Original Article
Bulletin of Pure and Applied Sciences.
Mathematics & Statistics Vol.45 E, No.2.
July-December 2026: P.1-7

Print version ISSN 0970 6577
Online version ISSN 2320 3226
Available online at www.bpasjournals.com

Fractional Black-Scholes Equation Using Sumudu Decomposition Method

[1]Eltayeb Abdeldaim and [2]Adil Mousa*

Author Affiliation:
[1]Mathematics Department, College of Education, Blue Nile University, Aldamazin, Sudan
E-mail: Eltayeb.mohammed3@gmail.com
[2]Department of Mathematics, Almanhal Academy of Science, Khartoum, Sudan
E-mail: aljarada@gmail.com

***Corresponding Author: Adil Mousa**, Department of Mathematics, Almanhal Academy of Science, Khartoum, Sudan.
E-mail: aljarada@gmail.com

ABSTRACT

In this work, the (ITDM) integral transform is used in combination with the decomposition tool to gain discrete analytical solution of (FBSEs) fractional Black-Scholes equations, the simplicity and efficiency of the combined approach are demonstrated through illustrative problems and the simplicity of the resulting solutions.

Keyword: (ITADM) integral Adomian decomposition method, (FBSEs) fractional Black-Scholes equations, linear and (NLDEs).

How to cite this article: Abdeldaima E. and Mousa A. Fractional Black-Scholes Equation Using Integral Transform Decomposition Method. *Bulletin of Pure and Applied Sciences- Math & Stat.*, 2026;45E (1):1-7.

Received on 15.04.2026, Revised on 29.06.2026, Accepted on 09.07.2026

1. INTRODUCTION

Fractional differential equations are widely used to describe dynamical phenomena engender in economy, physic, biology, scientific and engineering [1-5]
The (FBSEs) fractional Black-Scholes equations have analytical solutions in closed form therefore, finding new methods to solve linear and nonlinear equations.[9]
In recent years , the mathematical models used many tools have been developed to solution and analysis of linear and nonlinear (FBSEs),[4]

$$\frac{\partial \Phi}{\partial \hbar}+\frac{\alpha^2 \lambda\!\!\!^{-}{}^2}{2}\frac{\partial \Phi^2}{\partial \lambda\!\!\!^{-}{}^2}+\vartheta(\hbar)\lambda\!\!\!^{-}\frac{\partial \Phi}{\partial \lambda\!\!\!^{-}}-\vartheta(\hbar)\Phi=0,\ \ (\lambda\!\!\!^{-},\hbar)\in R^{+}\times(0,T)$$

where $\Phi(\lambda,\hbar)$ European call choice at asset amok $\lambda,\hbar$ time, $\vartheta(\hbar)$ is the risk free interest rate, $\alpha(\lambda,\hbar)$ represent volatility function of underlying asset, and the Payoff function are $\Phi_c(\lambda,\hbar)=\max(\lambda-E,0)=\mathrm{M}(\lambda-E,0),\qquad \Phi_k(\lambda,\hbar)=\max(E-\lambda,0)=\mathrm{M}(E-\lambda,0)$

Where E mean the expiration amok for the choice and the function $\max(\lambda,0)=\mathrm{M}(\lambda,0)$ gives the large value amidst λ and 0 .

Latterly, many structure methods, homotopy perturbation tool, the variable separation tool, Adomian decomposition tool, Laplace transform tool [6], Sumudu transform tool [3], Elzaki transform tool [13], and many others. these tools can produce different solutions. which are obtained from modeling in engineering applications. we present an problems of (FBSEs) fractional Black-Scholes equations.

The main idea, the new integral transform (ST) called Sumudu transform which introduced by Watugala [6], over the set of a function as

$$\mathbf{Y}=\left\{\Phi(\hbar)\setminus\exists k,s_1,s_2>0,\left|\Phi(\hbar)\right|<ke^{\frac{|\lambda|}{\tau_j}},if\ \hbar\in(-1)^j\times[0,\infty)\right\}$$

by

$$H(\rho)=\mathbf{S}\left[\Phi(\hbar)\right]=\frac{1}{\rho}\int_0^{\infty}e^{-\frac{\hbar}{\rho}}\Phi(\hbar)d\hbar,\quad \rho\in(-s,s)$$

And the Adomian decomposition method which first introduced by Georg Adomian [11], the (SDM) is combined of the powerful in solving many fractional differential equations in science and engineering, [4]

2. DEFINITION AND PRELIMINARIES OF CONCEPTS

In this section of, we give some theories and definitions which are useful in this study, of (FDEs) and integral transform.

Definition 1. [16]

The integral transform of the (CFD) Caputo fractional derivative defined as

$$\mathbf{S}\left[\mathbf{D}^{\lambda}\Phi(\hbar)\right]=\rho^{-\lambda}\mathbf{S}\left[\Phi(\hbar)\right]-\sum_{v=0}^{\sigma-1}\rho^{-\lambda+v}\Phi^{v}(0),\quad \sigma-1<\lambda\leq\sigma$$

Definition 2. [17]

The (CFD) Caputo fractional derivative of $\Phi\in C_{-1}^{\sigma},\ \sigma\in N$ is defined as

$$\mathbf{D}^{\lambda}\Phi(\hbar)=\frac{1}{\Gamma[\sigma-\lambda]}\int_0^{\lambda}(\lambda-\hbar)^{\sigma-\lambda-1}\Phi^{\sigma}(\hbar)d\hbar,\quad \sigma-1<\lambda\leq\sigma$$

Lemma 1: [8]

If $\sigma-1<\lambda\leq\sigma$, $\sigma\in N$, $\Phi\in C_{\zeta}^{\sigma}$, $\zeta>-1$ then the following two properties have,

First property : $\mathbf{D}^{\lambda}\left(J^{\lambda}\Phi(\lambda)\right)=\Phi(\lambda)$

Scand property : $J^{\lambda}\left(\mathbf{D}^{\lambda}\Phi(\lambda)\right)=\Phi(\lambda)-\sum_{v=1}^{\sigma-1}\Phi^{v}(0)(\frac{\lambda^{v}}{v!})$

Definition 3 : [8]

The (MLF) Mittag-Leffler function $\mathbf{E}_{\lambda}\left(\hbar\right)$ with $\lambda>0$ is defined by,

$$\mathbf{E}_{\lambda}\left(\hbar\right)=\sum_{v=0}^{\infty}\frac{\hbar^{v}}{\Gamma\left[\lambda v+1\right]}$$

3. IDEA OF (SDM) INTEGRAL TRANSFORM DECOMPOSITION METHOD

We Let us examine the (FBSEs) the basic aim of the (SDM).

$$\mathbf{D}^{\lambda}\Phi(\lambdabar,\hbar)+\mathrm{H}\left[\lambdabar\right]\Phi(\lambdabar,\hbar)+\mathrm{T}\left[\lambdabar\right]\Phi(\lambdabar,\hbar)=\mathrm{P}(\lambdabar,\hbar),\quad \hbar>0,\ \lambdabar\in R,\ 0<\lambda\leq 1$$

(1)

Where $\mathrm{H}\left[\lambdabar\right]$ is the general linear factor in $\lambdabar$, $\mathrm{T}\left[\lambdabar\right]$ is the nonlinear factor in $\lambdabar$ and $\mathrm{P}\left(\lambdabar,\hbar\right)$ are general continuous function.

By using (ST) integral transform of eq.(1) on both side,

$$\mathbf{S}\left[\mathbf{D}^{\lambda}\Phi(\lambdabar,\hbar)\right]+\mathbf{S}\left[\mathrm{H}\left[\lambdabar\right]\Phi(\lambdabar,\hbar)+\mathrm{T}\left[\lambdabar\right]\Phi(\lambdabar,\hbar)\right]=\mathbf{S}\left[\mathrm{P}(\lambdabar,\hbar)\right],$$

$$\frac{\Phi(\lambdabar,\rho)-\Phi(\lambdabar,0)}{\rho^{\lambda}}=\mathbf{S}\left[\mathrm{P}(\lambdabar,\hbar)\right]-\mathbf{S}\left[\mathrm{H}\left[\lambdabar\right]\Phi(\lambdabar,\hbar)+\mathrm{T}\left[\lambdabar\right]\Phi(\lambdabar,\hbar)\right],$$

$$\Phi(\lambdabar,\rho)=\Phi(\lambdabar,0)+\rho^{\lambda}\mathbf{S}\left[\mathrm{P}(\lambdabar,\hbar)\right]-\rho^{\lambda}\mathbf{S}\left[\mathrm{H}\left[\lambdabar\right]\Phi(\lambdabar,\hbar)+\mathrm{T}\left[\lambdabar\right]\Phi(\lambdabar,\hbar)\right],$$

$$\Phi(\lambdabar,\hbar)=\mathrm{P}(\lambdabar,\hbar)-\mathbf{S}^{-1}\left[\rho^{\lambda}\mathbf{S}\left[\mathrm{H}\left[\lambdabar\right]\Phi(\lambdabar,\hbar)+\mathrm{T}\left[\lambdabar\right]\Phi(\lambdabar,\hbar)\right]\right], \tag{2}$$

Where $\mathbf{G}(\lambdabar,\hbar)=\Phi(\lambdabar,0)+\mathbf{S}^{-1}\left[\rho^{\lambda}\mathbf{S}\left[\mathrm{P}(\lambdabar,\hbar)\right]\right]$,represent the exporter code and specified (IC) initial conditions.

The standard (SDT) integral transform decomposition method defined by,

$$\Phi(\lambdabar,\hbar)=\sum_{v=0}^{\infty}\Phi_{v}\left(\lambdabar,\hbar\right) \tag{3}$$

The nonlinear term defined by,

$$\mathrm{T}\Phi(\lambdabar,\hbar)=\sum_{v=0}^{\infty}\Upsilon_{v} \tag{4}$$

Where Υ_{v} is donated a domain polynomial of $\Phi_{0},\Phi_{1},\Phi_{2},...,\Phi_{n}$ can be calculated by,

$$\Upsilon_{v}=\frac{1}{v\,!}\frac{d^{v}}{d\lambdabar^{v}}\left(\mathrm{T}\left[\sum_{v=0}^{\infty}\lambdabar^{v}\Phi_{v}\right]\right)\Bigg|_{\lambdabar=0},\qquad v=0,1,2...$$

The first a domain polynomial we have ,

Applying eq.(3) and eq. (4) in to eq. (2) we have,

$$\sum_{v=0}^{\infty}\Phi_{v}=\mathbf{G}(\lambdabar,\hbar)-\mathbf{S}^{-1}\left[\rho^{\lambda}\mathbf{S}\left(\mathrm{H}\sum_{v=0}^{\infty}\Phi_{v}\left(\lambdabar,\hbar\right)+\mathrm{T}\sum_{v=0}^{\infty}\Upsilon_{v}\left(\lambdabar,\hbar\right)\right)\right],$$

(5)

Applying inverse (ST) integral transform to eq. (5)

$$\Phi_{0}=\mathbf{G}(\lambdabar,\hbar),\qquad \Phi_{v+1}=-\mathbf{S}^{-1}\left\{\rho^{\lambda}\mathbf{S}\left(\mathrm{H}\sum_{v=0}^{\infty}\Phi_{v}\left(\lambdabar,\hbar\right)+\mathrm{T}\sum_{v=0}^{\infty}\Upsilon_{v}\left(\lambdabar,\hbar\right)\right)\right\},$$

$$\Phi_{1}=-\mathbf{S}^{-1}\left\{\rho^{\lambda}\mathbf{S}\left(\mathrm{H}\sum_{v=0}^{\infty}\Phi_{0}(\lambdabar,\hbar)+\mathrm{T}\sum_{v=0}^{\infty}\Upsilon_{0}(\lambdabar,\hbar)\right)\right\},$$

$$\Phi_{2}=-\mathbf{S}^{-1}\left\{\rho^{\lambda}\mathbf{S}\left(\mathrm{H}\sum_{v=0}^{\infty}\Phi_{1}(\lambdabar,\hbar)+\mathrm{T}\sum_{v=0}^{\infty}\Upsilon_{1}(\lambdabar,\hbar)\right)\right\},$$

$$\Phi_3 = -\mathbf{S}^{-1}\left\{\rho^{\lambda}\mathbf{S}\left(\mathrm{H}\sum_{v=0}^{\infty}\Phi_2(\lambda,\hbar)+\mathrm{T}\sum_{v=0}^{\infty}\Upsilon_2(\lambda,\hbar)\right)\right\},$$

4. APPLICATIONS

In this section, the method of integral transform and decomposition method (SDTM) are joint to secure the solving of linear and nonlinear (FBSE). [1]

problem 4.1: Let us examine the (FBSE)

$$\frac{\partial^{\lambda}\Phi}{\partial\hbar^{\lambda}}=\frac{\partial^2\Phi}{\partial\lambda^2}+(\vartheta-1)\frac{\partial\Phi}{\partial\lambda}-\vartheta\Phi, \qquad 0<\lambda\le 1 \tag{6}$$

with the (IC) initial condition $\Phi(\lambda,0)=\max(e^{\lambda}-1,0)=\mathrm{M}(\Omega,0)$ (7)

Solution

Applying (ST) integral transform of eq. (6) on both side

$$\mathbf{S}\left[\frac{\partial^{\lambda}\Phi}{\partial\hbar^{\lambda}}\right]=\mathbf{S}\left[\frac{\partial^2\Phi}{\partial\lambda^2}+(\vartheta-1)\frac{\partial\Phi}{\partial\lambda}-\vartheta\Phi\right]$$

$$\frac{\Phi(\lambda,\rho)-\Phi(\lambda,0)}{\rho^{\lambda}}=\mathbf{S}\left[\frac{\partial^2\Phi}{\partial\lambda^2}+(\vartheta-1)\frac{\partial\Phi}{\partial\lambda}-\vartheta\Phi\right]$$

$$\Phi(\lambda,\rho)-\Phi(\lambda,0)=\rho^{\lambda}\mathbf{S}\left[\frac{\partial^2\Phi}{\partial\lambda^2}+(\vartheta-1)\frac{\partial\Phi}{\partial\lambda}-\vartheta\Phi\right]$$

$$\Phi(\lambda,\rho)=\Phi(\lambda,0)+\rho^{\lambda}\mathbf{S}\left[\frac{\partial^2\Phi}{\partial\lambda^2}+(\vartheta-1)\frac{\partial\Phi}{\partial\lambda}-\vartheta\Phi\right]$$

$$\Phi(\lambda,\hbar)=\mathbf{S}^{-1}\left[\mathrm{M}(\Omega,0)\right]+\mathbf{S}^{-1}\left[\rho^{\lambda}\mathbf{S}\left[\frac{\partial^2\Phi}{\partial\lambda^2}+(\vartheta-1)\frac{\partial\Phi}{\partial\lambda}-\vartheta\Phi\right]\right],$$

$$\Phi_0(\lambda,\hbar)=\mathbf{S}^{-1}\left[\mathrm{M}(\Omega,0)\right]=\mathrm{M}(\Omega,0)$$

$$\Phi_{v+1}(\lambda,\hbar)=\mathbf{S}^{-1}\left[\rho^{\lambda}\mathbf{S}\left[\frac{\partial^2\Phi_v}{\partial\lambda^2}+(\vartheta-1)\frac{\partial\Phi_v}{\partial\lambda}-\vartheta\Phi_v\right]\right],$$

$$\Phi_1(\lambda,\hbar)=\mathbf{S}^{-1}\left[\rho^{\lambda}\mathbf{S}\left[\frac{\partial^2\Phi_0}{\partial\lambda^2}+(\vartheta-1)\frac{\partial\Phi_0}{\partial\lambda}-\vartheta\Phi_0\right]\right],$$

$$\Phi_1(\lambda,\hbar)=\mathbf{S}^{-1}\left[\rho^{\lambda}\mathbf{S}\left(\frac{\partial^2}{\partial\lambda^2}\mathrm{M}(\Omega,0)+(\vartheta-1)\frac{\partial}{\partial\lambda}\mathrm{M}(\Omega,0)-\vartheta\mathrm{M}(\Omega,0)\right)\right]$$

$$\mathrm{M}(e^{\lambda}-1,0)=\mathrm{M}(\Omega,0),\mathrm{M}(e^{\lambda}-1,0)=\mathrm{M}(\hat{\Omega},0),\mathrm{M}(e^{\lambda},0)=\mathrm{M}(\hat{\Omega}+1,0)$$

$$=\mathbf{S}^{-1}\left[\rho^{\lambda}\mathbf{S}(\mathrm{M}(\hat{\Omega}+1,0)+(\vartheta-1)\mathrm{M}(\hat{\Omega}+1,0)-\vartheta\mathrm{M}(\hat{\Omega},0)\right]$$

$$=\mathbf{S}^{-1}\left[\rho^{\lambda}\mathbf{S}(\vartheta\mathrm{M}(\hat{\Omega}+1,0)-\vartheta\mathrm{M}(\hat{\Omega},0)\right]$$

$$\Phi_1(\lambda,\hbar)=\vartheta \mathrm{M}(\widehat{\Omega}+1,0)\frac{\hbar^{\lambda}}{\Gamma[\lambda+1]}-\ \vartheta \mathrm{M}(\widehat{\Omega},0)\frac{\hbar^{\lambda}}{\Gamma[\lambda+1]}$$

$$\mathrm{M}(e^{\lambda}-1,0)=\mathrm{M}(\widehat{\Omega},0),\mathrm{M}(e^{\lambda},0)=\mathrm{M}(\widehat{\Omega}+1,0)$$

$$=\mathrm{M}(\widehat{\Omega}+1,0)\frac{\vartheta\hbar^{\lambda}}{\Gamma[\lambda+1]}-\ \mathrm{M}(\widehat{\Omega},0)\frac{\vartheta\hbar^{\lambda}}{\Gamma[\lambda+1]}$$

$$\Phi_2(\lambda,\hbar)=\mathbf{S}^{-1}\left[\rho^{\lambda}\mathbf{S}\left[\frac{\partial^2\Phi_1}{\partial\lambda^2}+(\vartheta-1)\frac{\partial\Phi_1}{\partial\lambda}-\vartheta\Phi_1\right]\right],$$

$$\Phi_2(\lambda,\hbar)=\vartheta^2\mathrm{M}(\widehat{\Omega}+1,0)\frac{\hbar^{2\lambda}}{\Gamma(2\lambda+1)}+\vartheta^2\mathrm{M}(\widehat{\Omega},0)\frac{\hbar^{2\lambda}}{\Gamma(2\lambda+1)}$$

$$=\mathrm{M}(\widehat{\Omega}+1,0)\frac{\vartheta^2\hbar^{2\lambda}}{\Gamma(2\lambda+1)}+\mathrm{M}(\widehat{\Omega},0)\frac{\vartheta^2\hbar^{2\lambda}}{\Gamma(2\lambda+1)}$$

$$\Phi_3(\lambda,\hbar)=\mathbf{S}^{-1}\left[\rho^{\lambda}\mathbf{S}\left[\frac{\partial^2\Phi_2}{\partial\lambda^2}+(\vartheta-1)\frac{\partial\Phi_2}{\partial\lambda}-\vartheta\Phi_2\right]\right],$$

$$\Phi_3(\lambda,\hbar)=\vartheta^3\mathrm{M}(\widehat{\Omega}+1,0)\frac{\hbar^{3\lambda}}{\Gamma(3\lambda+1)}+\vartheta^3\mathrm{M}(\widehat{\Omega},0)\frac{\hbar^{3\lambda}}{\Gamma(3\lambda+1)}$$

$$=\mathrm{M}(\widehat{\Omega}+1,0)\frac{\vartheta^3\hbar^{3\lambda}}{\Gamma(3\lambda+1)}+\mathrm{M}(\widehat{\Omega},0)\frac{\vartheta^3\hbar^{3\lambda}}{\Gamma(3\lambda+1)}$$

The solution is given by the series,

$$\Phi=\Phi_0+\Phi_1+\Phi_2+\Phi_3+....$$

$$\Phi(\lambda,\hbar)=\mathrm{M}(\widehat{\Omega},0)+\mathrm{M}(\widehat{\Omega}+1,0)\frac{\vartheta\hbar^{\lambda}}{\Gamma[\lambda+1]}+\mathrm{M}(\widehat{\Omega}+1,0)\frac{\vartheta^2\hbar^{2\lambda}}{\Gamma[2\lambda+1]}+$$

$$\mathrm{M}(\widehat{\Omega},0)\frac{\vartheta^2\hbar^{2\lambda}}{\Gamma[2\lambda+1]}+\mathrm{M}(\widehat{\Omega}+1,0)\frac{\vartheta^3\hbar^{3\lambda}}{\Gamma[3\lambda+1]}+\mathrm{M}(\widehat{\Omega},0)\frac{\vartheta^3\hbar^{3\lambda}}{\Gamma[3\lambda+1]}+....$$

$$\Phi(\lambda,\hbar)=\sum_{v=0}^{\infty}\Phi_v(\lambda,\hbar)=\mathrm{M}(\widehat{\Omega},0)\mathbf{E}_{\lambda}\left(-\vartheta\hbar^{\lambda}\right)+\mathrm{M}(\widehat{\Omega}+1,0)\left(1-\mathbf{E}_{\lambda}\left(-\vartheta\hbar^{\lambda}\right)\right)$$

Anywhere $\mathbf{E}_{\lambda}(\hbar)$ noted the (MLF) Mettag-Leffler function.

problem 4.2: Let us examine the (FBSE)

$$\frac{\partial^{\lambda}\Phi}{\partial\hbar^{\lambda}}+8\times10^{-1}(2+\sin\lambda)^2\lambda^2\frac{\partial^2\Phi}{\partial\lambda^2}+6\times10^{-1}\lambda\frac{\partial\Phi}{\partial\lambda}-6\times10^{-1}\Phi=0,\qquad 0<\lambda\le1$$

(8)

With the initial condition $\Phi(\lambda,0)=\max(\lambda-25e^{-\left(6\times10^{-1}\right)},0)=\mathrm{M}(\mathrm{X},0)$ (9)

Solution

Applying (ST) integral transform of eq. (8), on both side

$$\mathbf{S}\left[\frac{\partial^{\lambda}\Phi}{\partial\hbar^{\lambda}}\right]=\mathbf{S}\left[-8\times10^{-1}(2+\sin\lambda)^2\lambda^2\frac{\partial^2\Phi}{\partial\lambda^2}-6\times10^{-1}\lambda\frac{\partial\Phi}{\partial\lambda}+0.06\Phi\right]$$

$$\Phi(\lambda,\rho)=\Phi(\lambda,0)+\rho^{\lambda}\mathbf{S}\left[6\times10^{-1}\Phi-8\times10^{-1}(2+\sin\lambda)^2\lambda^2\frac{\partial^2\Phi}{\partial\lambda^2}-6\times10^{-1}\lambda\frac{\partial\Phi}{\partial\lambda}\right]$$

$$= \mathbf{S}^{-1}\left[\mathrm{M}(X,0)\right] + \mathbf{S}^{-1}\left[\rho^{\lambda} S\left(6\times 10^{-1}\Phi - 8\times 10^{-1}(2+\sin\bar{\lambda})^2\bar{\lambda}^2\frac{\partial^2\Phi}{\partial\bar{\lambda}^2} - 6\times 10^{-1}\bar{\lambda}\frac{\partial\Phi}{\partial\bar{\lambda}}\right)\right]$$

$$\Phi_0\left(\bar{\lambda},\hbar\right) = \mathbf{S}^{-1}\left(\mathrm{M}(X,0)\right) = \mathrm{M}(X,0)$$

$$\Phi_{v+1}\left(\bar{\lambda},\hbar\right) = \mathbf{S}^{-1}\left[\rho^{\lambda}\mathbf{S}\left(6\times 10^{-1}\Phi_v - 8\times 10^{-1}(2+\sin\bar{\lambda})^2\bar{\lambda}^2\frac{\partial^2\Phi_v}{\partial\bar{\lambda}^2} - 6\times 10^{-1}\bar{\lambda}\frac{\partial\Phi_v}{\partial\bar{\lambda}}\right)\right]$$

$$\Phi_1\left(\bar{\lambda},\hbar\right) = \mathbf{S}^{-1}\left[\rho^{\lambda}\mathbf{S}\left(6\times 10^{-1}\Phi_0 - 8\times 10^{-1}(2+\sin\bar{\lambda})^2\bar{\lambda}^2\frac{\partial^2\Phi_0}{\partial\bar{\lambda}^2} - 6\times 10^{-1}\bar{\lambda}\frac{\partial\Phi_0}{\partial\bar{\lambda}}\right)\right]$$

$$= \mathbf{S}^{-1}\left[\rho^{\lambda}\mathbf{S}\left(6\times 10^{-1}\left(\mathrm{M}(X,0)\right) - 8\times 10^{-1}(2+\sin\bar{\lambda})^2\bar{\lambda}^2\frac{\partial^2\left(\mathrm{M}(X,0)\right)}{\partial\bar{\lambda}^2} - 6\times 10^{-1}\bar{\lambda}\frac{\partial\left(\mathrm{M}(X,0)\right)}{\partial\bar{\lambda}}\right)\right]$$

$$= 6\times 10^{-1}\left(\mathrm{M}(X,0)\right)\frac{\hbar^{\lambda}}{\Gamma(\lambda+1)} - 6\times 10^{-1}\bar{\lambda}\frac{\hbar^{\lambda}}{\Gamma(\lambda+1)}$$

$$\Phi_2\left(\bar{\lambda},\hbar\right) = \mathbf{S}^{-1}\left[\rho^{\lambda}\mathbf{S}\left(6\times 10^{-1}\Phi_1 - 8\times 10^{-1}(2+\sin\bar{\lambda})^2\bar{\lambda}^2\frac{\partial^2\Phi_1}{\partial\bar{\lambda}^2} - 6\times 10^{-1}\bar{\lambda}\frac{\partial\Phi_1}{\partial\bar{\lambda}}\right)\right]$$

$$= (6\times 10^{-1})^2\left(\mathrm{M}(X,0)\right)\frac{\hbar^{2\lambda}}{\Gamma(2\lambda+1)} - (6\times 10^{-1})^2\bar{\lambda}\frac{\hbar^{2\lambda}}{\Gamma(2\lambda+1)}$$

$$\Phi_3\left(\bar{\lambda},\hbar\right) = \mathbf{S}^{-1}\left[\rho^{\lambda}\mathbf{S}\left(6\times 10^{-1}\Phi_3 - 8\times 10^{-1}(2+\sin\bar{\lambda})^2\bar{\lambda}^2\frac{\partial^2\Phi_3}{\partial\bar{\lambda}^2} - 6\times 10^{-1}\bar{\lambda}\frac{\partial\Phi_3}{\partial\bar{\lambda}}\right)\right]$$

$$= (6\times 10^{-1})^3\left(\mathrm{M}(X,0)\right)\frac{\hbar^{3\lambda}}{\Gamma(3\lambda+1)} - (6\times 10^{-1})^3\bar{\lambda}\frac{\hbar^{3\lambda}}{\Gamma(3\lambda+1)}$$

The solution is given by

$$\Phi\left(\bar{\lambda},\hbar\right) = \Phi_0\left(\bar{\lambda},\hbar\right) + \Phi_1\left(\bar{\lambda},\hbar\right) + \Phi_2\left(\bar{\lambda},\hbar\right) + \Phi_3\left(\bar{\lambda},\hbar\right) + \ldots..$$

$$\Phi\left(\bar{\lambda},\hbar\right) = \mathrm{M}(X,0) + 6\times 10^{-1}\left(\mathrm{M}(X,0)\right)\frac{\hbar^{\lambda}}{\Gamma(\lambda+1)} - 6\times 10^{-1}\hbar\frac{\hbar^{\lambda}}{\Gamma(\lambda+1)} +$$

$$(6\times 10^{-1})^2\left(\mathrm{M}(X,0)\right)\frac{\hbar^{2\lambda}}{\Gamma(2\lambda+1)} - (6\times 10^{-1})^2\hbar\frac{\hbar^{2\lambda}}{\Gamma(2\lambda+1)} +$$

$$(6\times 10^{-1})^3\left(\mathrm{M}(X,0)\right)\frac{\hbar^{3\lambda}}{\Gamma(3\lambda+1)} - (6\times 10^{-1})^3\bar{\lambda}\frac{\hbar^{3\lambda}}{\Gamma(3\lambda+1)}$$

$$\Phi\left(\bar{\lambda},\hbar\right) = \sum_{v=0}^{\infty}\Phi_v(\bar{\lambda},\hbar) = \mathrm{M}(X,0)\mathbf{E}_{\lambda}\left(-6\times 10^{-1}\hbar^{\lambda}\right) + \hbar\left(1-\mathbf{E}_{\lambda}\left(-6\times 10^{-1}\hbar^{\lambda}\right)\right)$$

Where $\mathbf{E}_{\lambda}\left(\hbar\right)$ is the (MLF) Mettag-Leffler function.

5. CONCLUSION

In that work, the combined decomposition method together for Integral transform (STDM) has been effective applied in obtain solutions of the (FBSE) fractional Black Scholes equation. some problems are presented to demonstrate the simplicity and distinction of the suggest tool, showing this is plain to perform and effective in obtaining solution results.

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