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Some Results on Cordial Labeling and Variations of Cordial Labeling

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ABSTRACT

Rosa [14] pioneered graph labeling is assigning integers to vertices, edges or both under specific constraints. In 2025, Jeba Jesintha *et al*. [10], [11] have introduced two variations of cordial labeling namely *Fibonacci mean cordial labeling* and *Octa –cordial labeling*. This paper discusses the existence of cordial labeling of christmas tree, Fibonacci mean cordial labeling of comb graph and Octa-cordial labeling of arrow graph.

Keyword: Cordial Labeling, Fibonacci Mean Cordial Labeling, Octa-Cordial Labeling, Arrow Graph, Comb Graph.

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1. INTRODUCTION

Rosa [14] developed a labeling technique known as β-valuation in 1967 as a means of breaking down the complete graph into subgraphs that are isomorphic. The β – *valuation* is referred to as graceful labeling. *Cordial labeling* was defined by Cahit [3] as " A graph G is said to be *cordial* if there exists a function $\varphi : V(G) \rightarrow \{0,1\}$ such that each edge uv in G is assigned the label $|\varphi(u) - \varphi(v)|$ with the property $|v_\varphi(0) - v_\varphi(1)| \leq 1$ and $|e_\varphi(0) - e_\varphi(1)| \leq 1$, where $v_\varphi(i)$ for $i = 0,1$ denotes the number of vertices with label i and $e_\varphi(i)$ for $i = 0, 1$ denotes the number of edges with label i." The graph that admits cordial labeling is *cordial graph*. Andar [1] *et.al* showed helms and closed helms graph admit cordial labeling. Lee and Shee [13] have proved unicyclic graph is cordial. Divya Dharshini and Mary [4] have proved splitting graphs admit cordial labeling. Ghodasara and Rokad [6] have proved one- point union of graphs is cordial.

"*Mean cordial labeling* was defined by Somasundaram and Ponraj [15] and the edge uv assign the label $\left\lceil\frac{\varphi(u)+\varphi(v)}{2}\right\rceil$ is called mean cordial labeling if φ is $\left|v_{\varphi}(i)-v_{\varphi}(j)\right| \leq 1$ and $\left|e_{\varphi}(i)-e_{\varphi}(j)\right| \leq 1$ for $i,j = \{0,1,2\}$ where $v_{\varphi}(x)$ and $e_{\varphi}(x)$ denote the number of vertices and edges with label i". A graph that admits a labeling is called *mean cordial graph.* Somasundaram and Ponraj [15] have proved star and complete graph is mean cordial. Ansari and Ansari [2] have proved that ladder and shadow graph admit mean cordial labeling. For more results on Cordial and Mean Cordial labeling, one can refer to the Gallian survey [5].

"The *Fibonacci sequence*, $F = \{0,1,1,2,3,5,8,\dots\}$ is defined by the recursive formula the $F_n = F_{n-1} + F_{n-2}$ with the initial conditions F_0 = 0 and F_1= 1". "The concept of *Fibonacci Mean Cordial Labeling* was introduced by Jeba Jesintha, *et al.* [10] to incorporate the Fibonacci sequence in the labeling process and it is an injective function $\varphi: V(G) \to \{F_0, F_1, \dots, F_{2q}\}$, where $F_i's$ for $0 \leq i \leq 2q$ are the Fibonacci numbers and the induced edge labeling is defined as $\varphi^*: E(G) \to \{0,1\}$ is defined by

$$\varphi^*(uv) = \begin{cases} \left\lceil\dfrac{\varphi(u)+\varphi(v)}{2}\right\rceil (mod\ 2), & \text{if } [\varphi(u)+\varphi(v)] \text{ is even} \\ \left\lceil\dfrac{\varphi(u)+\varphi(v)+1}{2}\right\rceil (mod\ 2), & \text{if } [\varphi(u)+\varphi(v)] \text{ is odd} \end{cases}$$

such that $|e_{\varphi*}(0) - e_{\varphi*}(1)| \leq 1$. The graph which admits Fibonacci mean cordial labeling is called as *Fibonacci mean cordial graph"*. Jeba Jesintha, *et al*. [10] obtained the Fibonacci mean cordial labeling for the path graph.

The *Octa-Cordial Labeling* is introduced by Jeba Jesintha *et.al* [11]. "Let *G* (*p, q*) be a graph, an octa-cordial labeling is a function $\varphi: V(G) \to O_{8k}$, where $k = 1$, if $p \leq 8$ and $k = \lfloor\frac{p}{8}\rfloor + 1$, if $p > 8$. For any edge $e = uv$ the resulting edge labeling $\varphi^*: E(G) \to \{0,1\}$ is given by $\varphi^*(e) = [\varphi(u)+\varphi(v)](mod2)$ satisfying $\left|e_{\varphi}(0) - e_{\varphi}(1)\right| \leq 1$, where $e_{\varphi}(i)$ denotes the number of vertices receiving the label i, for i = 0,1. The graph admitting Octa-cordial labeling is called an *Octa-cordial graph."*

In this paper, the cordial labeling of christmas tree, the Fibonacci mean cordial labeling of comb graph and the Octa-cordial labeling of arrow graph is proved.

2. MAIN RESULTS

This section states few definition initially and theorems on cordial and variations of cordial labelling on certain graphs is proved.

Definition 2.1 [5] "A *slim tree* is a rooted binary tree with consecutive vertices in the last level being adjacent. The slim tree with s levels is called the s^{th} slim tree."

Definition 2.2 [5] "A *Christmas tree* is composed of 3 copies of s^{th} slim tree, the root node of each slim tree appended to a new node u_1. Also the right node at the level s of the first slim tree ST_1 is joined to the left node at level s of the second slim tree ST_2. The right node at level s of the second slim tree ST_2 is joined to the left node at the level s of the third slim tree ST_3. The left node at the level s of the first slim tree ST_1 is joined to the right node at level s of the third slim tree ST_3. The new node u_1 is the root of the Christmas tree."

Definition 2.3 [4] "An *arrow graph* A_n^t with width t and length n is obtained by joining a vertex v with superior vertices of $P_m \times P_n$ by m new edges from one end".

Definition 2.4. [5] "A *Comb graph* $P_n \odot K_1$ is obtained by attaching pendant edges to the each vertex of the path graph".

Definition 2.5 "An *Octal number* is a number in a base 8 system, which uses eight digits from 0 to 7 in order to represent numerical values. It is denoted by the system $O = \{0, 1, 2, \dots, 7\}$".

Definition 2.6 [11] "Let G = (p, q) be a graph. An *octa-cordial labeling* is a function $f: V(G) \rightarrow O_{8k}$, where $k = 1$, if $p \leq 8$ and $k = \lfloor\frac{p}{8}\rfloor + 1$, if $p > 8$. For any edge $e = uv$ the resulting edge labeling $\varphi^*: E(G) \rightarrow \{0,1\}$ is given by $\varphi^*(e) = [\varphi(u) + \varphi(v)](mod\ 2)$ satisfying $|e_\varphi(0) - e_\varphi(1)| \leq 1$, where $e_\varphi(i)$ denotes the number of vertices receiving the label i, for i = 0,1".

Theorem 2.1 The Christmas Tree admit cordial labeling.

Proof. Let G be a Christmas tree. Let v_0 be the root vertex of G. We denote the levels of Christmas tree as $l_0, l_2, \ldots, l_{n-1}$. Let v_i^j be vertices, for all $0 \leq i \leq n-1, 1 \leq j \leq (3.2^{i-1})$. The generalized Christmas tree is shown in Figure 1. The number of vertices and edges of G is given by $|V(G)| = 3(2^{n-1} - 1) + 1$ and $|E(G)|$ = 3($3.2^{n-2} - 2$) respectively.

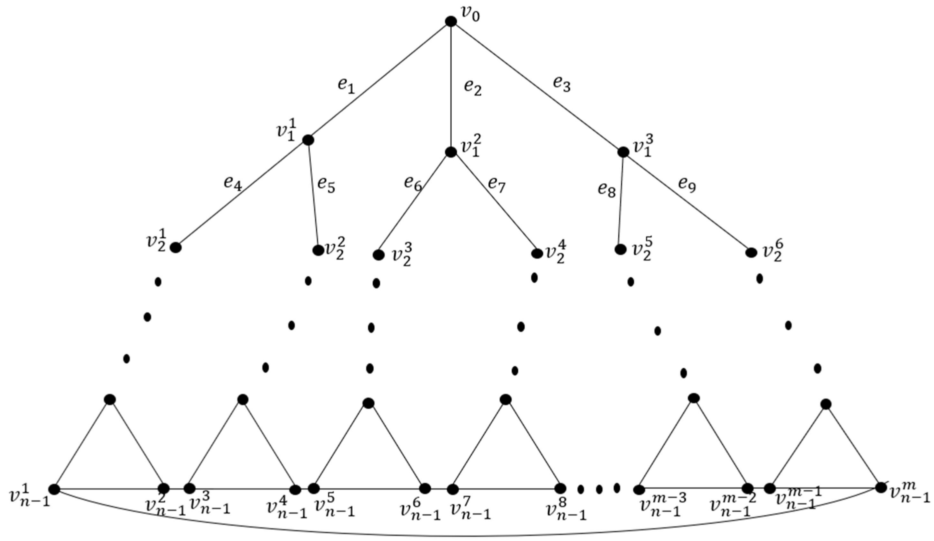

Figure 1: Generalized Christmas Tree

Define the vertex label $\varphi : V(G) \rightarrow$\{0,1\} by

$$\left.\begin{aligned} \varphi(v_o) &= 0 \\ \varphi(v_1^1) &= 0 \\ \varphi(v_1^2) &= 0 \\ \varphi(v_1^3) &= 0 \end{aligned}\right\} \qquad (1)$$

$$\varphi(v_i^j) = \begin{cases} 1, & if\ j = 0(mod\ 2) \\ 0, & if\ j = 1\ (mod\ 2) \end{cases} \quad for\ 3 \leq i \leq n-1,\ 1 \leq j \leq 3.2^{i-1} \qquad (2)$$

$$\varphi(v_n^j) = \begin{cases} 0, & if\ j = 1,0\ (mod\ 4) \\ 1, & if\ j = 2,3\ (mod\ 4) \end{cases} \quad for\ 1 \leq j \leq 3.2^{i-1} \qquad (3)$$

Number of vertices labeled with 0 and 1 is computed as follows:

$$\begin{aligned} v_\varphi(0) &= \left\lceil\frac{3(2^{n-1}-1)+1}{2}\right\rceil \\ v_\varphi(1) &= \left\lceil\frac{3(2^{n-1}-1)+1}{2}\right\rceil \end{aligned} \qquad (4)$$

Number of edges labeled with 0 and 1 is computed as follows:

$$\begin{aligned} e_\varphi(0) &= \left\lceil\frac{3(3.2^{n-2}-2)}{2}\right\rceil \\ e_\varphi(1) &= \left\lceil\frac{3(3.2^{n-2}-2)}{2}\right\rceil + 1 \end{aligned} \qquad (5)$$

From the above - computed equations in (4) and (5), we have

$$\left|v_\varphi(0) - v_\varphi(1)\right| \le \left\lceil\frac{3(2^{n-1}-1)+1}{2}\right\rceil - \left\lfloor\frac{3(2^{n-1}-1)+1}{2}\right\rfloor \le 1$$
$$\left|e_\varphi(0) - e_\varphi(1)\right| \le \left\lceil\frac{3(3.2^{n-2}-2)}{2}\right\rceil - \left\lfloor\frac{3(3.2^{n-2}-2)}{2}\right\rfloor + 1 \le 1$$

Therefore, Christmas tree admits cordial labeling.
Illustration for the Theorem 1 is depicted in Figure 2.

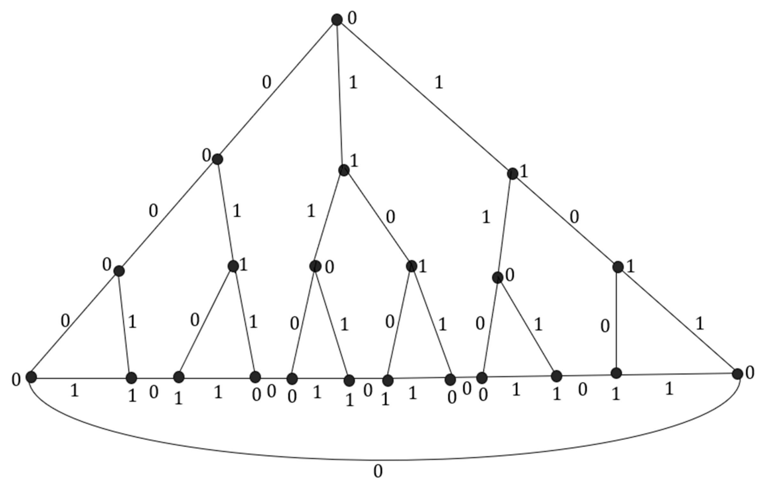

Figure 2: Christmas tree for $n = 4$

Theorem 2.2. The Comb graph $P_n \odot K_1$ admits Fibonacci mean cordial labeling.
Proof. Let $P_n \odot K_1$ denotes the Comb graph. The number of vertices and edges be denoted as p and q respectively. The path vertices of the Comb graph be denoted as $u_1, u_2, \dots u_n$ and the pendant vertices attached to the path vertices denoted as $v_1, v_2, \dots, v_n$. Let $p = |V(P_n \odot K_1,)| = 2n$ and $q = |E(P_n \odot K_1,)| = 2n - 1$. $P_n \odot K_1,$ is depicted in Figure 3.

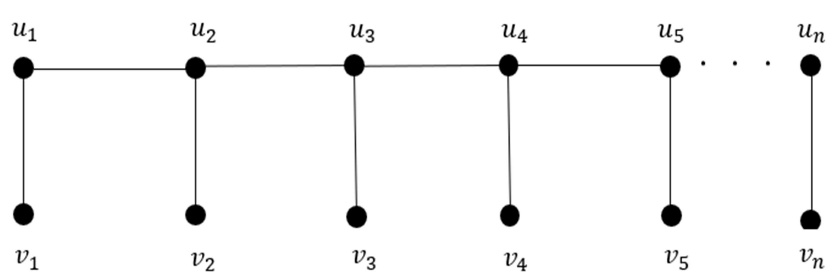

Figure 3: Comb graph $P_n \odot K_1$

Vertex label $\varphi: V(G) \rightarrow \{F_0, F_1, \dots, F_{2q}\}$ as follows:

$$\varphi(u_i) = F_{3i-2}\ ,\ \ 1 \le i \le n$$

$$\varphi(v_i) = F_{3i-3},\ \ 1 \le i \le n$$

Edge label $\varphi^*: E(G) \rightarrow \{0,1\}$ is defined by

$$e_{\varphi^*}(1) = \left\lfloor \frac{2n-1}{2} \right\rfloor + 1$$

$$e_{\varphi^*}(0) = \left\lfloor \frac{2n-1}{2} \right\rfloor$$

From the above equations, it is observed that the condition $|e_{\varphi^*}(0) - e_{\varphi^*}(1)| \le 1$ is satisfied. Therefore, the Comb graph admits FMCL. Illustration for the FMCL is depicted in Figure 4.

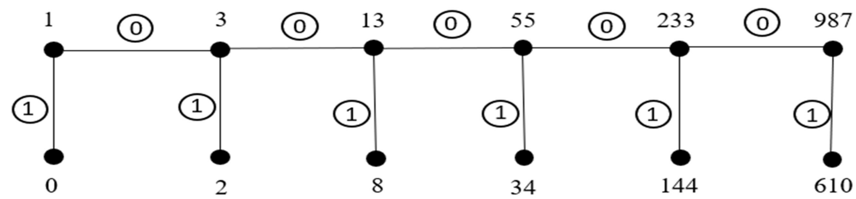

Figure 4: Fibonacci mean cordial labeling for Comb graph $P_6 \odot K_1$

Theorem 2.3 An arrow graph A_n^2 admits Octa-Cordial labeling.

Proof : Let A_n^2 denote an Arrow graph. The apex vertex is denoted as w. Let $u_1, u_2, u_3, u_4, \dots, u_n$ be the vertices above the apex of arrow graph. Let $v_1, v_2, v_3, v_4, \dots, v_n$ denote the vertices below the apex of the arrow graph. Arrow graph A_n^2 is shown in Figure 5.

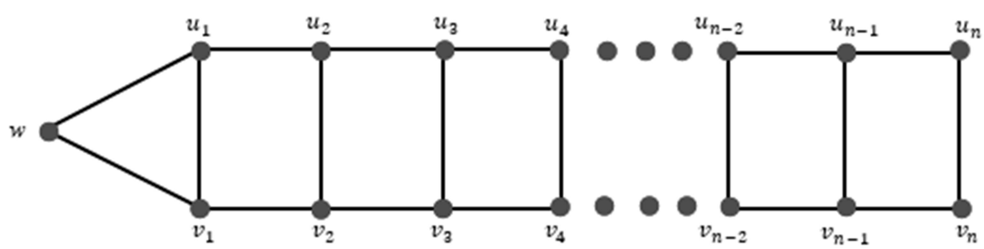

Figure 5: Arrow graph A_n^2

The vertex labeling $\varphi: V(A_n^2) \rightarrow O_{8k}$ is defined by

$$\varphi(w) = 0$$

$$\varphi(v_{4i-3}) = 3 \quad ; \quad 1 \le i \le \left\lfloor \frac{n}{3} \right\rfloor$$

$$\varphi(v_{4i-2}) = 1 \quad ; \quad 1 \le i \le \left\lfloor \frac{n}{3} \right\rfloor$$

$$\varphi(v_{4i-1}) = 2 \quad ; \quad 1 \le i \le \left\lfloor \frac{n}{3} \right\rfloor$$

$$\varphi(v_{4i}) = 0 \quad ; \quad 1 \le i \le \left\lfloor \frac{n}{3} \right\rfloor$$

$$\varphi(u_{4i-3}) = 4 \quad ; \quad 1 \le i \le \left\lfloor \frac{n}{3} \right\rfloor$$

$$\varphi(u_{4i-2}) = 7 \quad ; \quad 1 \le i \le \left\lfloor \frac{n}{3} \right\rfloor$$

$$\varphi(u_{4i-1}) = 5 \quad ; \quad 1 \le i \le \left\lfloor \frac{n}{3} \right\rfloor$$

$$\varphi(u_{4i}) = 6 \quad ; \quad 1 \le i \le \left\lfloor \frac{n}{3} \right\rfloor$$

The induced edge label $\varphi^* : E((A_n^2\,) \rightarrow (0,1)$ is defined as

$$\varphi^*(u_{2i-1}u_{2i}) = 1 \quad ; \quad 1 \leq i \leq \frac{n}{2}$$
$$\varphi^*(u_{2i}u_{2i+1}) = 0 \quad ; \quad 1 \leq i \leq \frac{n}{2}$$
$$\varphi^*\,(v_{2i-1}v_{2i}) = 1 \quad ; \quad 1 \leq i \leq \frac{n}{2}$$
$$\varphi^*(v_{2i}v_{2i+1}) = 0 \quad ; \quad 1 \leq i \leq \frac{n}{2}$$
$$\varphi^*(u_{2i-1}v_{2i-1}) = 1 \quad ; \quad 1 \leq i \leq \frac{n}{2}$$
$$\varphi^*(u_{2i}v_{2i}) = 0 \quad ; \quad 1 \leq i \leq \frac{n}{2}$$
$$\varphi^*(wu_1) = 0$$
$$\varphi^*(wv_1) = 1$$

Thus, $\left|e_\varphi(0) - \, e_\varphi(1)\right| \leq 1$ for an Arrow graph A_n^2. Thus an arrow graph A_n^2 admits Octa-Cordial labeling. An Illustration is depicted in Figure 6.

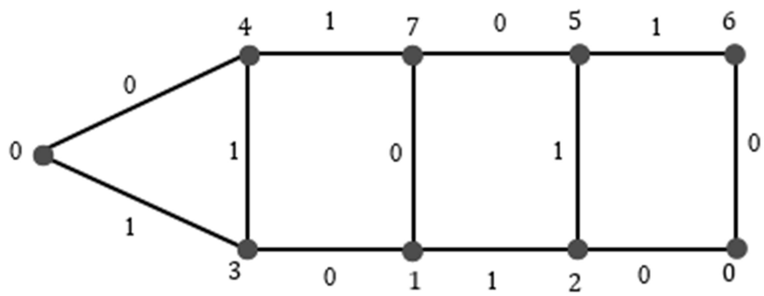

Figure 6: Octa-cordial labeling of arrow graph A_4^2

3. CONCLUSION

This study proves the cordial labeling for Christmas tree, the Fibonacci mean cordial labeling for comb graph and the Octa-cordial labeling for arrow graph. We intend to extend Fibonacci mean cordial labeling and Octa-cordial labeling to certain graphs like gear graph, helm graphs, double arrow graph and multiple arrow graph. Further we intend to arrive at more variations of cordial labeling.

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