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Accurate Domination in Fuzzy Digraphs Using Strong Arc

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ABSTRACT

In this paper, we introduce the concept of accurate dominating set in fuzzy digraphs using strong arc and also find the accurate domination number. In a fuzzy digraph, a subset D of V is an accurate fuzzy dominating set of a fuzzy digraph if every node $\sigma_D(v) \in V\text{-}D$ is not dominated by atleast one node $\sigma_D(u) \in D$ with same cardinality |D|. Also we have modified folydwarshall's Algorithm to fuzzy domination digraphs for finding an accurate dominating set.

Keywords: Fuzzy digraph, Strong arc, Dominating set, Accurate dominating set.

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1. Introduction

Ore [1] and Berge [2] introduced the study on dominating sets in graphs. The concept of accurate domination in graphs was initiated by Kulli and Kattimani [3]. Zadeh [6] introduced the concept of fuzzy set theory in 1965. Rosenfeld [6] explored fuzzy relations on fuzzy sets and fuzzy graphs in 1975. A.Somasundaram and N.Somasundaram [7] originated the concept of domination in fuzzy graphs using effective arcs.

NagoorGani and Chandrasekaran [8] discussed domination in fuzzy graphs using strong arcs. C.Y.Ponnayan and A.Selvam presented the concept of accurate domination in fuzzy graphs using strong arc[9]. The concept of domination in fuzzy digraphs was discussed by G.Nirmala and M.Sheela [10].

2. Preliminaries

Definition 2.1
A fuzzy graph is G: (V,σ,μ), V is the vertex set, $\sigma: V\rightarrow[0,1]$, $\mu: V\times V\rightarrow[0,1]$ with
$\mu(p,q) \leq \sigma(p) \wedge \sigma(q)$, for all $p,q \in V$.

Definition 2.2
Let G be a fuzzy graph. Let u and v be two distinct nodes of G. We say that u dominates v if (u,v) is a strong arc.

Definition 2.3
A subset D of V is called a dominating set of G if for every v ∈ V-D, there exists u ∈ D such that u dominates v.

Definition 2.4
An arc (u,v) is said to be a strong arc if $\mu(u,v) \geq \mu^{\infty}(u,v)$. If $\mu(u,v) = 0$ for every v ∈ V, then u is called as an isolated node.

Definition 2.5
A fuzzy digraph $G_D = (\sigma_D, \mu_D)$ is a pair of two functions $\sigma_D : V \to [0,1]$ and $\mu_D: A \to [0,1]$ such that $\mu_D(x,y) \leq \sigma_D(x) \wedge \sigma_D(y)$ for all x,y ∈ V.

Definition 2.6
An arc (x,y) of a fuzzy digraph is called an effective arc if $\mu_D(x,y) = \sigma_D(x) \wedge \sigma_D(y)$.

Definition 2.7
An arc (u,v) is said to be a strong arc or strong edge if $\mu_D(u,v) \geq \mu_D^{\infty}(u,v)$ and node v is said to be a strong neighbour of u.

Definition 2.8
Let x,y∈ V. The vertex $\sigma_D(x)$ dominates $\sigma_D(y)$ in a fuzzy digraph G_D if $\mu_D(x,y)$ is a strong arc.

Definition 2.9
A subset D of V is a fuzzy dominating set of G_D if every node $\sigma_D(v)$ ∈ V-D is dominated by atleast one node $\sigma_D(u)$ ∈ D.

Definition 2.10
The fuzzy domination number $\gamma_f(G_D)$ of a fuzzy digraph is the minimum cardinality of a fuzzy dominating set in G_D.

Definition 2.11
In fuzzy digraph, an isolated vertex refers to a vertex that has no incoming and outgoing edges, meaning that its membership degree in any relation or connection to other vertices is zero.

3.Accurate Domination In Fuzzy Digraphs

Definition 3.1
A subset D of V is an accurate fuzzy dominating set of a fuzzy digraph G_D if every node $\sigma_D(v)$ ∈V-D is not dominated by atleast one node $\sigma_D(u)$ ∈ D with same cardinality $|D|$.

An accurate fuzzy domination number $\gamma_{fa}(G_D)$ of a fuzzy digraph is the minimum cardinality of an accurate fuzzy dominating set in G_D. The upper domination number $\Gamma_{fa}(G_D)$ is the maximum cardinality of an accurate dominating set.

Example 3.2

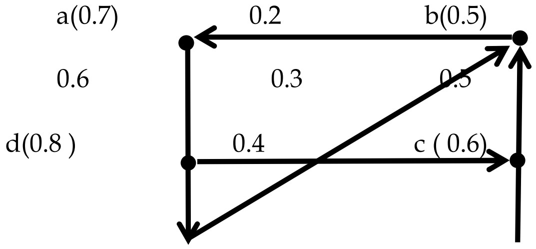

Figure 1

D = {a,c,d} is an accurate dominating set, since V-D is not dominated by atleast one node σ_D (u) Є D with cardinality |D|.

3.3 Folyd Warshall's Algorithm For Fuzzy Domination

It is used to find the accurate dominating set in a fuzzy digraph.

Step 1: Let us fix the initial matrix W_0 through arc weight.

Step 2: We can produce W_k from W_{k-1}.

Step 3: List the locations $v_1, v_2, \ldots$.. in column k of W_{k-1} where the entry is non zero and the locations $v_1, v_2, \ldots$in row k of W_{k-1} where the entry is non zero.

Step 4: Frame the possible combinations of non zero elements of column k with non zero elements of row k.

Step 5: If (v_i, v_j) is a strong arc and i = j put 1 otherwise 0.

Step 6: Proceed till W_n.

Step 7: We reach the row having maximum number of 1's which is a dominating set of a fuzzy digraph.

Step 8: Finally from the dominating set of a fuzzy digraph, a vertex u Є V-D is not dominated by atleast one vertex in D with same cardinality |D| is called as an accurate dominating set of a fuzzy digraph.

Example 3.4

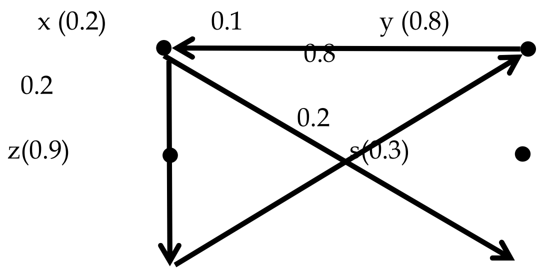

Figure 2

Consider the initial matrix,

$$W_0 = \begin{bmatrix} 0.2 & 0 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0 & 0 \\ 0 & 0.8 & 0.9 & 0 \\ 0 & 0 & 0 & 0.3 \end{bmatrix}$$

R(1) = { (x,x), (x,z), (x,s), (y,x), (y,z), (y,s) }

$$W_1 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0.8 & 0.9 & 0 \\ 0 & 0 & 0 & 0.3 \end{bmatrix}$$

R(2) = {(y,y), (z,y)}

$$W_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0.9 & 0 \\ 0 & 0 & 0 & 0.3 \end{bmatrix}$$

R(3) = { (x,y), (x,z), (z,y), (z,z) }

$$W_3 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0.3 \end{bmatrix}$$

R(4) = {(s,s)}

$$W_4 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

D = {x,z} is a dominating set of a fuzzy digraph and it also a accurate dominating set of a fuzzy digraph.

Theorem 3.5

Every accurate dominating set of a fuzzy digraph is a dominating set of a fuzzy digraph.

Proof :

Let D be an accurate dominating set of a fuzzy digraph.
By definition, V-D is not dominated by atleast one node in D with cardinality |D|.
Therefore V-D is adjacent to atleast one vertex in D.
This implies V-D is dominated by atleast one node in D.
Hence D is a dominating set of fuzzy digraph.

Theorem 3.6

For any fuzzy digraph $\gamma_f(G_D) \leq \gamma_{fa}(G_D)$

Proof :

Domination number of a fuzzy digraph is a minimum cardinality of a fuzzy dominating set.
For any fuzzy digraph, an accurate dominating set is a dominating set of a fuzzy digraph.
This implies a minimum accurate dominating set is also a dominating set of a fuzzy digraph.
Thus $\gamma_f(G_D) \leq \gamma_{fa}(G_D)$.

Theorem 3.7

In a fuzzy digraph, if there exists an isolated vertex then a minimal dominating set of a fuzzy digraph is an accurate dominating set of a fuzzy digraph.

Proof :

Assume that D is a dominating set of a fuzzy digraph.
Let u Є V be an isolated vertex of a fuzzy digraph which has no incoming and outgoing edges.
This means it does not dominate or dominated by any other vertices.
Therefore V-D is not dominated by u with cardinality |D|.
This implies D is an accurate dominating set of a fuzzy digraph.
Hence the result.

Theorem 3.8

For any fuzzy digraph, if D be an accurate dominating set of a fuzzy digraph then V-D is need not be accurate dominating set of a fuzzy digraph.

Proof :
Let G_D be a fuzzy digraph. Assume that u∈ D ia an accurate dominating set of a fuzzy digraph. Then V-D is not dominated by atleast one vertex in u ∈ D with cardinality |D|.
Case (i):
If V-D has no strong neighbour then V-D cannot be a dominating set of a fuzzy digraph.
So it cannot be an accurate dominating set of a fuzzy digraph.
Case (ii):
Suppose it has atleast one strong neighbour which dominates u, then V-D is a dominating set of a fuzzy digraph and we have |V-D| ≠ |D|.
Hence it can be an accurate dominating set of a fuzzy digraph.

4.Applications

Accurate domination helps to identify the minimum number of nodes which dominates other nodes provided if it is a strong arc. It supports to determine the best routes to ensure optimal flow throughout the city. Also it can be used in optimization problems to ensure that a minimal set of vertices accurately dominate the entire system which could be crucial in resource allocation, decision making and monitoring systems where resources are limited. Hence accurate domination in fuzzy digraphs provides a powerful tool for analyzing complex systems with imprecise or uncertain connections.

5. Conclusion

In this work, we have introduced the concept of accurate domination in fuzzy digraphs using strong arc. Some results on accurate dominating set of a fuzzy digraph are discussed. Also, We found modified warshall's algorithm for fuzzy domination to find the accurate dominating set of fuzzy digraphs.

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