

An application of the Laplace-Carson transform method for the solution of the generalized Abel's integral equation *

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Abstract In the current scenario integral transforms is an interesting field for research due to the wide applicability of the method of integral transforms in obtaining the analytical solution of many problems of engineering, physical sciences, and space science, etc. In this paper we determine the analytical primitive (solution) of a generalized Abel's integral equation by employing the Laplace-Carson transform method. For the purpose of the applicability of this method we illustrate five numerical problems which are solved with the help of the Laplace-Carson transform method.

Key words Laplace-Carson transform, Inverse Laplace-Carson transform, Generalized Abel integral equation, Upadhyaya transform.

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1 Introduction

The solutions to the problems of electric circuits, spectroscopy, motion of a particle, seismology, growth of the species, heat transfer, plasma physics, radioactive decay and vibration of strings can be easily determined by developing their mathematical model by using the appropriate integral equations [1–5]. Integral equations frequently appear when initial or boundary value problems are expressed in the terms of integrals. There are several analytical and numerical methods for obtaining the exact and approximate solutions of integral equations. Integral transforms is one of them which provides us with exact results. Chakrabarti [6] used direct function theoretic method and determined the solution of the generalized Abel integral equation in compact form. Gorenflo and Luchko [7] applied operation method on generalized Abel integral equation of second kind and determined its solution. Dixit et al. [8] used almost Bernstein operational matrix method for determining the solution of the generalized Abel integral equation. Zarei and Noeiaghdam [9] obtained the primitive of the generalized Abel's integral equations of the first and second kind by the use of the Taylor-Collocation method. Brunner [10]

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applied implicit interpolation on the generalized Abel integral equation and gave the global solution of this problem. Atkinson [11] defined the existence theorem for Abel integral equations. Chakrabarti and George [12] determined a formula for obtaining the solution of the general Abel integral equation. Mahgoub [13] *reintroduced (perhaps, mistakenly without being aware of its previous existence in the literature) the Laplace-Carson transform by the name of the Mahgoub transform* (see also, Upadhyaya [32, subsection 4.10, p. 478]). Gupta [14] used the so-called Mahgoub transform (in fact the Laplace-Carson transform) to solve the famous problem of mechanics, namely, the Abel problem. Aggarwal and other scholars [15–21] applied different integral transforms (like, the Laplace, the Kamal, the Mohand, the Aboodh, the Sumudu, the Shehu and the Sadik transforms) and solved the Abel problem of mechanics. Chauhan and Aggarwal [22] considered linear partial integro-differential equations and solved it using the Mahgoub (Laplace-Carson) transform. Linear ordinary differential equations with variable coefficients were solved by Aggarwal et al. [23] using the Mahgoub transform. Aggarwal et al. [24, 25] fruitfully used the Mahgoub (Laplace-Carson) transform for obtaining the primitives of the first and the second kind of Volterra integral equations. Aggarwal et al. [26] also studied the second kind linear Volterra integro-differential equations and solved them completely by employing the Mahgoub (Laplace-Carson) transform. The Mahgoub (Laplace-Carson) transform was also used by Aggarwal et al. [27] for obtaining the solution of the famous problem of population growth and decay. Aggarwal and other researchers [28, 29] determined the Mahgoub (Laplace-Carson) transforms of the Bessel and the error functions. Aggarwal [30] also studied the Mohand and the Mahgoub (Laplace-Carson) transforms by obtaining the solution of a system of simultaneous differential equations. Chauhan et al. [31] studied the Laplace-Carson transform and determined its duality relations with the other existing integral transforms. We also remark here that the Laplace-Carson transform and most of the other currently existing variants of the classical Laplace transform in the mathematics research literature are the special cases of the Upadhyaya transform developed by the second author [32] and further updated by Upadhyaya et al. in [33].

The present study focuses on obtaining the exact solution of the generalized Abel's integral equation with the help of the Laplace-Carson transform method. We explain below the notations used by us in this paper:

Nomenclature of symbols

$\mathcal{L}_C$, the Laplace-Carson transform operator;	$J_0(t), J_1(t), J_2(t)$, the Bessel functions of orders zero, one and two respectively;
$\in$, belongs to;	$\operatorname{erf}\sqrt{t}$, the Error function;
$\mathbb{N}$, the set of natural numbers ;	$\mathcal{L}_C^{-1}$, the inverse Laplace-Carson transform operator.
$\mathbb{R}$, the set of real numbers;	
!, the usual factorial notation;	
Γ , the classical gamma function;	

Definition 1.1. Laplace-Carson transform of a function: The Laplace-Carson transform (*which was later mistakenly reintroduced as the Mahgoub transform* (see [13], also see Upadhyaya [32]) of a sectionally continuous function $H(t)$ of exponential order for all $t \geq 0$ is defined by the relation

$$\mathcal{L}_C \{H(t)\} = p \int_0^\infty H(t) e^{-pt} dt = h(p), \quad (1.1)$$

provided the integral on the right side of (1.1) exists.

Definition 1.2. Inverse Laplace-Carson transform of a function: If the Laplace-Carson transform of a function $H(t)$ is $h(p)$ then the function $H(t)$ is known as the inverse Laplace-Carson transform of $h(p)$ and it is denoted by $H(t) = \mathcal{L}_C^{-1}\{h(p)\}$.

The useful properties of the Laplace-Carson transform, the Laplace-Carson transform of some basic mathematical functions and their inverse Laplace-Carson transforms which are frequently encountered in various applications and some of these will also be required by us in this paper are summarized below for ready reference in the Tables 1–3 respectively.

Table 1: Some useful properties of the Laplace-Carson transform.

S. No.	Name of the Property/ Theorem	Mathematical Form
1.	Linearity	$\mathcal{L}_C \{aH_1(t) + bH_2(t)\} = a\mathcal{L}_C \{H_1(t)\} + b\mathcal{L}_C \{H_2(t)\}$
2.	Change of Scale	$\mathcal{L}_C \{H(at)\} = h\left(\frac{p}{a}\right)$
3.	Translation	$\mathcal{L}_C \{e^{at}H(t)\} = \left(\frac{p}{p-a}\right)h(p-a)$
4.	The First Derivative	$\mathcal{L}_C \{H'(t)\} = ph(p) - pH(0)$
5.	The Second Derivative	$\mathcal{L}_C \{H''(t)\} = p^2h(p) - p^2H(0) - pH'(0)$
6.	The n^{th} Derivative	$\mathcal{L}_C \{H^{(n)}(t)\} = p^nh(p) - p^nH(0) - p^{n-1}H'(0) - \dots - pH^{(n-1)}(0)$
7.	Convolution	$\mathcal{L}_C \{H_1(t) * H_2(t)\} = \frac{\mathcal{L}_C \{H_1(t)\}\mathcal{L}_C \{H_2(t)\}}{p}$

Table 2: The Laplace-Carson transform of some elementary mathematical functions.

S. No.	$H(t)$	$\mathcal{L}_C \{H(t)\} = h(p)$
1.	$t^n, n \in \mathbb{N} \cup \{0\}$	$\frac{n!}{p^n}$
2.	$t^n, n > -1, n \in \mathbb{R}$	$\frac{\Gamma(n+1)}{p^n}$
3.	e^{at}	$\frac{p}{p-a}$
4.	$\sin at$	$\frac{ap}{p^2+a^2}$
5.	$\cos at$	$\frac{p^2}{p^2+a^2}$
6.	$\sinh at$	$\frac{ap}{p^2-a^2}$
7.	$\cosh at$	$\frac{p^2}{p^2-a^2}$
8.	$J_0(t)$	$\frac{p}{\sqrt{p^2+1}}$
9.	$J_1(t)$	$p - \frac{p^2}{\sqrt{p^2+1}}$
10.	$J_2(t)$	$\frac{p+2p^3-2(\sqrt{p^2+1})p^2}{\sqrt{p^2+1}}$
11.	$\text{erf}\sqrt{t}$	$\frac{1}{\sqrt{1+p}}$

2 The Laplace-Carson transform method for the solution of the generalized Abel's integral equation

The general form of generalized Abel's integral equation is defined as

$$F(x) = \int_0^x \frac{u(t)}{(x-t)^\alpha} dt, 0 < \alpha < 1, \quad (2.1)$$

Table 3: The inverse Laplace-Carson transform of some elementary mathematical functions.

S. No.	$h(p)$	$H(t) = \mathcal{L}_C^{-1}\{h(p)\}$
1.	$\frac{1}{p^n}, n \in \mathbb{N} \cup \{0\}$	$\frac{t^n}{n!}$
2.	$\frac{1}{p^n}, n > -1, n \in \mathbb{R}$	$\frac{t^n}{\Gamma(n+1)}$
3.	$\frac{p}{p-a}$	e^{at}
4.	$\frac{p}{p^2+a^2}$	$\frac{\sin at}{a}$
5.	$\frac{p^2}{p^2+a^2}$	$\cos at$
6.	$\frac{p}{p^2-a^2}$	$\frac{\sinh at}{a}$
7.	$\frac{p^2}{p^2-a^2}$	$\cosh at$
8.	$\frac{p}{\sqrt{p^2+1}}$	$J_0(t)$
9.	$p - \frac{p^2}{\sqrt{p^2+1}}$	$J_1(t)$
10.	$\frac{p+2p^3-2(\sqrt{p^2+1})p^2}{\sqrt{p^2+1}}$	$J_2(t)$
11.	$\frac{1}{\sqrt{1+p}}$	$\text{erf}\sqrt{t}$

where,

$$\begin{cases} F(x) = \text{the known function,} \\ u(x) = \text{the unknown function,} \\ \text{and} \\ \alpha = \text{a constant.} \end{cases}$$

Operating $\mathcal{L}_C$ on both sides of (2.1), we obtain

$$\begin{aligned} \mathcal{L}_C \{F(x)\} &= \mathcal{L}_C \left\{ \int_0^x \frac{u(t)}{(x-t)^\alpha} dt \right\}, 0 < \alpha < 1, \\ \Rightarrow \mathcal{L}_C \{F(x)\} &= \mathcal{L}_C \{u(x) * x^{-\alpha}\}, 0 < \alpha < 1. \end{aligned} \quad (2.2)$$

Use of the convolution theorem in (2.2) gives

$$\begin{aligned} \mathcal{L}_C \{F(x)\} &= \frac{1}{p} \mathcal{L}_C \{u(x)\} \mathcal{L}_C \{x^{-\alpha}\}, 0 < \alpha < 1, \\ \Rightarrow \mathcal{L}_C \{F(x)\} &= \frac{1}{p} \mathcal{L}_C \{u(x)\} \frac{\Gamma(1-\alpha)}{p^{-\alpha}}, 0 < \alpha < 1, \\ \Rightarrow \mathcal{L}_C \{u(x)\} &= \mathcal{L}_C \{F(x)\} \frac{p^{1-\alpha}}{\Gamma(1-\alpha)}, 0 < \alpha < 1, \\ \Rightarrow \mathcal{L}_C \{u(x)\} &= \frac{1}{p} \mathcal{L}_C \{F(x)\} \frac{p}{\Gamma(\alpha) \Gamma(1-\alpha)} \left(\frac{\Gamma(\alpha)}{p^{\alpha-1}} \right), 0 < \alpha < 1, \\ \Rightarrow \mathcal{L}_C \{u(x)\} &= \frac{p}{\Gamma(\alpha) \Gamma(1-\alpha)} \left[\frac{1}{p} \mathcal{L}_C \{F(x)\} \mathcal{L}_C \{x^{\alpha-1}\} \right], 0 < \alpha < 1, \\ \Rightarrow \mathcal{L}_C \{u(x)\} &= \frac{p}{\Gamma(\alpha) \Gamma(1-\alpha)} \mathcal{L}_C \{F(x) * x^{\alpha-1}\}, 0 < \alpha < 1, \\ \Rightarrow \mathcal{L}_C \{u(x)\} &= \frac{p}{\Gamma(\alpha) \Gamma(1-\alpha)} \mathcal{L}_C \left\{ \int_0^x \frac{F(t)}{(x-t)^{1-\alpha}} dt \right\}, 0 < \alpha < 1, \end{aligned}$$

$$\Rightarrow \mathcal{L}_C \{u(x)\} = \frac{p}{\Gamma(\alpha)\Gamma(1-\alpha)} \mathcal{L}_C \{\mu(x)\}, 0 < \alpha < 1, \quad (2.3)$$

where,

$$\mu(x) = \int_0^x \frac{F(t)}{(x-t)^{1-\alpha}} dt, 0 < \alpha < 1. \quad (2.4)$$

Now,

$$\begin{aligned} \mathcal{L}_C \{\mu'(x)\} &= p \mathcal{L}_C \{\mu(x)\} - p\mu(0), \\ \Rightarrow \mathcal{L}_C \{\mu'(x)\} &= p \mathcal{L}_C \{\mu(x)\} - 0 = p \mathcal{L}_C \{\mu(x)\}, \\ \Rightarrow \mathcal{L}_C \{\mu(x)\} &= \frac{1}{p} \mathcal{L}_C \{\mu'(x)\}. \end{aligned} \quad (2.5)$$

Now, from (2.3) and (2.5), we have

$$\mathcal{L}_C \{u(x)\} = \frac{1}{\Gamma(\alpha)\Gamma(1-\alpha)} \mathcal{L}_C \{\mu'(x)\}, 0 < \alpha < 1. \quad (2.6)$$

Operating $\mathcal{L}_C^{-1}$ on both sides of (2.6), we obtain

$$\begin{aligned} u(x) &= \frac{1}{\Gamma(\alpha)\Gamma(1-\alpha)} \mu'(x), \\ \Rightarrow u(x) &= \frac{1}{\Gamma(\alpha)\Gamma(1-\alpha)} \left[\frac{d}{dx} \mu(x) \right], 0 < \alpha < 1. \end{aligned}$$

Use (2.4) in the above equation, we have

$$u(x) = \frac{1}{\Gamma(\alpha)\Gamma(1-\alpha)} \left[\frac{d}{dx} \int_0^x \frac{F(t)}{(x-t)^{1-\alpha}} dt \right], 0 < \alpha < 1.$$

Using the result $\Gamma(\alpha)\Gamma(1-\alpha) = \left(\frac{\pi}{\sin \alpha\pi}\right)$, $0 < \alpha < 1$ in the last equation, we get

$$u(x) = \frac{\sin \alpha\pi}{\pi} \left[\frac{d}{dx} \int_0^x \frac{F(t)}{(x-t)^{1-\alpha}} dt \right], 0 < \alpha < 1,$$

which gives the exact solution of (2.1).

3 Illustrative applications

We now present below five illustrative examples for explaining the applicability of the proposed method.

Example 3.1. Consider (2.1) with $F(x) = \frac{9}{4}(x^{4/3})$ and $\alpha = 2/3$ as

$$\frac{9}{4}(x^{4/3}) = \int_0^x \frac{u(t)}{(x-t)^{2/3}} dt. \quad (3.1)$$

On taking the Laplace-Carson transform on both sides of (3.1), we have

$$\begin{aligned} \frac{9}{4} \mathcal{L}_C \{x^{4/3}\} &= \mathcal{L}_C \left\{ \int_0^x \frac{u(t)}{(x-t)^{2/3}} dt \right\} \\ \Rightarrow \frac{9}{4} \mathcal{L}_C \{x^{4/3}\} &= \mathcal{L}_C \{u(x) * x^{-(2/3)}\}. \end{aligned}$$

Use of the convolution theorem in above equation gives

$$\begin{aligned} \frac{9}{4} \frac{\Gamma(7/3)}{p^{4/3}} &= \frac{1}{p} \mathcal{L}_C \{u(x)\} \mathcal{L}_C \{x^{-(2/3)}\} \\ \Rightarrow \frac{\Gamma(1/3)}{p^{4/3}} &= \frac{1}{p} \mathcal{L}_C \{u(x)\} \frac{\Gamma(1/3)}{p^{-(2/3)}} \end{aligned}$$

$$\Rightarrow \mathcal{L}_C \{u(x)\} = \frac{1}{p}$$

On taking the inverse Laplace-Carson transform of the above equation we obtain

$$u(x) = \mathcal{L}_C^{-1} \left\{ \frac{1}{p} \right\} = x,$$

which gives the exact solution of (3.1).

Example 3.2. Consider (2.1) with $F(x) = \frac{24}{5} (x^{5/6})$ and $\alpha = 1/6$ as

$$\frac{24}{5} (x^{5/6}) = \int_0^x \frac{u(t)}{(x-t)^{1/6}} dt. \quad (3.2)$$

Operating by the Laplace-Carson transform operator on both the sides of (3.2), we have

$$\begin{aligned} \frac{24}{5} \mathcal{L}_C \{x^{5/6}\} &= \mathcal{L}_C \left\{ \int_0^x \frac{u(t)}{(x-t)^{1/6}} dt \right\} \\ \Rightarrow \frac{24}{5} \mathcal{L}_C \{x^{5/6}\} &= \mathcal{L}_C \{u(x) * x^{-(1/6)}\}. \end{aligned}$$

The use of the convolution theorem in the above equation yields

$$\begin{aligned} \frac{24}{5} \frac{\Gamma(11/6)}{p^{5/6}} &= \frac{1}{p} \mathcal{L}_C \{u(x)\} \mathcal{L}_C \{x^{-(1/6)}\}, \\ \Rightarrow 4 \frac{\Gamma(5/6)}{p^{5/6}} &= \frac{1}{p} \mathcal{L}_C \{u(x)\} \frac{\Gamma(5/6)}{p^{-(1/6)}} \\ \Rightarrow \mathcal{L}_C \{u(x)\} &= 4 \end{aligned}$$

Operating now the last equation by $\mathcal{L}_C^{-1}$ we obtain

$$u(x) = \mathcal{L}_C^{-1} \{4\} = 4 \mathcal{L}_C^{-1} \{1\} = 4,$$

which gives the exact solution of (3.2).

Example 3.3. Consider (2.1) with $F(x) = \frac{25}{36} (x^{9/5})$ and $\alpha = 1/5$ as

$$\frac{25}{36} (x^{9/5}) = \int_0^x \frac{u(t)}{(x-t)^{1/5}} dt. \quad (3.3)$$

On taking the Laplace-Carson transform of both the sides of (3.3), we have

$$\begin{aligned} \frac{25}{36} \mathcal{L}_C \{x^{9/5}\} &= \mathcal{L}_C \left\{ \int_0^x \frac{u(t)}{(x-t)^{1/5}} dt \right\}, \\ \Rightarrow \frac{25}{36} \mathcal{L}_C \{x^{9/5}\} &= \mathcal{L}_C \{u(x) * x^{-(1/5)}\}. \end{aligned}$$

An application of the convolution theorem to the above equation provides

$$\begin{aligned} \frac{25}{36} \frac{\Gamma(14/5)}{p^{9/5}} &= \frac{1}{p} \mathcal{L}_C \{u(x)\} \mathcal{L}_C \{x^{-(1/5)}\}, \\ \Rightarrow \frac{\Gamma(4/5)}{p^{9/5}} &= \frac{1}{p} \mathcal{L}_C \{u(x)\} \frac{\Gamma(4/5)}{p^{-(1/5)}}, \\ \Rightarrow \mathcal{L}_C \{u(x)\} &= \frac{1}{p}, \end{aligned}$$

which on inverting with the inverse Laplace-Carson transform yields the following exact solution of (3.3).

$$u(x) = \mathcal{L}_C^{-1} \left\{ \frac{1}{p} \right\} = x.$$

Example 3.4. Consider (2.1) with $F(x) = \frac{128}{231} (x^{11/4}) + \frac{16}{21} (x^{7/4})$ and $\alpha = 1/4$ as

$$\frac{128}{231} (x^{11/4}) + \frac{16}{21} (x^{7/4}) = \int_0^x \frac{u(t)}{(x-t)^{1/4}} dt. \quad (3.4)$$

Taking the Laplace-Carson transform of both the sides of (3.4) gives

$$\begin{aligned} \frac{128}{231} \mathcal{L}_C \{x^{11/4}\} + \frac{16}{21} \mathcal{L}_C \{x^{7/4}\} &= \mathcal{L}_C \left\{ \int_0^x \frac{u(t)}{(x-t)^{1/4}} dt \right\}, \\ \Rightarrow \frac{128}{231} \mathcal{L}_C \{x^{11/4}\} + \frac{16}{21} \mathcal{L}_C \{x^{7/4}\} &= \mathcal{L}_C \{u(x) * x^{-(1/4)}\}, \end{aligned}$$

which on utilizing the convolution theorem gives

$$\begin{aligned} \frac{128}{231} \frac{\Gamma(15/4)}{p^{11/4}} + \frac{16}{21} \frac{\Gamma(11/4)}{p^{7/4}} &= \frac{1}{p} \mathcal{L}_C \{u(x)\} \mathcal{L}_C \{x^{-(1/4)}\}, \\ \Rightarrow 2 \frac{\Gamma(3/4)}{p^{11/4}} + \frac{\Gamma(3/4)}{p^{7/4}} &= \frac{1}{p} \mathcal{L}_C \{u(x)\} \frac{\Gamma(3/4)}{p^{-(1/4)}}, \\ \Rightarrow \mathcal{L}_C \{u(x)\} &= \frac{2}{p^2} + \frac{1}{p}. \end{aligned}$$

Inverting the above equation by the inverse Laplace-Carson transform yields the following exact solution of (3.4)

$$\begin{aligned} u(x) &= \mathcal{L}_C^{-1} \left\{ \frac{2}{p^2} + \frac{1}{p} \right\} \\ u(x) &= 2 \mathcal{L}_C^{-1} \left\{ \frac{1}{p^2} \right\} + \mathcal{L}_C^{-1} \left\{ \frac{1}{p} \right\} = x^2 + x. \end{aligned}$$

Example 3.5. Consider (2.1) with $F(x) = 3 (x^{1/3}) \pi$ and $\alpha = 2/3$ as

$$3 (x^{1/3}) \pi = \int_0^x \frac{u(t)}{(x-t)^{2/3}} dt. \quad (3.5)$$

As above an application of the Laplace-Carson transform to both the sides of (3.5) gives

$$\begin{aligned} 3\pi \mathcal{L}_C \{x^{1/3}\} &= \mathcal{L}_C \left\{ \int_0^x \frac{u(t)}{(x-t)^{2/3}} dt \right\}, \\ \Rightarrow 3\pi \mathcal{L}_C \{x^{1/3}\} &= \mathcal{L}_C \{u(x) * x^{-(2/3)}\}. \end{aligned}$$

Now the use of the convolution theorem in the previous equation gives

$$\begin{aligned} 3\pi \frac{\Gamma(4/3)}{p^{1/3}} &= \frac{1}{p} \mathcal{L}_C \{u(x)\} \mathcal{L}_C \{x^{-(2/3)}\}, \\ \Rightarrow \frac{\pi \Gamma(1/3)}{p^{1/3}} &= \frac{1}{p} \mathcal{L}_C \{u(x)\} \frac{\Gamma(1/3)}{p^{-(2/3)}}, \\ \Rightarrow \mathcal{L}_C \{u(x)\} &= \pi, \end{aligned}$$

which on inverting with the operator $\mathcal{L}_C^{-1}$ gives the following exact solution of (3.5)

$$u(x) = \mathcal{L}_C^{-1} \{\pi\} = \pi L^{-1} \{1\} = \pi.$$

4 Conclusions

In this paper, the authors used Laplace-Carson transform technique for finding the analytical solution of the generalized Abel's integral equation. The practical applicability of the method is demonstrated by considering five illustrative examples, which shows that the Laplace-Carson transform method is a handy technique for dealing with the generalized Abel's integral equation and in future efforts can be made for obtaining the primitive of a system of generalized Abel's integral equations by employing this tool.

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