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Antimagic Labeling of Pumpkin Graph

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ABSTRACT

A graph with q edges is called antimagic if its edges can be labeled with $1, 2, \dots, q$ without repetition such that the sums of the labels of the edges incident to each vertex are distinct. A graph which admits an antimagic labeling is called an antimagic graph. In this paper, we prove that the Pumpkin graph admit antimagic labeling.

KEYWORDS: Antimagic labeling, Pumpkin Graph.

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1. Introduction

Graph theory is a branch of Mathematics, concerned with network of points called vertices which are connected by the lines called edges. In graph theory [4], Network theory is the study of graphs as a representation of either symmetric or asymmetric relation. It is useful for analysing complex data by converting them into a network graph. A labeling [5] of a graph is any mapping that sends some set of graph elements to a set of numbers (usually positive integers). Specially, if we have bijections $g : V \to \{1, 2, \dots, p\}$ or $h : E \to \{1, 2, \dots, q\}$ then the labelings are called a vertex labeling or an edge labeling respectively, where $p = |V(G)|$ and $q = |E(G)|$. The first graph labeling was given by Rosa [10] in 1967 termed as β-valuation. In 1972, Golomb renamed β-valuation as graceful labeling. Labeled graphs serve as a useful models for a broad range of applications such as coding theory, x-ray crystallography, radar, astronomy, circuit design, communication network addressing and data base management.[8] Hartsfield and Ringel [6]introduced the concept of antimagic labeling in 1990. A graph with q edges is called antimagic if its edges can be labeled with $1, 2, \dots,$ q without repetition such that the sums of the labels of the edges incident to each vertex are distinct. A graph which admits an antimagic labeling is called an antimagic graph. Several results on antimagic labelingis given in the Gallian survey [5]. In this paper, we prove that the Pumpkin graph admit antimagic labeling.

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Hartsfield and Ringel [6] conjectured that every tree other than K_2 is antimagic and more strongly, that every connected graph other than K_2 is antimagic. Alon *et.al* [1] use probabilistic methods and analytic number theory to show that this conjecture is true for all graphs with n vertices and minimum degree $\Omega(logn)$. Hartsfield and Ringel [6] showed that paths P_n $(n \geq 3)$, cycles, wheels and complete graphs K_n $(n \geq 3)$ are antimagic. Basher M [2] has found the k-Zumkeller labeling of super subdivision of some graphs. Hong Yang *et.al* [7] has showed that the Pumpkin graph admit cycle super magic labeling. Antimagic labeling is widely used in the model of security systems or surveillance, electrical switchboards, developing a cipher block chain, circuit designs, cryptography, communication networks etc.[3,9]

2. Definitions

In this section, we provide all the fundamental notations and definitions which serve as prerequisites for the advancement of the topic.

Definition 2.1: [6]
Let $\alpha$and βbe two integers with $\beta \geq 3$. Let $y_1, y_2, \dots, y_\alpha$be fixed vertices. We connect the vertices $y_i$and $y_{i+1}$by means of βinternally disjoint paths $p_i^j$of length $i+1$ each, $1 \leq i \leq \alpha - 1$, $1 \leq j \leq \beta$. Let $y_i, x_{i,j,1}, x_{i,j,2}, \dots, x_{i,j,i}, y_{i+1}$be the vertices of the path p_i^j . The resulting graph embedded in the plane is denoted by P_α^β and is called the **pumpkin graph**.

3. Main Results

In this section, we prove that the Pumpkin graph admit antimagic labeling.

Theorem 1:
Let $\alpha, \beta \geq 3$ be two integers. Then the Pumpkin Graph P_α^βadmit antimagiclabeling.

Proof:
Let P_α^β be a Pumpkin Graph with α fixed vertices $y_1, y_2, \dots, y_\alpha$ and βinternally disjoint paths $p_i^j$of length $i+1$ each, $1 \leq i \leq \alpha - 1$, $1 \leq j \leq \beta$.
Let n denote the sum of the length of the paths p_i^j for any $j$and $1 \leq i \leq \alpha - 1$.

$$n = \sum_{i=1}^{\alpha-1} length of p_i^j = (\alpha^2 + \alpha - 2)/2$$

Let $E(P_\alpha^\beta)$ and $V(P_\alpha^\beta)$ denote the edge set and vertex set of the graph respectively. Let$E(P_\alpha^\beta) = \{E_1 \cup E_2 \cup E_3 \cup \dots \cup E_{\alpha-1}\}$ where $E_i = \{e_{i,j,k}, 1 \leq i \leq \alpha - 1, 1 \leq j \leq \beta$ and $1 \leq k \leq i\}$ denote the edgeset of the graph P_α^β between the vertices y_i and y_{i+1}.Let $V(P_\alpha^\beta) = \left\{ Y_1 \cup Y_\alpha \cup Y_{i_{i \neq 1,\alpha}} \cup V_1 \cup V_2 \cup V_3 \cup \dots \cup V_{\alpha-1} \right\}$where$Y_1 = \{y_1\}$ denote the first fixed vertex, $Y_\alpha = \{y_\alpha\}$ denote the last fixed vertex and $Y_i = \{y_i : 1 \leq i \leq \alpha - 1\}$ denote the internal fixed vertices and $V_i = \{x_{i,j,k}\}$ denote the internal vertices between y_i and y_{i+1} for $1 \leq i \leq \alpha - 1$, $1 \leq j \leq \beta$ and $1 \leq k \leq i$.
Total number of edges in the graph P_α^β is given by $|E(P_\alpha^\beta)| = n\beta$ and the total number of vertices is given by $|V(P_\alpha^\beta)| = \frac{\alpha}{2}(2 + \beta(\alpha - 1))$.We define the edge labelling $\phi : E(P_\alpha^\beta) \to \{1,2,3, \dots, |E(P_\alpha^\beta)|\}$ as follows:

$$\phi(e_{i,j,k}) = n(j-1) + k + \frac{i^2 + i - 2}{2}$$

for $1 \leq i \leq \alpha - 1, 1 \leq j \leq \beta$ and $1 \leq k \leq i + 1$.
Thus the entire edge set is labelled with distinct integers from 1 to $|E(P_\alpha^\beta)|$.
The induced vertex labels$\psi : V(P_\alpha^\beta) \to Z^+$ are as follows:

$$\psi(y_i) = \begin{cases} \beta + \dfrac{n\beta(\beta-1)}{2}, & for i = 1 \\ \dfrac{n\beta(\beta+1)}{2}, & for i = \alpha \\ n\beta(\beta-1) + \beta(i^2+i-1), & for otherwise \end{cases}$$

$$\psi(x_{i,j,k}) = 2n(j-1) + (i-1)(i+2),$$
$$for\ 1 \leq i \leq \alpha - 1, 1 \leq j \leq \beta, 1 \leq k \leq i$$

So we observe that all the vertex labels are distinct and hence the Pumpkin Graph P_α^β admit Antimagic Labeling.

Illustration

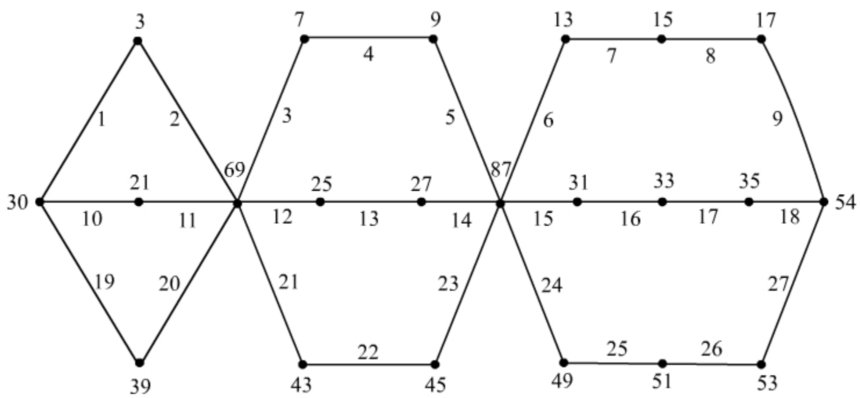

Figure1: Antimagic labeling of Pumpkin Graph P_{10}^3

4. Conclusion

Thus in this paper we have obtained the antimagic labeling for Pumpkin graph.

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