of the problems are given as follows:
The table below shows that the value of $x_1(\delta)$ and $x_2(\delta)$for a sequence of parameters. We see that $X = (x_1(\delta), x_2(\delta))$ exhibits a linear rate of convergence to the optimal solution.

Table 1:

No.	δ^k	x_1	x_2
1	10	(0.385,0.595,0.805)	(1.365,1.595,1.815)
2	10^2	(0.3985,0.5995,0.8005)	(1.3965,1.5995,1.8015)
3	10^3	(0.39985,0.59995,0.80005)	(1.39965,1.59995,1.80015)
4	10^4	(0.399985,0.599995,0.800005)	(1.399965,1.599995,1.800015)
5	10^5	(0.3999985,0.5999995,0.8000005)	(1.3999965,1.5999995,1.8000015)
6	10^6	(0.39999985,0.59999995,0.80000005)	(1.39999965,1.59999995,1.80000015)
7	10^7	(0.399999985,0.599999995,0.800000005)	(1.399999965,1.599999995,1.800000015)
8	10^8	(0.399999999,0.600000000,0.800000000)	(1.399999997,1.600000000,1.800000002)
9	10^9	(0.400000000,0.6,000000000.800000000)	(1.400000000,1.600000000,1.800000000)

Therefore $X = [(0.4,0.6,0.8), (1.4,1.6,1.8)]$ is the solution and the minimum value of the function is z= (4,7.2,11.2).

Problem 4.2:
Consider the problem
Minimize $z = (0.3,0.4,0.5)x_1 + (0.4,0.5,0.6)x_2$
$0.3x_1 + 0.1x_2 \geq (2.6,2.7,2.8)$,
$0.5x_1 + 0.5x_2 = (5,6,7)$,
$x_j \geq 0, j = 1,2$.

Solution:
Let Minimize $z = (0.3,0.4,0.5)x_1 + (0.4,0.5,0.6)x_2$
$-0.3x_1 - 0.1x_2 \leq -(2.6,2.7,2.8)$,
$0.5x_1 + 0.5x_2 = (5,6,7)$.
$\tilde{p}(x) = [-0.3x_1 - 0.1x_2 + (2.6,2.7,2.8)]^2 + [0.5x_1 + 0.5x_2 - (5,6,7)]^2$
$$\tilde{p}(x,\delta) = (0.3,0.4,0.5)x_1 + (0.4,0.5,0.6)x_2 + \delta[-0.3x_1 - 0.1x_2 + (2.6,2.7,2.8)]^2 + \delta[0.5x_1 + 0.5x_2 - (5,6,7)]^2$$
The problem can be changed into the standard form of unconstrained problems
$$\text{Min } \tilde{p}(x,\delta) = (0.3,0.4,0.5)x_1 + (0.4,0.5,0.6)x_2 + \delta[-0.3x_1 - 0.1x_2 + (2.6,2.7,2.8)]^2 + \delta[0.5x_1 + 0.5x_2 - (5,6,7)]^2$$
Applying the first order necessary condition for optimality, the derivatives of $\tilde{P}(x,\delta)$ with respect to x_1 and x_2 respectively give as follows
$$\frac{\partial\tilde{p}(x,\delta)}{x_1} = 0.68\delta x_1 + 0.56\delta x_2 - \delta(6.56,7.62,8.68) + (0.3,0.4,0.5), \quad (5)$$
$$\frac{\partial\tilde{p}(x,\delta)}{x_2} = 0.56\delta x_1 + 0.5\delta x_2 - \delta(5.52,6.54,7.56) + (0.4,0.5,0.6). \quad (6)$$
Solving (5) and (6) we get,

$$x_1 = \frac{(5.44,5.10,4.76)}{0.68} + \frac{(1.156,1.224,1.292)}{0.68\delta}, \qquad x_2 = \frac{(0.08,0.18,0.28)}{0.04} - \frac{(0.104,0.116,0.128)}{0.04\delta}$$

Taking the limit $\delta \to \infty$ we get ,

$$x_1 = \lim_{\delta\to\infty} \frac{(5.44,5.10,4.76)}{0.68} + \frac{(1.156,1.224,1.292)}{0.68\delta} = \frac{(5.44,5.10,4.76)}{0.68},$$
$$x_2 = \lim_{\delta\to\infty} \frac{(0.08,0.18,0.28)}{0.04} - \frac{(0.104,0.116,0.128)}{0.04\delta} = \frac{(0.08,0.18,0.28)}{0.04}$$

When we look at the few iterations of the optimal of the problems they are given as follows:
The following table shows the value of $x_1(\delta)$ and $x_2(\delta)$for a sequence of parameters. We see that $X = (x_1(\delta), x_2(\delta))$ exhibits a linear rate of convergence to the optimal solution.

Table 2:

No.	δ^k	x_1	x_2
1	10	(7.17,7.68,8.19)	(1.74,4.210,7.32)
2	10^2	(7.017,7.518,8.019)	(1.974,4.471,7.032)
3	10^3	(7.0017,7.5018,8.0019)	(1.9974,4.4971,7.0032)
4	10^4	(7.00017,7.50018,8.00019)	(1.99974,4.49971,7.00032)
5	10^5	(7.000017,7.500018,8.000019)	(1.999974,4.499971,7.000032)
6	10^6	(7.0000017,7.5000018,8.0000019)	(1.9999974,4.4999971,7.0000032)
7	10^7	(7.00000017,7.50000018,8.00000019)	(1.99999974,4.49999971,7.00000032)
8	10^8	(7.000000017,7.500000018,8.000000019)	(1.999999974,4.499999971,7.000000032)
9	10^9	(7.000000002,7.500000002,8.000000002)	(1.999999997,4.499999997,7.0000000032)
10	10^{10}	(7.000000000,7.500000000,8.000000000)	(2.000000000,4.5000000000,7.0000000000)

Therefore $X = [(7,7.5,8),(2,4.5,7)]$ is the solution and the minimum value of the function is $z = (2.9,5.25,8.2)$.

Problem 4.3:
Consider the problem
Minimize $z = (2,3,4)x_1 + (7,8,9)x_2$
$3x_1 + 4x_2 \le (19,20,21)$,
$x_1 + 3x_2 \ge (11,12,13)$,
$x_j \ge 0, j = 1,2$.
Solution:
Let Minimize $z = (2,3,4)x_1 + (7,8,9)x_2$
$3x_1 + 4x_2 \le (19,20,21)$,
$-x_1 - 3x_2 \le -(11,12,13)$,
$\tilde{p}(x) = [3x_1 + 4x_2 - (19,20,21)]^2 + [-x_1 - 3x_2 + (11,12,13)]^2$,
$\tilde{p}(x,\delta) = (2,3,4)x_1 + (7,8,9)x_2 + \delta[3x_1 + 4x_2 - (19,20,21)]^2 + \delta[-x_1 - 3x_2 + (11,12,13)]^2$
The problem can be changed into the standard form of unconstrained problems
Min $\tilde{p}(x,\delta) = (2,3,4)x_1 + (7,8,9)x_2 + \delta[3x_1 + 4x_2 - (19,20,21)]^2 + \delta[-x_1 - 3x_2 + (11,12,13)]^2$

Applying the first order necessary condition for optimality, the derivatives of $\tilde{p}(x,\delta)$with respect to x_1 and x_2 respectively give as follows

$$\frac{\partial\tilde{p}(x,\delta)}{x_1} = 20\delta x_1 + 30\delta x_2 - \delta(136,144,152) + (2,3,4), \quad (7)$$

$$\frac{\partial\tilde{p}(x,\delta)}{x_2} = 30\delta x_1 + 50\delta x_2 - \delta(218,232,246) + (7,8,9). \quad (8)$$

Solving (7) and (8) and on taking the limit $\delta \to \infty$ we get,

$$x_1 = \lim_{\delta\to\infty} \frac{(52,48,44)}{20} + \frac{(22,18,14)}{20\delta} = \frac{(52,48,44)}{20},$$

$$x_2 = \lim_{\delta\to\infty} \frac{(280,320,360)}{100} + \frac{(60,70,80)}{100\delta} = \frac{(280,320,360)}{100}.$$

When we look at the few iterations of the optimal of the problems they given as follows:
The table below shows the value of $x_1(\delta)$ and $x_2(\delta)$ for a sequence of parameters. We see that $X = (x_1(\delta), x_2(\delta))$ exhibits a linear rate of convergence to the optimal solution.

Table 3:

No.	δ^k	x_1	x_2
1	10	(2.27,2.49,2.710)	(2.74,3.130,3.52)
2	10^2	(2.207,2.409,2.611)	(2.794,3.1930,3.592)
3	10^3	(2.2007,2.4009,2.6011)	(2.7994,3.19930,3.5992)
4	10^4	(2.20007,2.40009,2.60011)	(2.79994,3.199930,3.59992)
5	10^5	(2.200007,2.400009,2.600011)	(2.799994,3.1999930,3.599992)
6	10^6	(2.2000007,2.4000009,2.6000011)	(2.7999994,3.19999930,3.5999992)
7	10^7	(2.20000007,2.40000009,2.60000011)	(2.79999994,3.199999930,3.59999992)
8	10^8	(2.200000007,2.400000009,2.600000011)	(2.799999994,3.1999999930,3.599999992)
9	10^9	(2.200000001,2.400000001,2.6000000001)	(2.7999999994,3.19999999930,3.5999999992)

Therefore $X = [(2.2,2.4,2.6),(2.8,3.2,3.6)]$ is the solution of the minimum value of the function is $z = (24.0,32.8,42.8)$.

5. CONCLUSION

In this paper our algorithm gives the better solution. The polynomial penalty function with the penalty terms in even order for solving the fuzzy linear programming problem by using the proposed algorithm to obtain the better optimal solution is discussed. The three tables for the three problems considered above show that computational procedure for the algorithm developed by us when β is an even number, gives a better the rate of convergence to the optimal solution.

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