Bulletin of Pure and Applied Sciences
Section - E - Mathematics & Statistics
Vol. 41E, No.2, July-December 2022.P.159-166

Print version ISSN 0970 6577
Online version ISSN 2320 3226
DOI: 10.5958/2320-3226.2022.00022.4.

Original Article

Content Available online at: https://bpasjournals.com/math-and-stat/

Exact Solution of Non-Linear Volterra Integral Equation of First Kind Using Rishi Transform

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How to cite this article: Aggarwal S., Kumar R., Chandel J. (2022). Exact Solution of Non-Linear Volterra Integral Equation of First Kind Using Rishi Transform. *Bull. Pure Appl. Sci. Sect. E Math. Stat.* 41E(2), 159-166.

ABSTRACT

The problems of Engineering and Science can easily represent by developing their mathematical models in the terms of integral equations. Various analytical and numerical methods are available that can be used for solving integral equations of different kinds. In this paper, authors have considered recently developed integral transform "Rishi Transform" for obtaining the exact solution of non-linear Volterra integral equation of first kind (NLVIEFK). Four numerical problems have considered for demonstrating the complete procedure of determining the exact solution. Results of these problems depict that Rishi transform is very effective integral transform and it provides the exact solution of NLVIEFK without doing complicated calculation work.

KEYWORDS: Analytical Solution; Rishi Transform; Inverse Rishi Transform; Convolution; Volterra Integral Equation.

Mathematics Subject Classification: 35A22; 44A05; 44A35; 45D05; 45G10

1. Introduction

Volterra integral equations have a number of applications in the vast field of Mechanics, linear visco-elasticity, hereditary phenomena, renewal theory, particle size statistics, theory of superfludity, damped vibration of a string, heat transfer problem, geometric probability, visco-elastic stress analysis, population dynamics and study of epidemics [1-14]. Differential and partial differential equations (liner or non-linear) with initial conditions can be transformed into a single or multiple Volterra integral equations (linear or non-linear) [15]. There are numerous analytical and numerical methods available for treatment of Volterra integral equations such as Laplace transform

Received on 12.09.2022/ Revised on 20.11.2022/ Accepted on 30.11.2022
Online First Published on Dec 15, 2022 at https://www.bpasjournals.com/

[16]; Mohand transform [29]; Aboodh transform [30]; Kamal transform [27]; Laplace-Carson transform (Mahgoub transform) [28]; Anuj transform [46]; successive approximation method [1]; Taylor's series method [22, 32]; Runge-Kutta methods [17-18]; piecewise polynomial collocation method [19]; spline method [20, 25]; implicit methods [21]; backward differentiation type method [23]; modified Runge-Kutta method [24]; finite difference method [26]; Adomian decomposition method [1] and variational iteration method [1].

Aggarwal with other researchers [16, 27-31] used various integral transformations, Laplace; Kamal; Mahgoub (Laplace-Carson); Mohand; Aboodh and Shehu for determining the solution of LVIESK. Aggarwal et al. [32] gave the numerical treatment of non-homogeneous LVIESK using Taylor's series method. In the recent years, researchers developed numerous new integral transforms (Sumudu [33]; Natural [34]; Elzaki [35]; Aboodh [36]; Mahgoub [37]; Kamal [38]; ZZ [39]; Mohand [40]; Sadik [41]; Shehu [42]; Sawi [43]; Upadhyay [44]; Jafari [45]; Anuj [46]) and used them to handle the problems of Science and Engineering. Higazy et al. [47] introduced a new decomposition method "Sawi decomposition method" to determine the solution of Volterra integral equation. Aggarwal with other scholars [48-55] introduced the relations of duality among the established integral transformations. Ali et al. [56] determined the solution of fractional Volterra-Fredholm integro-differential equations under mixed boundary conditions by using the HOBW method. Padder et al. [57] analyzed the tumor-immune response model by differential transformation method.

Ali et al. [58] used HOBW method and determined the solution of the problem of nonlinear Volterra integral equations with weakly singular kernel. Higazy and Aggarwal [59] used Sawi transform and solved the complex problem of chain reaction in chemical kinetics by representing it into a system of ordinary differential equations. El-Mesady et al. [60] completely solved the problem of medical science by using Jafari transform. Higazy et al. [61] studied infections model of HIV-1 by the help of Shehu transform. Priyanka and Aggarwal [63] recently solved the model of the bacteria growth via Rishi transform.

The motive of the present paper is to determine the exact solution of non-linear Volterra integral equation of first kind by using recently developed integral transform "Rishi Transform".

The residual of the present paper is planned in seven sections. Section 2 gives the nomenclature of the symbols. Section 3 provides the definition of Rishi transform. Section 4 deals the inverse Rishi transform. In section 5, solution of non-linear Volterra integral equation of first kind is given using Rishi transform. Section 6 contains four numerical examples for better explaining the complete procedure of determining the solution of NLVIEFK in detail. The paper ends in section 7 with the conclusion.

2. Nomenclature of Symbols

Υ, Rishi transform operator;
Υ^{-1}, inverse Rishi transform operator;
N, the set of natural numbers;
$\in$, belongs to;
!, the usual factorial notation;
Γ, the classical Gamma function;
L, Laplace transform operator;
R, the set of real numbers

3. Definition of Rishi Transform

The Rishi transform of a piecewise continuous exponential order function $F(t), t \geq 0$ is given by [62]

$$\Upsilon\{F(t)\} = \left(\frac{\sigma}{\varepsilon}\right)\int_0^\infty F(t)e^{-\left(\frac{\varepsilon}{\sigma}\right)t}dt = T(\varepsilon,\sigma),\ \varepsilon > 0, \sigma > 0 \tag{1}$$

4. Inverse Rishi Transform [62]

The inverse rishi transform of $T(\varepsilon,\sigma)$, designated by $\Upsilon^{-1}\{T(\varepsilon,\sigma)\}$, is another function $F(t)$ having the property that $\Upsilon\{F(t)\} = T(\varepsilon,\sigma)$.

Some useful operational characteristics of Rishi transform, Rishi transforms of some fundamental functions and their inverse Rishi transforms are summarized in the Tables1-3 respectively.

Table 1: Some operational characteristics of Rishi transform [62]

S.N.	Name of Characteristic	Mathematical Form
1	Linearity	$Y\{\sum_{i=1}^{n} k_i F_i(t)\} = \sum_{i=1}^{n} k_i Y\{F_i(t)\}$, where k_i are arbitrary constants
2	Change of Scale	If $Y\{F(t)\} = T(\varepsilon,\sigma)$ then $Y\{F(kt)\} = \frac{1}{k^2} T\left(\frac{\varepsilon}{k},\sigma\right)$
3	Translation	If $Y\{F(t)\} = T(\varepsilon,\sigma)$ then $\left\{Y\{e^{kt}F(t)\} = \left(\frac{\varepsilon - k\sigma}{\varepsilon}\right) T(\varepsilon - k\sigma,\sigma)\right\}$
4	Convolution	If $Y\{F_1(t)\} = T_1(\varepsilon,\sigma)$ and $Y\{F_2(t)\} = T_2(\varepsilon,\sigma)$ then $\left\{Y\{F_1(t) * F_2(t)\} = \left[\left(\frac{\varepsilon}{\sigma}\right) T_1(\varepsilon,\sigma)\, T_2(\varepsilon,\sigma)\right]\right\}$

Table 2: Some fundamental functions and their Rishi transform [62]

S.N.	$F(t), t > 0$	$Y\{F(t)\} = T(\varepsilon,\sigma)$
1	1	$\left(\frac{\sigma}{\varepsilon}\right)^2$
2	e^{lt}	$\frac{\sigma^2}{\varepsilon(\varepsilon - l\sigma)}$
3	$t^\rho, \rho \in N$	$\rho!\left(\frac{\sigma}{\varepsilon}\right)^{\rho+2}$
4	$t^\rho, \rho > -1, \rho \in R$	$\left(\frac{\sigma}{\varepsilon}\right)^{\rho+2} \Gamma(\rho+1)$
5	$sin(lt)$	$\frac{l\sigma^3}{\varepsilon(\varepsilon^2 + \sigma^2 l^2)}$
6	$cos(lt)$	$\frac{\sigma^2}{(\varepsilon^2 + \sigma^2 l^2)}$
7	$sinh(lt)$	$\frac{l\sigma^3}{\varepsilon(\varepsilon^2 - \sigma^2 l^2)}$
8	$cosh(lt)$	$\frac{\sigma^2}{(\varepsilon^2 - \sigma^2 l^2)}$
9	$J_0(t)$	$\frac{\sigma^2}{\varepsilon\left(\sqrt{\varepsilon^2 + \sigma^2}\right)}$

Table 3: Inverse Rishi transformations of some fundamental functions [63]

S.N.	$T(\varepsilon,\sigma)$	$F(t) = Y^{-1}\{T(\varepsilon,\sigma)\}$
1	$\left(\frac{\sigma}{\varepsilon}\right)^2$	1
2	$\frac{\sigma^2}{\varepsilon(\varepsilon - l\sigma)}$	e^{lt}
3	$\left(\frac{\sigma}{\varepsilon}\right)^{\rho+2}, \rho \in N$	$\frac{t^\rho}{\rho!}$
4	$\left(\frac{\sigma}{\varepsilon}\right)^{\rho+2}, \rho > -1, \rho \in R$	$\frac{t^\rho}{\Gamma(\rho+1)}$
5	$\frac{\sigma^3}{\varepsilon(\varepsilon^2 + \sigma^2 l^2)}$	$\frac{sin(lt)}{l}$

6	$\frac{\sigma^2}{(\varepsilon^2+\sigma^2 l^2)}$	$cos(lt)$
7	$\frac{\sigma^3}{\varepsilon(\varepsilon^2-\sigma^2 l^2)}$	$\frac{sinh(lt)}{l}$
8	$\frac{\sigma^2}{(\varepsilon^2-\sigma^2 l^2)}$	$cosh(lt)$
9	$\frac{\sigma^2}{\varepsilon\left(\sqrt{\varepsilon^2+\sigma^2}\right)}$	$J_0(t)$

5. Solution of Non-Linear Volterra Integral Equation of First Kind Using Rishi Transform

In this work, we have assumed the following form of non-linear Volterra integral equation of first kind

$$F(t)=\int_0^t \Theta(t-\zeta)\,\Theta(\zeta)d\zeta\ , \tag{2}$$

where $\Theta(t)$ and $F(t)$ are unknown and known functions respectively.
Operating Rishi transform on equation (2), we get

$$\Upsilon\{F(t)\}=\Upsilon\left\{\int_0^t \Theta(t-\zeta)\,\Theta(\zeta)d\zeta\right\}$$

$$\Rightarrow \Upsilon\{F(t)\}=\Upsilon\{\Theta(t)*\Theta(t)\} \tag{3}$$

Use of convolution theorem in equation (3) gives

$$\Upsilon\{F(t)\}=\left(\frac{\varepsilon}{\sigma}\right)\Upsilon\{\Theta(t)\}\Upsilon\{\Theta(t)\}$$

$$\Upsilon\{\Theta(t)\}=\pm\sqrt{\left(\frac{\sigma}{\varepsilon}\right)\Upsilon\{F(t)\}} \tag{4}$$

After operating inverse Rishi transform on equation (4), the solutions of equation (2) are given by

$$\Theta(t)=\pm\Upsilon^{-1}\left\{\sqrt{\left(\frac{\sigma}{\varepsilon}\right)\Upsilon\{F(t)\}}\right\} \tag{5}$$

6. Numerical Problems

This section contains four numerical problems for better explaining the complete procedure of determining the solution of NLVIEFK in detail.

Problem 1: Consider the following NLVIEFK given by

$$\frac{t^3}{6}=\int_0^t \Theta(t-\zeta)\,\Theta(\zeta)d\zeta \tag{6}$$

Operating Rishi transform on equation (6), we get

$$\Upsilon\left\{\frac{t^3}{6}\right\}=\Upsilon\left\{\int_0^t \Theta(t-\zeta)\,\Theta(\zeta)d\zeta\right\}$$

$$\Rightarrow \frac{1}{6}\Upsilon\{t^3\}=\Upsilon\{\Theta(t)*\Theta(t)\}$$

$$\Rightarrow \frac{1}{6}\left[3!\left(\frac{\sigma}{\varepsilon}\right)^5\right]=\Upsilon\{\Theta(t)*\Theta(t)\} \tag{7}$$

Using convolution theorem in equation (7), we have

$$\left[\left(\frac{\sigma}{\varepsilon}\right)^5\right]=\left(\frac{\varepsilon}{\sigma}\right)[\Upsilon\{\Theta(t)\}]^2$$

$$\Rightarrow \left[\left(\frac{\sigma}{\varepsilon}\right)^6\right]=[\Upsilon\{\Theta(t)\}]^2$$

$$\Rightarrow \Upsilon\{\Theta(t)\}=\pm\left[\left(\frac{\sigma}{\varepsilon}\right)^3\right] \tag{8}$$

After operating inverse Rishi transform on equation (8), the solutions of equation (6) are given by

$$\Theta(t)=\pm\Upsilon^{-1}\left\{\left[\left(\frac{\sigma}{\varepsilon}\right)^3\right]\right\}$$

$$\Rightarrow \Theta(t)=\pm t.$$

Problem 2: Consider the following NLVIEFK given by

$$sint = \int_0^t \Theta(t-\zeta)\,\Theta(\zeta)d\zeta \quad (9)$$

Operating Rishi transform on equation (9), we get

$$\Upsilon\{sint\} = \Upsilon\left\{\int_0^t \Theta(t-\zeta)\,\Theta(\zeta)d\zeta\right\}$$

$$\Rightarrow \left[\frac{\sigma^3}{\varepsilon(\varepsilon^2+\sigma^2)}\right] = \Upsilon\{\Theta(t)*\Theta(t)\} \quad (10)$$

Using convolution theorem in equation (10), we have

$$\left[\frac{\sigma^3}{\varepsilon(\varepsilon^2+\sigma^2)}\right] = \left(\frac{\varepsilon}{\sigma}\right)[\Upsilon\{\Theta(t)\}]^2$$

$$\Rightarrow \Upsilon\{\Theta(t)\} = \pm\left[\frac{\sigma^2}{\varepsilon\left(\sqrt{\varepsilon^2+\sigma^2}\right)}\right] \quad (11)$$

After operating inverse Rishi transform on equation (11), the solutions of equation (9) are given by

$$\Theta(t) = \pm\Upsilon^{-1}\left\{\left[\frac{\sigma^2}{\varepsilon\left(\sqrt{\varepsilon^2+\sigma^2}\right)}\right]\right\}$$

$$\Rightarrow \Theta(t) = \pm J_0(t).$$

Problem 3: Consider the following NLVIEFK given by

$$e^t sint = \int_0^t \Theta(t-\zeta)\,\Theta(\zeta)d\zeta \quad (12)$$

Operating Rishi transform on equation (12), we get

$$\Upsilon\{e^t sint\} = \Upsilon\left\{\int_0^t \Theta(t-\zeta)\,\Theta(\zeta)d\zeta\right\}$$

$$\Rightarrow \left[\frac{\sigma^3}{\varepsilon\{(\varepsilon-\sigma)^2+\sigma^2\}}\right] = \Upsilon\{\Theta(t)*\Theta(t)\} \quad (13)$$

Using convolution theorem in equation (13), we have

$$\left[\frac{\sigma^3}{\varepsilon\{(\varepsilon-\sigma)^2+\sigma^2\}}\right] = \left(\frac{\varepsilon}{\sigma}\right)[\Upsilon\{\Theta(t)\}]^2$$

$$\Rightarrow \Upsilon\{\Theta(t)\} = \pm\left[\frac{\sigma^2}{\varepsilon\sqrt{\{(\varepsilon-\sigma)^2+\sigma^2\}}}\right] \quad (14)$$

After operating inverse Rishi transform on equation (14), the solutions of equation (12) are given by

$$\Theta(t) = \pm\Upsilon^{-1}\left\{\left[\frac{\sigma^2}{\varepsilon\sqrt{\{(\varepsilon-\sigma)^2+\sigma^2\}}}\right]\right\}$$

$$\Rightarrow \Theta(t) = \pm e^t J_0(t).$$

Problem 4: Consider the following NLVIEFK given by

$$e^{-t}\left(\frac{t^3}{6}\right) = \int_0^t \Theta(t-\zeta)\,\Theta(\zeta)d\zeta \quad (15)$$

Operating Rishi transform on equation (15), we get

$$\Upsilon\left\{e^{-t}\left(\frac{t^3}{6}\right)\right\} = \Upsilon\left\{\int_0^t \Theta(t-\zeta)\,\Theta(\zeta)d\zeta\right\}$$

$$\Rightarrow \frac{1}{6}\Upsilon\{e^{-t}t^3\} = \Upsilon\left\{\int_0^t \Theta(t-\zeta)\,\Theta(\zeta)d\zeta\right\}$$

$$\Rightarrow \frac{1}{6}\left[3!\frac{\sigma^5}{\varepsilon(\varepsilon+\sigma)^4}\right] = \Upsilon\{\Theta(t)*\Theta(t)\}$$

$$\Rightarrow \left[\frac{\sigma^5}{\varepsilon(\varepsilon+\sigma)^4}\right] = \Upsilon\{\Theta(t)*\Theta(t)\} \quad (16)$$

Using convolution theorem in equation (16), we have

$$\left[\frac{\sigma^5}{\varepsilon(\varepsilon+\sigma)^4}\right] = \left(\frac{\varepsilon}{\sigma}\right)[\Upsilon\{\Theta(t)\}]^2$$

$$\Rightarrow \Upsilon\{\Theta(t)\} = \pm\left[\frac{\sigma^3}{\varepsilon(\varepsilon+\sigma)^2}\right] \quad (17)$$

After operating inverse Rishi transform on equation (17), the solutions of equation (15) are given by

$$\Theta(t) = \pm \mathrm{Y}^{-1}\left\{\left[\frac{\sigma^3}{\varepsilon(\varepsilon+\sigma)^2}\right]\right\}$$
$$\Rightarrow \Theta(t) = \pm te^{-t}.$$

7. Conclusions

In the presented paper, authors successfully obtained the exact solution of NLVIEFK using Rishi transform. The findings of the presented paper indicate that the Rishi transform is a very efficient integral transform for obtaining the exact solution of NLVIEFK without doing tedious and large computational work. In future, Rishi transform can be use for solving the complex problems of Science and Engineering which can be transformed into a single or multiple non-linear Volterra integral equations of first kind.

Data Availability: Authors of this paper confirm that the datasets that are used in this paper are available from the author upon request.

Conflicts of Interest: There are no conflicts of interest between the authors.

Funding Statement: This research has no funding.

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