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On δ_{IJ}^*-semi-homeomorphisms in ideal Topological spaces

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ABSTRACT
In this paper, the notions of δ_I- semi-open, δ_I- semi-closed, δ_{IJ}- semi-homeomorphism and δ_{IJ}^*- semi-homeomorphism functions are introduced and investigated some characterizations of these functions in ideal topological spaces.

KEYWORDS: δ_I- semi-open function, δ_I-semi-closed function, $\delta_{IJ} - semi - homeomorphism, \delta_{IJ}^*$-semi-homeomorphism.
Mathematics Subject Classification: 54A05.

1. INTRODUCTION

In 1968, N. V. Velicko [5] introduced δ -open set in topological spaces. In 2005, δ -open sets are introduced by S. Yuksel et al. [6] in ideal topological spaces. Kuratowski [3] and Vaidyanathasamy [4] introduced and studied an ideal concept in topological spaces. In this paper, the notions δ_I-semi-open functions, δ_I-semi-closed functions, δ_{IJ}- semi-homeomorphisms and δ_{IJ}^*- semi-homeomorphisms are introduced and study some of its properties in ideal topological spaces.

2. PRELIMINARIES

In this paper, X, Y and Z are always mean ideal topological spaces. Fora subset Aof a space X, $\mathrm{sint}_{\delta_I}(A)$ and $\mathrm{scl}_{\delta_I}(A)$ denote δ_I-semi-interior and δ_I-semi-closure of A.

Definition 2.1. [1] A subset A of an ideal topological space (X, τ, I) is said to be δ_I-semi-open if $A \subseteq cl^*(int_\delta(A))$.

Definition 2.2. [2] A function $f : (X,\tau,I) \to (Y,\sigma)$ is said to be δ_I -semi- continuous if $f^{-1}(V)$ is δ_I-semi-open in (X,τ,I) for each open set V of (Y,σ) .

Definition 2.3. [2] A function $f : (X,\tau,I) \to (Y,\sigma,I)$ is said to be δ_I-semi- irresolute if inverse image of every δ_I -semi-open set in Yis δ_I-semi-open set in X.

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3. δ_I -SEMI-OPEN AND δ_I -SEMI-CLOSED FUNCTIONS

Definition 3.1. *A function $f : (X,\tau) \to (Y,\sigma,I)$ is called δ_I -semi-open (briefly $\delta_I s_{op}$) if for each open set U of X, $f(U)$ is δ_I -semi-open in Y.*

Definition 3.2. *A function $f : (X,\tau) \to (Y,\sigma,I)$ is called δ_I -semi-closed (briefly $\delta_I s_{clo}$) if for each closed set U of X, $f(U)$ is δ_I -semi-closed in Y.*

Example 3.3. *Let $X = \{a,b,c,d\}$, $\tau = \{\emptyset,\{d\},\{a,c\},\{a,c,d\},X\}$ and let $Y = \{p,q,r\}$, $\sigma = \{\emptyset,\{p\},\{q\},\{p,q\},\{q,r\},Y\}$ and $I = \{\emptyset,\{p\},\{q\},\{p,q\}\}$. Let $f:(X,\tau) \to (Y,\sigma,I)$ be a function defined as $f(a) = f(b) = q, f(c) = r$ and $f(d) = p$. Then f is $\delta_I s_{op}$.*

Example 3.4. *Let $X = \{a,b,c,d\},\tau = \{\emptyset,\{d\},\{a,c\},\{a,c,d\},X\}$ and let $Y = \{p,q,r\}$, $\sigma = \{\emptyset,\{q\},\{r\},\{q,r\},Y\}$ and $I = \{\emptyset,\{p\}\}$. Let $f:(X,\tau) \to (Y,\sigma,I)$ be a function defined as $f(a) = f(c) = r, f(b) = p$ and $f(d) = q$. Then f is $\delta_I s_{clo}$.*

Theorem 3.5. *A function $f : (X,\tau) \to (Y,\sigma,I)$ is $\delta_I s_{op}$ if and only if for each $x \in X$ and each neighborhood U of x, there exists $V \in \delta_I SO(Y)$ containing $f(x)$ such that $V \subseteq f(U)$.*

Proof. Suppose that f is $\delta_I s_{op}$ For each $x \in X$ and each neighborhood U of x, there exists an open set, U_0 such that $x \in U_0 \subseteq U$.Since f is $\delta_I s_{op}, V = f(U_0) \in \delta_I SO(Y)$ and $f(x) \in V \subseteq f(U)$. Conversely, let U be an open set of X. For each x $\in U$, *there* exists $V_x \in \delta_I SO(Y)$ such that $f(x) \in V_x \subseteq f(U)$.

Therefore, we obtain $f(U) = \{V_x | x \in U\}$ and hence $f(U)$ is $\delta_I s_{op}$ in Y, *by* Theorem 3.13 of [1]. This implies that f is $\delta_I s_{op}$.

Theorem 3.6. *A bijective function $f : (X,\tau) \to (Y,\sigma,I)$ is $\delta_I s_{op}$ if and only if for each subset $W \subseteq Y$ and each closed set F of X containing $f^{-1}(W)$ there exists a δ_I -semi-closed set $H \subseteq Y$ containing W such that $f^{-1}(H) \subseteq F$.*

Proof.

Necessity: Suppose that f is a $\delta_I s_{op}$ function. Let W be any subset of Y and F is a closed subset of X containing $f^{-1}(W)$. Then F^C is open and since f *is* δ_I -semi-open, $f(F^C)$ is δ_I -semi-open in Y. Hence $H = [f(F^C)]^C$ is δ_I -semi-closed in Y. $f^{-1}(W) \subseteq F$ Implies that $W \subseteq H$. Moreover, we obtain $f^{-1}(H) = f^{-1}([f(F^C)]^C) = f^{-1}(f(F)) = F$. Hence, $f^{-1}(H) = F$.

Sufficiency: Let U be any open set of X and $W = [f(U)]^c$. Then $f^{-1}(W) = f^{-1}([f(U)]^c) = f^{-1}(f(U^c)) = U^c$ and U^c is closed. By hypothesis, there exists a δ_I -semi-closed set H of Y containing W *such* that $f^{-1}(H) \subseteq U^c$. Then we have $f^{-1}(H) \cap U = \emptyset$ and $H \cap f(U) = \emptyset$. Therefore we obtain $[f(U)]^C \supseteq H \supseteq W = [f(U)]^C$ and $f(U)$ is δ_I -semi-open in Y. This shows that f*is* $\delta_I s_{op}$.

Theorem 3.7. *For a function $f : (X,\tau) \to (Y,\sigma,I)$, the following are equivalent:*

1. f *is* $\delta_I s_{clo}$.
2. *For every subset A of X, $scl_{\delta_I}(f(A)) \subseteq f(cl(A))$.*

Proof. 1 ⇒ 2: Suppose that f *is* $\delta_I s_{clo}$ and $A \subseteq X$. then $f(cl(A))$ is $\delta_I s_{clo}$ in Y. We have $f(A) \subseteq f(cl(A))$ and $scl_{\delta_I}(f(A)) \subseteq scl_{\delta_I}\left(f(cl(A))\right) = f(cl(A))$.

2⇒1: Let A be any closed set in X. Then $A = cl(A)$ and so $f(A) = f(cl(A)) \supseteq scl_{\delta_I}\left(f(cl(A))\right)$, by assumption. Therefore $f(A) = scl_{\delta_I}(f(A))$. Thus $f(A)$ is $\delta_I s_{clo}$. Hence f *is* $\delta_I s_{clo}$.

Theorem 3.8. *For a function $f:(X,\tau) \to (Y,\sigma,I)$, the following are equivalent:*

1. f is $\delta_I s_{\text{clo}}$.
2. For every subset A of X, $f\ (int(A)) \subseteq sint_{\delta_I}\ (f\ (A))$.

Proof. Obvious from the Theorem 3.7 and Theorem 3.36 [1].

Theorem 3.9. *For any bijective function* $f : (X,\tau,I) \rightarrow (Y,\sigma,J)$, *the following are equivalent:*
1. f^{-1} *is* δ_I *-semi-continuous.*
2. f is $\delta_I s_{\text{op}}$.
3. f is $\delta_I s_{\text{clo}}$.

Proof. 1 ⇒ 2: Let U be a open set of X. Then, by assumption $(f^{-1})^{-1}(U) = f\ (U)$ is $\delta_I s_{\text{op}}$ in Y. Hence f is $\delta_I s_{\text{op}}$.
2 ⇒ 3: Let F be a closed set of X. Then F^c is open in X. Since f is $\delta_I s_{\text{op}}$, $f\ (F^c)$ is $\delta_I s_{\text{op}}$ in Y and so $f\ (F)$ is $\delta_I s_{\text{clo}}$. Hence f is $\delta_I s_{\text{clo}}$.
3 ⇒ 1: Let F be a closed set of X. Then by assumption $f\ (F) = (f^{-1})^{-1}(F)$ is δ_I -semi-closed set in Y. Hence f^{-1} is δ_I -semi-continuous.

Definition 3.10. *A function* $f: (X,\tau) \rightarrow (Y,\sigma,I)$ *is called* δ_I^**-semi-open (briefly* $\delta_I^* s_{op}$*) if* $f\ (U)$ *is* $\delta_I s_{op}$ *in* Y *for every* δ *-semi-open set* U *of* X.

Definition 3.11. *A function* $f : (X,\tau) \rightarrow (Y,\sigma,J)$ *is called* δ_I^* *-semi-closed (briefly* $\delta_I^* s_{clo}$*) if* $f\ (U)$ *is* $\delta_I s_{clo}$ *in* Y *for every* δ *-semi-closed set* U *of* X.

Example 3.12. *Let* $X = \{a,b,c\}, \tau = \{\emptyset,\{a\},\{b\},\{a,b\},\{b,c\},X\}$ *and*
$Y = \{p,q,r\},\ \sigma = \{\emptyset,\{q\},\{r\},\{q,r\},Y\},\ I = \{\emptyset,\{p\}\}$. *Let* $f: (X,\tau) \rightarrow (Y,\sigma,I)$
be a function defined as $f(a) = q, f(b) = p$ *and* $f(c) = r$. *Then* f *is* $\delta_I^* s_{op}$ *and* $\delta_I^* s_{clo}$.

Theorem 3.13. *For a function* $f : (X,\tau) \rightarrow (Y,\sigma,I)$, *the following are equivalent:*

1. f is $\delta_I^* s_{clo}$.
2. For every subset A of X, $scl_{\delta_I}\ (f\ (A)) \subseteq f\ (\delta cl_S(A))$.

Proof. Similar to Theorem 3.7.

Theorem 3.14. *For a function* $f : (X,\tau) \rightarrow (Y,\sigma,I)$, *the following are equivalent:*

1. f is $\delta_I^* s_{op}$.
2. For every subset A of X, $f\ (\delta int_S(A)) \subseteq sint_{\delta_I}\ (f\ (A))$.

Proof. Obvious from the Theorem 3.13 and Theorem 3.36 [1].

Theorem 3.15. *For any bijective function* $f: (X,\tau) \rightarrow Y,\sigma,J)$, *the following statements are equivalent:*

1. f^{-1} *is* δ_I *-semi-irresolute.*
2. f is $\delta_I^* s_{op}$.
3. f is $\delta_I^* s_{clo}$.

Proof. 1 ⇒ 2: Let U be a δ-semi-open set of X. Then, by assumption $(f^{-1})^{-1}(U) = f\ (U)$ is δ_I -semi-open in Y. Hence f is $\delta_I^* s_{op}$.
2 ⇒ 3: Let V be a δ-semi-closed set of X. Then V^C is δ_I -semi-open and by assumption, $f\ (V^C) = [f\ (V)]^C$ is $\delta_I s_{op}$ in Y. That is $f\ (V)$ is $\delta_I s_{clo}$ in Y and so f is $\delta_I^* s_{clo}$.

3 ⇒ 1: Let W be δ-semi-closed set of X. Then by assumption $f(W) = (f^{-1})^{-1}(W)$ is δ_I-semi-closed in Y. Hence f^{-1} is δ_I-semi-irresolute.

4. δ_{IJ} -SEMI-HOMEOMORPHISMS AND δ^*_{IJ}-SEMI-HOMEOMORPHISMS

Definition 4.1. *A bijective function $f : (X,\tau,I) \to (Y,\sigma,J)$ is called δ_{IJ}-semi-homeomorphism (briefly $\delta_{IJ}s_{hom}$) if f is δ_I-semi-continuous and f^{-1} is δ_I-semi-continuous.*
The family of all δ_I-semi-homeomorphisms of an ideal topological space (X,τ,I) onto itself is denoted by $S_{\delta_I}H(X,\tau,I)$ (or) $S_{\delta_I}H(X)$.

Example 4.2. *Let $X = Y = \{a,b,c,d\}$, $\tau = \{\emptyset,\{d\},\{a,c\},\{a,c,d\},X\}$, $\sigma = \{\emptyset,\{a\},\{b,d\},Y\}$ and $I = \{\emptyset,\{a\},\{d\},\{a,d\}\}$, $J = \{\emptyset,\{a\}\}$ respectively. Let $f : (X,\tau,I) \to (Y,\sigma,J)$ be a function defined by $f(a) = d$, $f(b) = c$, $f(c) = b$ and $f(d) = a$. Then f is $\delta_{IJ}s_{hom}$.*

Theorem 4.3. Let $f:(X,\tau,I) \to (Y,\sigma,J)$ *be a bijective and δ_I-semi-continuous function. Then the following are equivalent.*

1. *f is $\delta_J s_{op.}$*
2. *f is $\delta_{IJ}s_{hom}$.*
3. *f is $\delta_J s_{clo.}$*

Proof. 1 ⇒ 2. Let V be open in X. Then, by assumption $f(V) = (f^{-1})^{-1}(V)$ is δ_J-semi-open in Y. This shows that f^{-1} is δ_J-semi-continuous. Hence f is $\delta_{IJ}s_{hom}$.
2 ⇒ 3. Let F be a closed set in X. Then, by assumption $(f^{-1})^{-1}(F) = f(F)$ is δ_J-semi-closed in Y. Thus f is $\delta_J s_{clo.}$
3 ⇒ 1. Let V be open in X. Then V^C is closed in X. By assumption, $f(V^C)$ is δ_J-semi-closed in Y. This implies that $(f(V))^C$is δ_J-semi-closed in Y and so $f(V)$ is δ_J-semi-open in Y. Hence f is $\delta_J s_{op.}$.

Definition 4.4. *A bijective function $f:(X,\tau,I) \to (Y,\sigma,J)$ is called δ^*_{IJ}-semi-homeomorphism(briefly $\delta^*_{IJ}s_{hom}$) if and only if f is δ_I-semi-irresolute and f^{-1} is δ_J-semi- irresolute.*

Example 4.5. *Let $X = \{a,b,c\}$, $\tau = \{\emptyset,\{a\},\{b\},\{a,b\},\{b,c\},X\}$, $I = \{\emptyset,\{a\},\{b\},\{a,b\}\}$ and let $Y = \{p,q,r\}$, $\sigma = \{\emptyset,\{q\},\{r\},\{q,r\},Y\}$, $J = \{\emptyset,\{p\}\}$. Let $f:(X,\tau,I) \to (Y,\sigma,J)$ be a function defined by $f(a) = r$ and $f(b) = f(c) = q$. Then f is $\delta^*_{IJ}s_{hom}$.*

Remark 4.6. 1.The spaces (X,τ,I) and (Y,σ,J) are δ^*_{IJ}-semi-homeomorphic if there exists a $\delta^*_{IJ}s_{hom}$ from(X,τ,I)onto (Y,σ,J).
2. The family of all δ^*_{IJ}-semi-homeomorphisms of an ideal topological Space (X,τ,I) onto itself is denoted by $S^*_{\delta_I}H(X,\tau,I)$ (or) $S^*_{\delta_I}H(X)$).

Proposition 4.7. If the bijective function $f:(X,\tau,I) \to (Y,\sigma,J)$ *is a $\delta^*_{IJ}s_{hom}$ then, $scl_{\delta_I}(f^{-1}(B)) \subseteq f^{-1}(\delta cl_S(B))$ for every $B \subseteq Y$.*
Proof: Since f *is* $\delta^*_{IJ}s_{hom}$ then f is δ_I-semi-irresolute and f^{-1} is δ_J-semi-irresolute. Let B *be* a subset of Y. Since $\delta cl_S(B)$ is δ-semi-closed in Y, $f^{-1}(\delta cl_S(B))$ is δ_I-semi-closed in X *and* so $scl_{\delta_I}((f^{-1})^{-1}(B) \subseteq (f^{-1})^{-1}(\delta cl_S(B))$, by Theorem 3.17 of [2].

Corollary 4.8. *If $f : (X,\tau,I) \to (Y,\sigma,J)$ is a $\delta^*_{IJ}s_{hom}$, then $scl_{\delta_J}(f(B)) \subseteq f(\delta cl_S(B))$ for every $B \subseteq X$.*
Proof. Since f is $\delta^*_{IJ}s_{hom}$, $f^{-1}$is also $\delta^*_{IJ}s_{hom}$. Then by Proposition 4.8 $scl_{\delta_J}((f^{-1})^{-1}(B) \subseteq (f^{-1})^{-1}(\delta cl_S(B))$ for every $B \subseteq X$. This implies, $scl_{\delta_J}(f(B)) \subseteq f(\delta cl_S(B))$.

Corollary 4.9. *If $f : (X,\tau,I) \to (Y,\sigma,J)$ is a $\delta^*_{IJ}s_{hom}$, then $sint_{\delta_J}(f(B)) \subseteq f(\delta int_S(B))$ for every*

$B \subseteq X$.

Proof. For any set $B \subseteq X$, $\delta int_S(B) = (\delta cl_s(B^c))^c$. $f\left(\delta int_s(B)\right) = f\left(\left(\delta cl_s(B^c)\right)^c\right) = \left(f\left(\delta cl_s(B^c)\right)\right)^c$. *Then by corollary* 4.8, we see that $f\,(\delta int_S(B)) = (f(\delta cl_S(B^c)))^c \supseteq (scl_{\delta_J}(f\,(B^c)))^c = sint_{\delta_J}\,(f\,(B))$.

Corollary 4.10. *If* $f : (X,\tau,I) \to (Y,\sigma,J)$ *is a* $\delta^*_{IJ}s_{hom}$, *then* $sint_{\delta_J}(f^{-1}(B)) \subseteq f^{-1}(\delta int_S(B))$ *for every* $B \subseteq Y$.
Proof. Since f^{-1} is also a $\delta^*_{IJ}s_{hom}$, the proof follows from Corollary 4.9.

Proposition 4.11. *If* $f:(X,\tau,I) \to (Y,\sigma,J)$ *is* $\delta^*_{IJ}s_{hom}$ *and* $g : (Y,\sigma,J) \to (Z,\gamma,K)$ *is* $\delta^*_{JK}s_{hom}$, *then the composition* $g_o f:(X,\tau,I) \to (Z,\gamma,K)$ *is* $\delta^*_{IKs_{hom}}$.

Proof. Let *U be* δ -semi-open in *Z*. Since f and g are $\delta^*_{IJ}s_{hom}$ and $\delta^*_{JK}s_{hom}$ respectively and every δ_I -semi-open set is δ -semi- we have $f^{-1}\,(g^{-1}\,(U)) = (g_o f)^{-1}(U)$ is δ_I -semi-open in X. This implies that $g_o f$ is δ_I -semi-irresolute. Also, $(g^{-1})^{-1}((f^{-1})^{-1}(G)) = ((g_o f)^{-1})^{-1}(G)$ is δ_K -semi-open in *Z*. This implies that $(g_o f)^{-1}$is δ_K -semi-irresolute. Since f and g are $\delta^*_{IJ}s_{hom}$ and $\delta^*_{JK}s_{hom}$. f and g are bijective and so $g_o f$ is bijective. Hence $g_o f$ is $\delta^*_{IK}s_{hom}$.

Proposition 4.12. *The set* $S^*_{\delta_I}\,H(X,\tau,I)$ *is a group under composition of functions.*

Proof. Define a binary operation $* : S^*_{\delta_I}\,H(X,\tau,I)\; X\; S^*_{\delta_I}\,H(X,\tau,I) \to S^*_{\delta_I}\,H(X,\tau,I)$
$by\; f * g = (g_o f)\; for\; all\; f,g \in S^*_{\delta_I}\,H(X,\tau,I)$ and o is the usual operation of composition of maps. Then by Proposition 4.11, $(g_o f) \in S^*_{\delta_I}H(X,\tau,I)$. We know that the composition of maps is associative and the identity map $I : (X,\tau,I) \to (X,\tau,I)$ belonging to $S^*_{\delta_I}H(X,\tau,I)$ serves as the identity element. For any $f \in S^*_{\delta_I}\,H(X,\tau,I)\,, f o f^{-1} = f^{-1} o f = I$. Hence inverse exists for each element of $S^*_{\delta_I}\,H(X,\tau,I)$. Thus $S^*_{\delta_I}\,H(X,\tau,I)$ forms a group under the operation, composition of functions.

Proposition 4.13. *If* $f : (X,\tau,I) \to (Y,\sigma,J)$ *is a* $\delta^*_{IJ}s_{hom}$. *Then* f *induces an isomorphism from the group* $S^*_{\delta_I}\,H(X,\tau,I)$ *onto the group* $S^*_{\delta_J}H(Y,\sigma,J)$.

Proof. Let$f \in S^*_{\delta_I}H(X,\tau,I)$. Then define a map $\chi_f : S^*_{\delta_I}\,H(X,\tau,I) \to S^*_{\delta_I}\,H(Y,\sigma,J)\; by\; \chi_f(h) = f_o h_o f^{-1}\; for\; every\; h \in S^*_{\delta_I}\,H(X,\tau,I)$. Then χ_f is bijective. Let $h_1,h_2 \in S^*_{\delta_I}H(X,\tau,I)$. Then $\chi_f(h_{1_o}h_2) = f_o(h_{1_o}h_2)_o f^{-1} = f_0(h_{1_o}f^{-1}{}_o f_o h_2)_o f^{-1} = (f_o h_{1_o} f^{-1})_o(f_o h_{2_o} f^{-1}) = \chi_f(h_1)_o\,\chi_f(h_2)$. this shows that χ_f is an isomorphism.

Proposition 4.14. $\delta^*_{IJ}s_{hom}$ is an *equivalence relation in the collection of all ideal topological spaces.*

Proof. Reflexive and Symmetric properties are obvious and Transitive property follows from Proposition 4.11.

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