Original Article
Bulletin of Pure and Applied Sciences.
Mathematics & Statistics Vol.45 E, No.1.
January-June 2026: P.1-11

Print version ISSN 0970 6577
Online version ISSN 2320 3226
Available online at www.bpasjournals.com

An Investigation on Dynamics and Stability of a Predator-Prey Model with Logistic prey growth and Interspecific Competition

[1]Raju Kumar* and [2]Dr. P.K. Choudhary

Author Affiliation:
[1]Research scholar, Department of Mathematics, Patna University, Patna, Bihar-800005, India
[2]Associate professor, Department of Mathematics, Patna University, Patna, Bihar-800005, India

***Corresponding Author:** Raju Kumar, Department of Mathematics, Patna University, Patna, Bihar-800005, India
E-mail: rajukumar-rsmaths@pup.ac.in

ABSTRACT
This paper investigates a mathematical analysis of an extended predator- prey model that includes logistic growth for prey population and interspecific competition influencing both species. To incorporate more realistic ecological interaction, the classical Lotka-Volterra model is extended by incorporating a carrying capacity for the prey and competitive terms. The equilibrium point of the extended model is derived and derive stability analysis using linearization and Jacobian matrix. Numerical simulations are performed to demonstrates the dynamic behavior of the system under various initial conditions. This study highlights that including logistic growth and interspecific competition yield a more precise and robust representation of the predator –prey dynamics. This result provides valuable understanding ecological balance and help in managing biological populations in competitive environments.

Keywords: Predator-prey model; Logistic prey growth; Interspecific competition; Stability; Non- linear dynamics; Mathematical biology

How to cite this article: Kumar R. and Choudhary P.K. An Investigation on Dynamics and Stability of a Predator-Prey Model with Logistic prey growth and Interspecific Competition. *Bulletin of Pure and Applied Sciences- Math & Stat.*, 2026;45E (1): 1-11

Received on 25.07.2025, Revised on 21.09.2025, Accepted on 06.11.2025

1. INTRODUCTION

Mathematical modelling plays an important role in understanding and analyzing the dynamic of predator-prey interaction in ecological system [1,2]. The relationship between predator and their prey are crucial for balancing ecosystem [3,4]. The earliest and most important contributions to theoretical population biology were made by Alfred J. Lotka in 1920V. Volterra in 1927 during the early 20th century. Their works represent one of the first mathematical model to explain the interactions between predator and prey population [5]. This classical model, now known as Lotka- Volterra model or predator-prey model, has become a cornerstone of mathematical biology and population ecology [6,7].

In particular, the predator-prey model is a crucial tool in mathematical biology, and plays an important role in enhancing our understanding of how populations interact with real world environment. Lotka-

Volterra model is a set of paired non-linear ordinary differential equation used to represent the interactions of two populations out of which one is taken as prey and another is predator. The Lotka-Volterra models are developed from experimental data and observations and use to make predictions. Biological or ecological or ecological phenomenon is influenced not only by its current value but it also depends on past history. The Population is an ecological interaction where a predator feeds on a prey. In ecological prediction happens across various scenario like in interaction among wild animals, between herbivores and plants and between host and parasite. If the predator species is to persist over many generations, it must consume enough prey otherwise its population its population will decline over time and may eventually drive to extinction. On the other side if predator consume more prey the prey population will decline and predator will also die out from starvation. The population of prey grows exponentially in the absence of predators while the population of predators depends entirely on prey availability for survival and reproduction [4,7,8]. Although highly influential, the classical model assumes unlimited resources and ignores ecological constraints such as intra-species competition, environmental carrying capacity, and external mortality factors, which makes it unrealistic in many real-world contexts [3,4,9,10,11,12].

To address these limitations, researchers have extended the model by introducing logistic growth for the prey, which imposes a carrying capacity and restricts unbounded growth [8–10]. Other extensions include interspecific competition [6,10,13] and variable carrying capacity. Such refinements provide a more accurate description of biological and ecological systems [11, 12, 13, 14].

In this study, we propose a modified predator–prey model that incorporates logistic growth for the prey and interspecific competition between species. This formulation overcomes two limitations of the classical Lotka–Volterra equations by constraining population growth and accounting for competitive effects. The resulting nonlinear system is analysed by first formulating the model under ecological assumptions, determining equilibrium points, and studying their local stability through eigenvalue analysis of the Jacobian matrix. Furthermore, global behaviour is explored using phase-plane analysis and numerical simulations. These results provide deeper insight into ecological balance and may serve as a tool for managing biological populations in competitive environments.

2. CLASSICAL PREY-PREDATOR MODEL

Let $x(t)$ and $y(t)$ be the population of prey and predator at time t. According to the assumption this prey-predator dynamic is governed by system of non-linear differential equations

$$\frac{dx}{dt} = ax - bxy \quad \ldots(1) \quad a, b > 0 \quad \text{(prey)}$$

$$\frac{dy}{dt} = -cy - dxy \quad \ldots(2) \quad c, d > 0 \quad \text{(predator)}$$

Where:

- a represents growth rate of prey
- b represents rate at which predators destroy prey
- c represents death rate of predators
- d represents rate at which predators increase by consuming prey

3. ANALYTICAL SOLUTION OF THE MODEL

When the prey population x(t) is large the predator population has plenty of food, which cause their population y(t) increase. As the number of predator increase more and more of the prey x(t) is consumed as food leading to sharp decline in prey population. As prey becomes scare, y(t) stops increasing due to lack of food. Therefore, giving the remaining prey a chance to recover and grow [4].

When $y(t) = 0$ and $x(t) > 0$, then the equation (1) becomes

$\frac{dx}{dt} - ax$

On integrating this gives the solution

$\Rightarrow x = ce^{at}$ (here c is constant of integration)

Let $x(0) = x_0$

As initial condition we get, $x_0 = c$

Therefore we have $x(t) = x_0 e^{at}$

Which shows that prey population grow exponentially in the absence of predators. Further when $x(t) = 0$ and $y(t) > 0$ then equation (2) becomes

$\frac{dy}{dt} = -cy$

On integrating we get solution

$\Rightarrow y = c_1 e^{-ct}$ (c_1 being constant of integration)

Let $y(0) = y_0$ as initial condition , therefore we get

$\Rightarrow y(t) = y_0 e^{-ct}$

Which shows that the predators population driven to extinction in the absence of prey

Now from two non-linear differential equations (1) and (2) above to produce a single differential equation as

$\frac{dy}{dx} = \frac{dxy - cy}{ax - bxy}$

$\Rightarrow \frac{dy}{dx} = \frac{y(dx - c)}{x(a - by)}$

On separating the variable we get

$\frac{a - by}{y} dy = \frac{dx - c}{x} dx$

$\Rightarrow \frac{ady}{y} - dy = d\,dx - \frac{c}{x} dx$

On *integrating* we get

$\Rightarrow a\log y \quad by = dx \quad c\log x \mid v$ … (3) where v is constant

This is analytical solution of the system (1) & (2)

A complete phase plane diagram of the predator prey system is show in fig. (1a)

With initial condition is given below

$(x(0), y(0)) = (12,6)$; $(x(0), y(0)) = (16,12)$;

$(x(0), y(0)) = (6,17)$; $(x(0), y(0)) = (25,7)$

And parameter value $a = 1.0, b = 0.1, c = 1.5, d = 0.075$

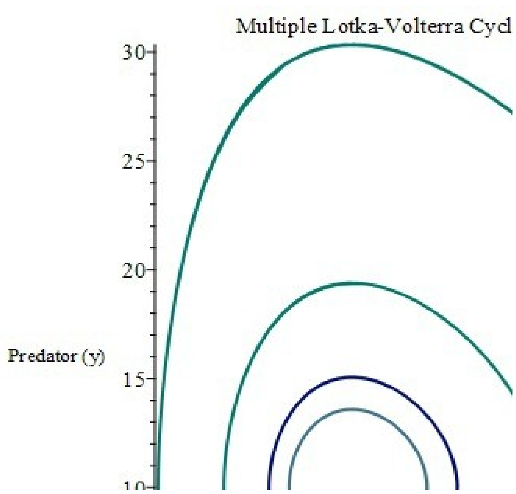

Figure 1a: phase plane diagram of the predator prey system

Meanwhile the relationship between the predator-prey versus time is represented in the figure (1b) below

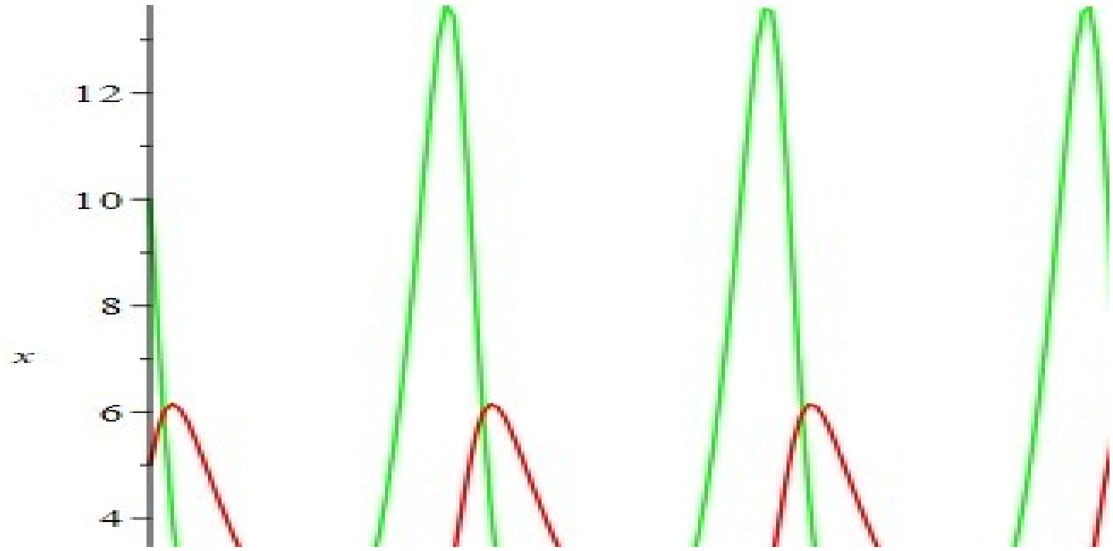

Figure 1b: prey-predator dynamics of (1) & (2) for $a = 1.1, b = 0.4, c = 0.1,$
$d = 0.4;\ x(0) = 10, y(0) = 5$

From the above graph it is shown that there is an interaction between two species, indicating that as the population of other changes accordingly. This increase or decrease in prey population will also increase or decrease in predator population and conversely.

4. MODIFIED MODEL WITH LOGISTIC GROWTH AND COMPETITION

To study the interaction between prey and predator populations under realistic ecological constraints, we extend the classical Lotka–Volterra framework [4,7,8]. The following assumptions are made:

- **Prey dynamics**: In the absence of predators, the prey population grows logistically with intrinsic growth rate r and carrying capacity K[1,2,5,8]
- **Predator–prey interaction**: Encounters between predators and prey occur at a rate proportional to their product. Each such interaction reduces the prey population and contributes to predator growth [5,6,17].
- **Competition and mortality**: In addition to natural mortality, both species experience extra losses due to interspecific competition. These effects are modelled by the terms δ_1(extra prey mortality) and δ_2(reduction in predator growth efficiency)[7,12,13].
- **Predator dynamics**: The predator population declines naturally at rate $d$in the absence of prey, but benefits from prey consumption with conversion efficiency β[17,18]

Based on these assumptions, the system is described by the following nonlinear differential equations

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right) - \alpha xy - \delta_1 x \qquad \text{............ (4)}$$

$$\frac{dy}{dt} = \beta xy - dy - \delta_2\, xy \qquad \text{.......... (5)}$$

where:

- $x(t)$ represents the prey population at time t
- $y(t)$ represents the predator population at time t
- r represent the intrinsic growth rate of the prey
- K is the environmental carrying capacity of the prey
- α is the predation rate
- β represent the conversion efficiency of consumed prey into predator reproduction

- d represent the natural predator death rate
- δ_1 represents additional mortality in prey due to interaction stresses,
- δ_2 represents reduction in predator benefit due to ecological or environmental constraints.

The model extends the classical Lotka–Volterra system [4,7,8] by including a logistic term in the prey dynamics [5,6,8,10] and additional interspecific competition effects [6,11,12,13]. These modifications not only constrain unbounded growth but also reflect realistic ecological scenarios where both prey and predators are affected by external pressures such as limited resources, disease, or habitat restrictions [12,14,15].

5. EQUILIBRIUM POINTS AND STABILITY ANALYSIS

For equilibrium points we set

$\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$

Then the system (4) & (5) becomes

$\Rightarrow rx(1 - \frac{x}{k}) - (\alpha + \delta_1)xy = 0$ …(A)

$\Rightarrow \beta xy - dy - \delta_2 xy = 0$ …(B)

We proceed step by step, from equation

$x\left(r(1 - \frac{x}{k}) - (\alpha + \delta_1)y \right) = 0$

this gives , $x = 0$ or $r\left(1 - \frac{x}{k}\right) - (\alpha + \delta_1)y = 0$

$\Rightarrow r(1 - \frac{x}{k}) = (\alpha + \delta_1)y$ …(c)

From equation (B)

$y(\beta x - d - \delta_2 x) = 0$

This gives $y = 0$ or $\beta x - d - \delta_2 x = 0$ …(D)

First equilibrium point of the system (4) & (5) is trivial equilibrium

Set $x = 0$ & $y = 0$

$E_0 - (0,0)$, this corresponds to extinction of the prey species

Second equilibrium point of the system (4) & (5) is prey only equilibrium

Let y = 0 using equation (A)

$rx\left(1 - \frac{x}{K}\right) = 0$

$x = 0$ & $x = K$

We already have $x = 0$ in E_0

So another point is $E_1 = (K, 0)$

Third equilibrium point is coexistence equilibrium

This occurs when $x \neq 0, y \neq 0$

From equation (B)

$y(\beta x - d - \delta_2 x) = 0$

$\Rightarrow \beta x - \delta_2 x = d$ as $y \neq 0$

$\Rightarrow x(\beta - \delta_2) = d$

$\Rightarrow x^* = \frac{d}{(\beta - \delta_2)}$

Now from equation (A)

$rx(1 - \frac{x}{k}) - (\alpha + \delta_1)xy = 0$

$\Rightarrow rx(1 - \frac{x}{k}) - (\alpha + \delta_1)xy = 0$ as $x \neq 0$

$\Rightarrow r\left(1 - \frac{x^*}{K}) \right) - (\alpha + \delta_1)y^* = 0$

$\Rightarrow r\left(1-\frac{x^*}{K})\right) = (\alpha + \delta_1)y^*$

$\Rightarrow y^* = \dfrac{r\left(1-\frac{x^*}{K}\right)}{(\alpha+\delta_1)}$

$\Rightarrow y^* = \dfrac{r\left(1-\frac{d}{K(\beta-\delta_2)}\right)}{(\alpha+\delta_1)}$

Thus $E^* = \left(\frac{d}{(\beta-\delta_2)}, \frac{r\left(1-\frac{d}{K(\beta-\delta_2)}\right)}{(\alpha+\delta_1)}\right)$

Existence condition for E^*(positive coordinates)

$(\beta-\delta_2) > 0, d < K(\beta-\delta_2)$

(In simple term, predator must have positive net conversion $(\beta-\delta_2)$ and predator death rate must be small enough relative to available prey at carrying capacity for persistence.) See standard stability/invasion results [3,11,12,13].

To determine the local stability of the equilibrium points, we need to compute the linearization of the system (4) & (5), which is obtained from the Jacobian matrix of the system (4) & (5), See [3,9,10] for more information about such calculations. For the system (4) & (5), the Jacobian is the following:

$$J(x,y) = \begin{bmatrix} r\left(1-\frac{2x}{K}\right)-(\alpha+\delta_1)y & -(\alpha+\delta_1)x \\ (\beta-\delta_2)y & (\beta x-\delta_2 x-d) \end{bmatrix}$$

5.1 Equilibrium at (0,0)

$$J(0,0) = \begin{bmatrix} r & 0 \\ 0 & -d \end{bmatrix}$$

The corresponding eigenvalue of the matrix are $\lambda_1 = r > 0$ and $\lambda_2 = -d$

This shows saddle point

5.2 Equilibrium at E_1(K,0)

$$E_1(K,0) = \begin{bmatrix} r(1-\frac{2K}{K}) & -(\alpha+\delta_1)K \\ 0 & (\beta-\delta_2)K-d \end{bmatrix}$$

$$J(K,0) = \begin{bmatrix} -r & -(\alpha+\delta_1)K \\ 0 & (\beta-\delta_2)K-d \end{bmatrix}$$

The corresponding eigenvalues are $\lambda_1 = -r < 0$, $\lambda_2 = K(\beta-\delta_2)-d$

The stability depends on

- If $K(\beta-\delta_2) < d$ then $\lambda_2 < 0$ also $\lambda_1 < 0$ this shows E_1 is locally asymptotically stable (predators cannot invade).
- If $K(\beta-\delta_2) > d$ *then* $\lambda_2 > 0$ this shows E_1 is a **saddle** (unstable against predator invasion). saddle

5.3 Equilibrium at coexistence point $E^*\left(\frac{d}{(\beta-\delta_2)}, \frac{r\left(1-\frac{d}{K(\beta-\delta_2)}\right)}{(\alpha+\delta_1)}\right)$

$$J(E^*) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Where $a_{11} = r\left(1-\frac{2d}{K(\beta-\delta_2)}\right) - r\left(1-\frac{d}{K(\beta-\delta_2)}\right)$

$$a_{11} = r\left(1-\frac{2d}{K(\beta-\delta_2)}-1+\frac{d}{K(\beta-\delta_2)}\right)$$

$$= r\left(1-\frac{2d}{k(\beta-\delta_2)}-1+\frac{d}{(\beta-\delta_2)}\right)$$

$$\Rightarrow a_{11} = \frac{-rd}{K(\beta-\delta_2)}$$

$a_{12} = \frac{-(\alpha+\delta_1)d}{(\beta-\delta_2)}$ and $a_{21} = (\beta-\delta_2)r\left(1-\frac{d}{\kappa(\beta-\delta_2)}\right)$

$\Rightarrow a_{21} = r\left((\beta-\delta_2)-\frac{d}{\kappa}\right)$

$\Rightarrow a_{22} = \frac{d}{\beta-\delta_2}(\beta-\delta_2)-d$

$\Rightarrow a_{22} = 0$

Now to compute trace and determinant of $J(E^*)$

- Trace

$tr(J) = a_{11} + a_{22}$

$$\Rightarrow tr(J) = \frac{-rd}{(\beta-\delta_2)} < 0$$

- Determinant

$\det(J) = a_{11}a_{22} - a_{21}a_{12}$

$$\det(J) = 0 + r\left((\beta-\delta_2)-\frac{d}{k}\right)\left(\frac{(\alpha+\delta_1)d}{(\beta-\delta_2)}\right)$$

$$\det(J) = r\left(1-\frac{d}{k(\beta-\delta_2)}\right)(\alpha+\delta_1)d > 0$$

Whenever E^* exist$\left(\frac{d}{k(\beta-\delta_2)} < 1\right)$. Thus $\det(J) > 0$ & $tr(J) < 0$,this gives both eigenvalues have negative real parts and therefore the coexistence equilibrium E^* is locally asymptotically stable (either a stable node or a stable spiral) whenever it exists. This is consistent with standard coexistence/stability results for logistic predator–prey models with competition [3,13,16]

Node vs. spiral (real vs. complex eigenvalues):

If det(J)> 0 *and* Trace(J) – 4det(J)< 0

Let the characteristic equation of the matrix J be $\lambda^2 - (tr(J))\lambda + det(J) = 0$. The qualitative nature of this stability depends on the discriminant: discriminant $\Delta = (tr(J))^2 - 4\det(J)$

- if$\Delta > 0$: eigenvalues real → stable node
- if $\Delta < 0$: complex conjugate eigenvalues → stable spiral with damped oscillations.
- if $\Delta = 0$: repeated root → degenerate node

Now to compute the discriminant at the coexistence equilibrium

$$\Delta = \left(\frac{-rd}{(\beta-\delta_2)}\right)^2 - 4\left(r\left(1-\frac{d}{k(\beta-\delta_2)}\right)(\alpha+\delta_1)d\right)$$

Since the existence condition for coexistence requires $\frac{d}{(\beta-\delta_2)K} < 1$

the second term is positive and dominates the squared term. Hence obviously $\Delta < 0$

which implies that the coexistence equilibrium is always a locally asymptotically stable spiral whenever it exists.

6. NUMERICAL SIMULATIONS

To validate the analytical findings, now we perform numerical simulations for the proposed predator-prey model under logistic prey growth and interspecific competition between both species for the model (4) & (5) let

$r = 0.85, K = 100, \alpha = 0.03, \delta_1 = 0.02, \beta = 0.03, \delta_2 = 0.006, d = 0.5$

initial condition $x(0) = 40, y(0) = 30$

The simulation results of this case are shown in figure population vs time and Phase Diagram given below

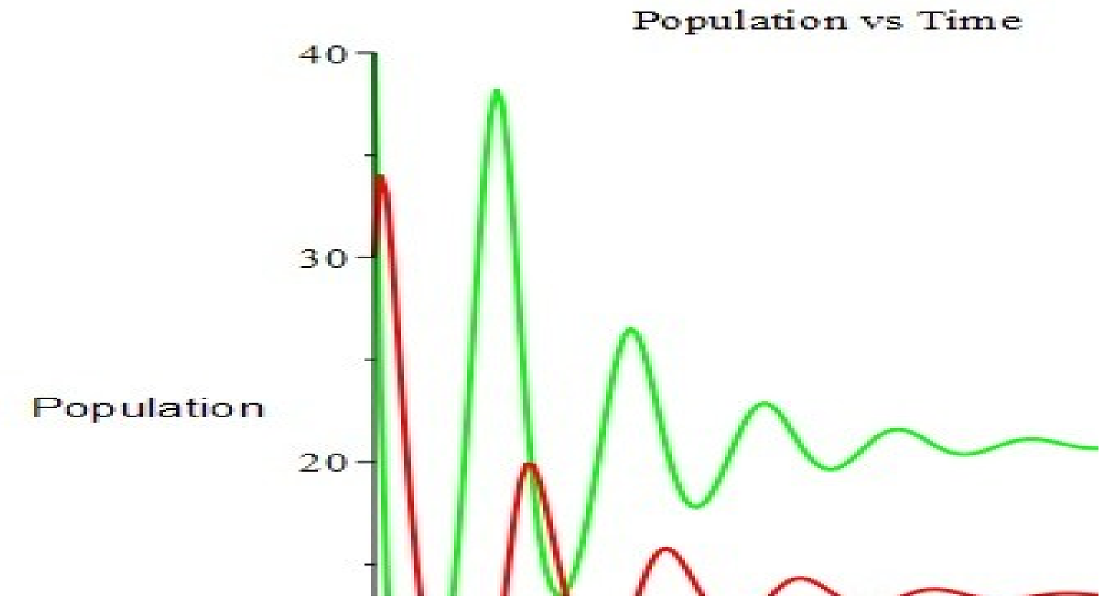

Figure 2a:

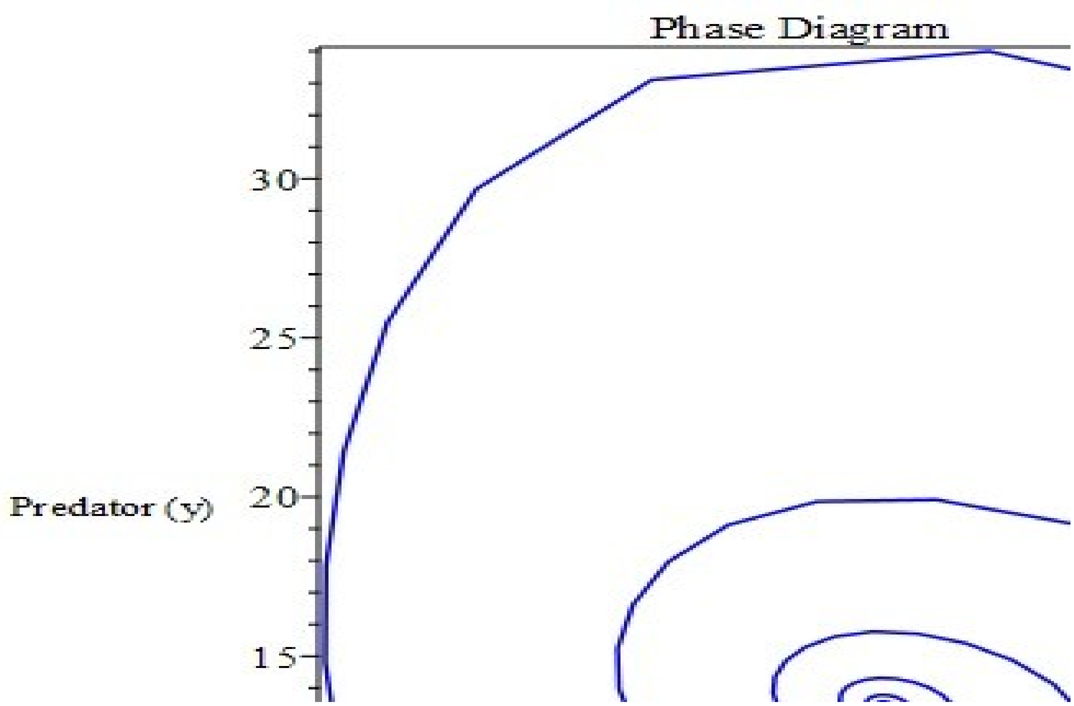

Figure 2b:

For the model (4) & (5) let $r = 0.80, K = 100, \alpha = 0.02, \delta_1 = 0.01, \beta = 0.03, \delta_2 = 0.005, d = 0.4$, initial condition $x(0) = 40, y(0) = 20$

The simulation results of this case are shown in figure population vs time and Phase Diagram given below

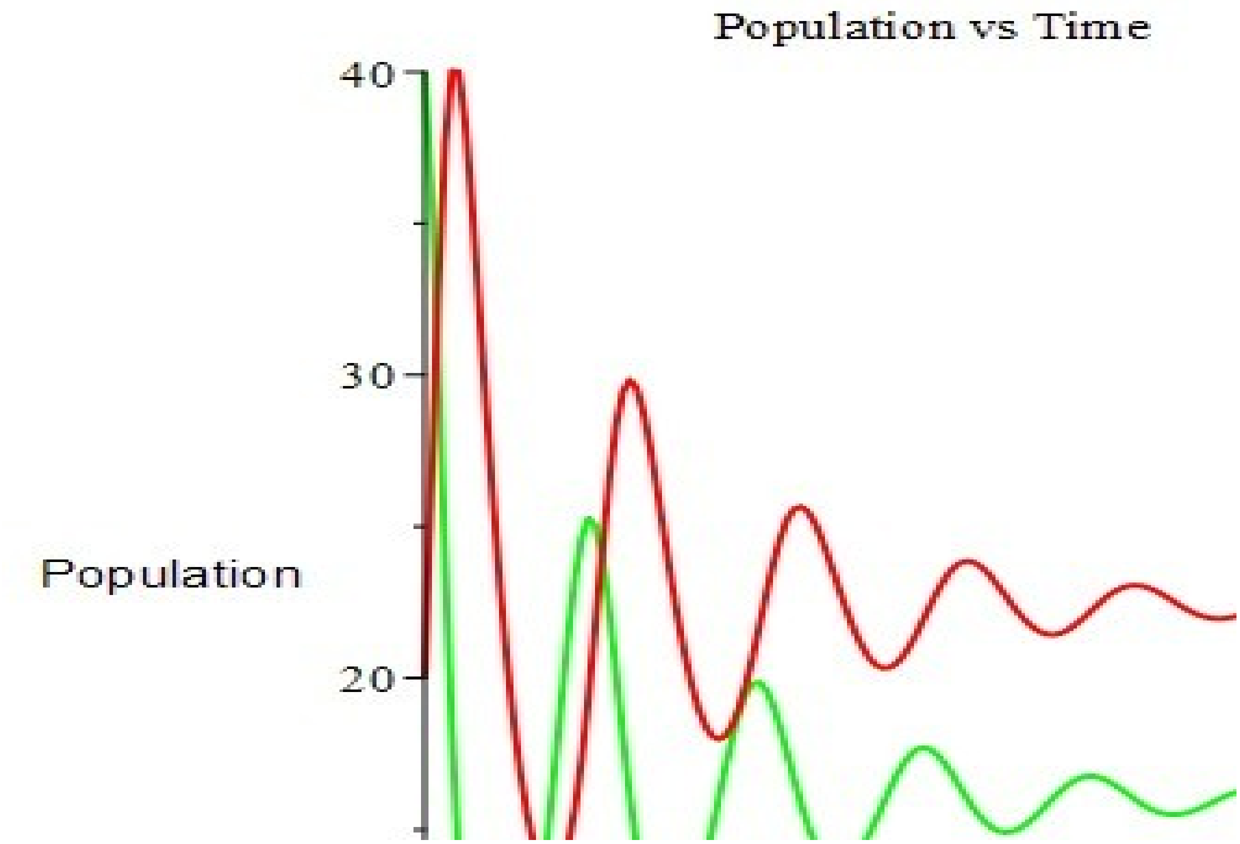

Figure 3a:

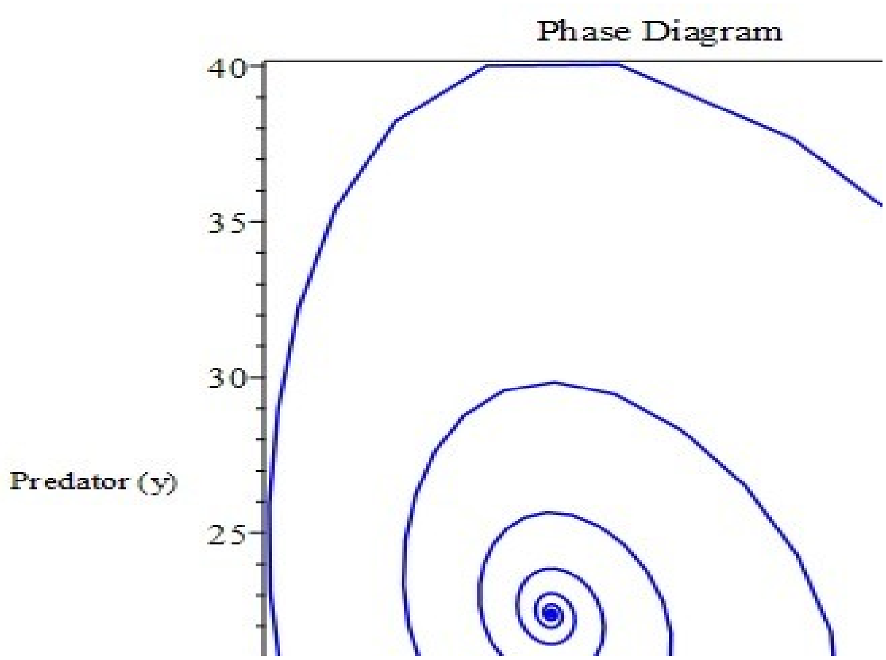

Figure 3b:

7. CONCLUSION

This paper presents the dynamic of a predator-prey model and modified with logistic prey growth and interspecific competition. Incorporating a logistic prey growth and interspecific competition provides a more realistic model in ecological scenario. Logistic prey growth means the prey population does not grow infinitely meaning it increases initially but slow down and stabilizes due to limited resources. Interspecific competition indicates each species affect the other's growth negatively

Numerical simulations of the system (4) & (5) indicate that the system exhibits damped oscillation eventually reaching a stable state at coexistence equilibrium where both populations survive over time. The phase diagram of the system (4) & (5) plotting predator population against prey population. The solution curve is spiral and converges to a stable interior equilibrium indicating that the system moves towards a stable coexistence equilibrium. These graphical results align with linear stability analysis where the Jacobean matrix is evaluated at the equilibrium points has complex conjugate eigenvalue with negative real parts, indicating a stable focus. The plot of population versus time of system (4) & (5) indicates the predator-prey populations oscillate initially with large amplitude. However, after some time the oscillation decline and converges to steady state values. This shows damped oscillation in the presence of locally asymptotically stable equilibrium. The extended model offers both theoretical and practical understanding of the population dynamics in real ecosystem, contributing to ecological stability, biodiversity conservation and environmental sustainability.

REFERENCES

1. Dym CL. *Principles of mathematical modeling*. Academic Press; 2006.
2. Kapur JN. Family of difference equations models in population dynamics. *Bull Math Assoc India*. 1991;24:23–34.
3. Murray JD. *Mathematical biology: an introduction*. Berlin Heidelberg New York: Springer-Verlag; 2003.
4. Pundir SK, Pundir R. *Bio-mathematics*. Pragati Prakashan; 2006.
5. Shepherd JJ, Stojkov L. The logistic population model with slowly varying carrying capacity. *ANZIAM J*. 2007;47:492–506.
6. Slatkin M. Competition and regional coexistence. *Ecology*. 1974;55(1):128–134.
7. Wangersky PJ. Lotka–Volterra population models. *Annu Rev Ecol Syst*. 1978;9:189–218.
8. Volterra V. Variation and fluctuation in the numbers of coexisting animal species. In: Scudo FM, Ziegler R, editors. *Lecture notes in biomathematics*. No. 22. Berlin: Springer-Verlag; 1978.
9. Kapur JN. Stability analysis of continuous and discrete population models. *Indian J Pure Appl Math*. 1976;9:702–708.
10. Lotka AJ. *Elements of mathematical biology*. New York: Dover; 1956.
11. Doust RMH, Haghighifar F. The Lotka–Volterra predator–prey system: having interspecific interactions or harvested factor. *J Intell Syst Res*. 2011;5(2):105–111.
12. Sardara AK, Hanif M, Asaduzzaman M, Biswas MHA. Mathematical analysis of the two-species Lotka–Volterra predator–prey inter-specific game theoretic competition model. *AMO Adv Model Optim*. 2016;18(2).

13. Yang Y, Li Z, Wang F. Global stability of a predator–prey model with the logistic term. *IOP Conf Ser Mater Sci Eng*. 2019;611:012027. doi:10.1088/1757-899X/611/1/012027.
14. Al-Moqbali MKA, Al-Salti NS, Elmojtaba IM. Prey–predator models with variable carrying capacity. *Mathematics*. 2018;6(6):102. doi:10.3390/math6060102.
15. Meyer P. Bi-logistic growth. *Technol Forecast Soc Change*. 1994;47:82–102.
16. Hofbauer J, Sigmund K. *Evolutionary games and population dynamics*. Cambridge: Cambridge University Press; 1998.
17. Adamu HA. Mathematical analysis of predator–prey model with two preys and one predator. *Int J Eng Appl Sci (IJEAS)*. 2018;5(11):2394–3661.
18. Freedman HI. *Deterministic mathematical models in population ecology*. New York: Marcel Dekker; 1980.