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Collatz Conjecture: The Mathematician and his $3n + 1$ Conjecture

Padmanabh S. Sarpotdar[1*] and Vivek V. Bhide[2]

Author Affiliation:
[1]Department of Physics, Khare Dhere Bhosale College, Guhagar, Dist. Ratnagiri, Maharashtra-415703, India. E-mail: padphy@gmail.com
[2]Department of Physics, Gogate Jogalekar College (Autonomous), Ratnagiri, Dist. Ratnagiri, Maharashtra- 415612, India. E-mail: vivek.bhide@gjcrtn.ac.in

***Corresponding Author: Padmanabh S. Sarpotdar**, Department of Physics, Khare Dhere Bhosale College, Guhagar, Dist. Ratnagiri, Maharashtra-415703, India.
E-mail: padphy@gmail.com

ABSTRACT
The Collatz conjecture (popularly known as the 3n+1 conjecture) has fascinated mathematicians worldwide for decades due to its simple structure and elusive nature. In this article, we briefly introduce the Collatz conjecture and explore patterns within the sequence while discussing its potential implications.

Keyword: Conjecture, Iterative Process, Loop 4 → 2 → 1, Hailstone, Stopping Time, Collatz Tree, Greedy Branch

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1. The Collatz conjecture: The German mathematician Lothar Collatz first proposed this conjecture in 1937, where, during a presentation at the Halle Congress of German Mathematical Society, he described a sequence of numbers generated by a simple iterative process dealing with positive integers. The conjecture is formally stated as below [1]:
We Define a function f on the positive integers by $f(n) = 3n + 1$ if n is odd, $f(n) = \frac{n}{2}$ if n is even. Given any integer m, define a sequence by putting $a(1) = m$ and, for $i \geq 1, a(i + 1) = f(a(i))$.
And, the problem asks if, for every starting value m, the sequence $a(i)$ always reaches 1? To put it in a layman's language, for any positive integer n, one has to perform following transformation:
1. If n is even, divide it by 2. *i.e.* $n/2$
2. If n is odd, multiply it by 3 and add 1 *i.e.* $3n + 1$
3. Repeat the process with the resulting number, continuing to apply the same rules. *e.g.* if one begins with the number 6, the sequence generated is:

$$6 \to 3 \to 10 \to 5 \to 16 \to 8 \to 4 \to 2 \to 1 \qquad (1)$$

Regardless of the number (a positive integer) with which one begins, would the sequence eventually reach the number 1 through the loop $4 \to 2 \to 1$? Various mathematical techniques, computer simulations, and theoretical approaches have been employed so far by the mathematicians to analyse the behaviour of the sequence, yet, the problem remains unsolved.

2. Lothar Collatz: Born in Arnsberg, Westphalia on July 6, 1910, Collatz (See, Figure 1) studied at different Universities *viz*. Greifswald, Gottingen, ¨ Munich, and Berlin from 1928 to 1933. In 1933, he completed his *Staatsexamen* (a German government licensing examination people have to pass to be allowed to work in their profession) under the direction of R. von Mises and E. Schrodinger [2]. He received his doctorate in 1935 under the supervision of Alfred Klose for the dissertation entitled 'The finite difference method with higher approximation for linear differential equations'. Collatz earned his doctorate officially with Klose, due to the changing political scenario that forced Mises (who actually supervised his work) to leave Germany, being Jewish. In his early years Collatz worked as a Professor at the Technical University of Hanover, and later (1952) moved to the University of Hamburg. Most of his research work dealt with applied mathematics, especially, numerical analysis (He founded the Institute of Applied Mathematics at the University of Hamburg). His work in the fifties and early sixties dealt with error analysis, monotonicity methods, approximation theory in one and several dimensions, and applications of functional analysis to numerical analysis [2]. Collatz has authored many important books throughout his career of which, to list a few are: Numerical Treatment of Eigenvalue problems (5 volumes), The Numerical Treatment of Differential Equations, Differential Equations: An Introduction with Applications, Functional Analysis and Numerical Mathematics, Optimization Problems *etc.*

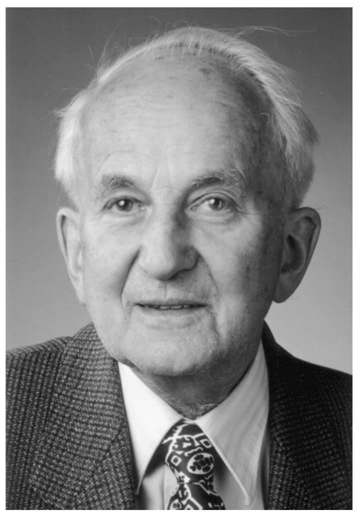

Figure 1: Lothar Collatz [3]

3. The 'random' structure of $3n+1$ problem: The difficulty of settling the $3n+1$ problem seems to be connected to the fact that it is a deterministic process that simulates 'random' behaviour [4]. The function $f(n)$ generates a sequence, say, Collatz sequence $C(n)$, similar to equation (1), which if, eventually reaches the value 1, it will then cycle through the loop of values $1 \rightarrow 4 \rightarrow 2 \rightarrow 1$ indefinitely. This behaviour has been verified empirically for all starting values of n up to 2^{68} [5]. Such sequences are commonly referred to as 'hailstone sequences' owing to the bouncing nature of the values in these sequences (A hailstone in a cloud was thought to bounce up and down, eventually falling to the ground). One can define the terms *viz.*

1. 'Stopping time T_n': the number of steps required for the number 1
2. 'Maximum value M_n': the largest value among the elements of $C(n)$.

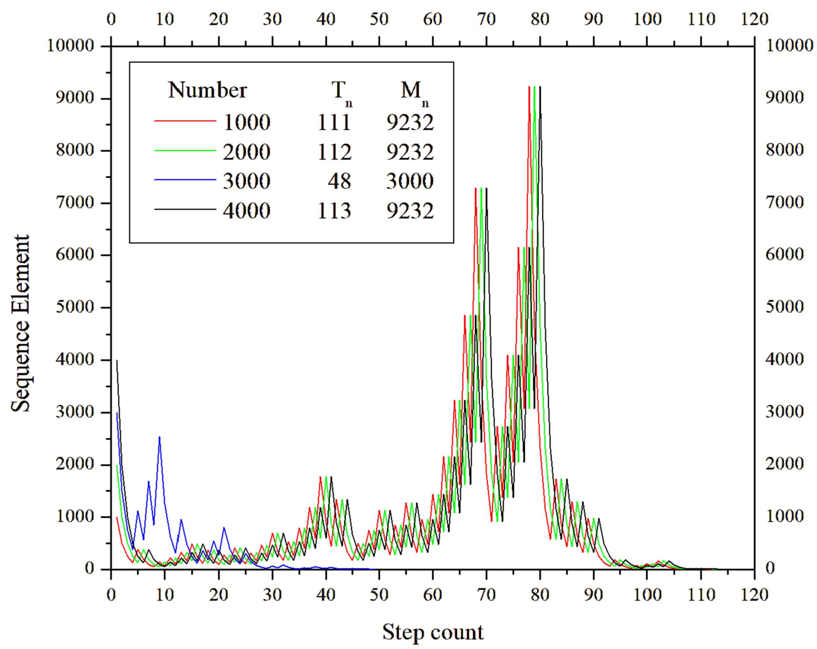

Figure 2: Hailstone pattern for some numbers

A lot of reliable resources are available online (such as [6], [7] and [8]) to find out $C(n)$, T_n and M_n, or else, one could write a code snippet to check for themselves. Researchers have so far not been able to establish any specific correlation between the value n and the stopping time and maximum value of the sequence $C(n)$. Table 1 gives the values of $C(n)$, T_n and M_n for some sample numbers.

Table 1: $C(n)$, T_n and M_n for different values of n

n	$C(n)$	T_n	M_n
11	11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1	14	52
13	13, 40, 20, 10, 5, 16, 8, 4, 2, 1	9	40
17	17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1	12	52
20	20, 10, 5, 16, 8, 4, 2, 1	7	20
30	30, 15, 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1	18	160
5376	5376, 2688, 1344, 672, 336, 168, 84, 42, 21, 64, 32, 16, 8, 4, 2, 1	15	5376

One could represent the same sequence graphically, as is shown in the Figure 2 for numbers 1000, 2000, 3000 and 4000. Values as high as 9232 are reached eventually following the loop $4 \rightarrow 2 \rightarrow 1$. The mechanism behind this pattern could be better understood with the help of a 'Collatz tree' (See, Figure 3) created using the bottom-up approach by drawing all the possible branches arising from the root of the tree (*i.e.* 1).

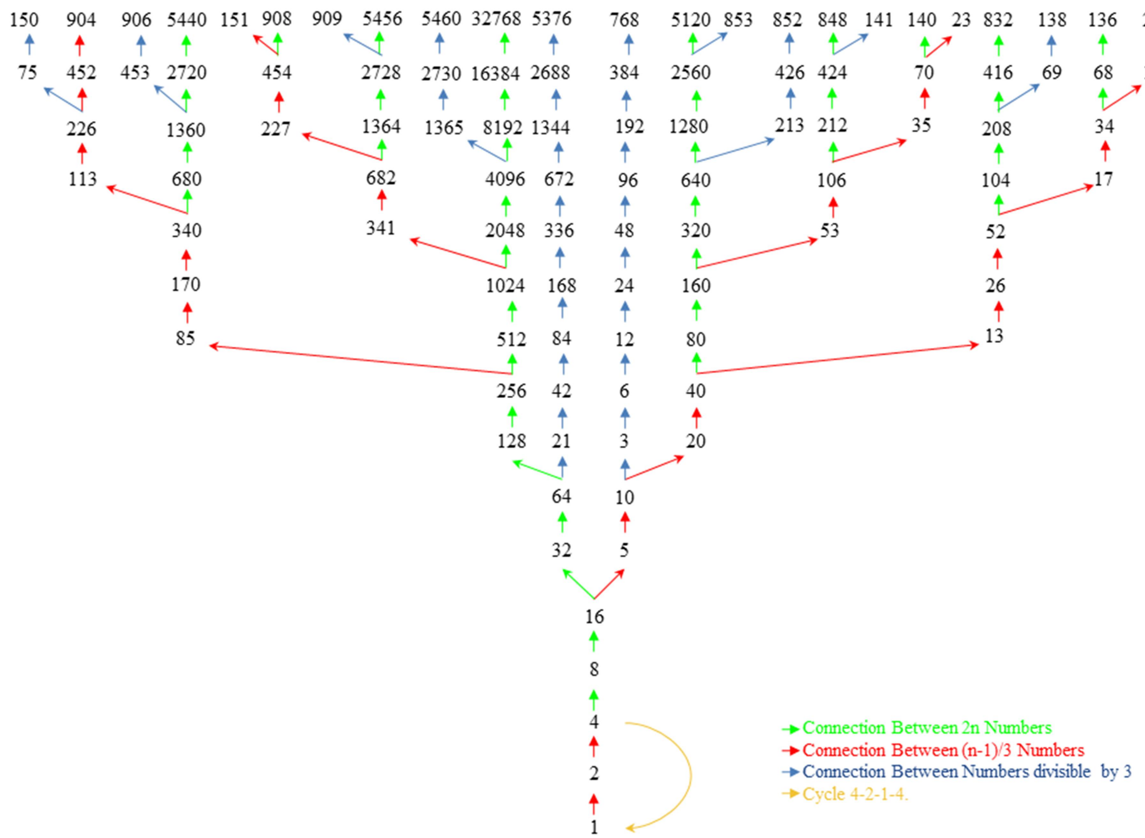

Figure 3: Collatz tree

The tree is a visual representation of the conjecture, where each number in the sequence generates a tree-like structure based on the iterations following the conjecture's rules. The starting positive integer serves as the root of the tree, and, the tree branches out creating sub-trees, as the sequence progresses. Numbers such as 16, 64, 10, 256, 40, ... are the 'nodes' of the tree. A node is the vertex of a tree having two or more branches. In case of a Collatz tree, every node has exactly two branches. Each node is either $(n - 1)/3$ or $2n$ owing to the rules of the Collatz conjecture. In Figure 3, three types of branches could be identified:

1. Green branches: which connect the integers of the form $2n$.
2. Red branches: which connect the integers of the form $(n-1)/3$.
3. Blue branches: Which are also called as 'greedy branches' (since, the stopping time is least for the integers on these branches as compared to their neighbouring integers). Here, all the vertices are not nodes and also are divisible by 3.

Despite its apparent simplicity, the Collatz conjecture remains unsolved. Hence, it finds implications in Number theory, computational explorations and is also used as an educational tool. It highlights the challenges in proving seemingly simple mathematical conjectures, where, the quest to find a counter-example or a proof helps in the understanding of mathematical reasoning in a broader way. It is often used as an educational tool to introduce students to mathematical thinking and exploration. In a sense, this conjecture bridges the theoretical mathematics and computational techniques.

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