Bull. Pure Appl. Sci. Sect. E Math. Stat.
40E(2), 164–166 (2021)
e-ISSN:2320-3226, Print ISSN:0970-6577
DOI: 10.5958/2320-3226.2021.00018.7

Bulletin of Pure and Applied Sciences
Section - E - Mathematics & Statistics

Website : https : //www.bpasjournals.com/

Erratum to the paper "Fifty Years of Kurepa's !n Hypothesis" by Žarko Mijajlović *

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Abstract In this short note we prove that the Kurepa (K) function is different from the Smarandache-Kurepa (SK) function, therefore, these functions are not the same, as Mijajlović has unfoundedly accused the prestigious Encyclopedia of Mathematics and this author. This note is an answer to Mijajlović's paper (Žarko Mijajlović, Fifty years of Kurepa's !n hypothesis, *Bulletin T.CLIV de l'Académie serbe des sciences et des arts – 2021 Classe des Sciences mathématiques et naturelles Sciences mathématiques*, No. 46, 169–181 (2021). http://elib.mi.sanu.ac.rs/pages/browse_issue.php?db=bltn&rbr=21, http://elib.mi.sanu.ac.rs/files/journals/bltn/46/bltnn46p169-181.pdf).

Key words Kurepa (K) function, Smarandache-Kurepa (SK) function, Encyclopedia of Mathematics.

2020 Mathematics Subject Classification 05A10, 11B65.

1 Introduction

In the paper [1, p. 172], Mijajlović asserts that: "We have to mention also that there are inappropriate names assigned related to Kurepa's left factorial function. The most remarkable example is that !n is also called Smarandache-Kurepa function at the rather reputed Wolfram MathWorld portal [5]." Although the author cited the prestigious Encyclopedia of Mathematics and the link to the SK function [5], he either overlooked its entry (although it has only five lines), or he did not understand it. Therefore, he jumped to attacks and an unfounded accusation.

2 Proposition

We propose here that the K and SK functions are different from each other and we prove this below.

Definition 2.1. The Kurepa K left factorial function [2] is defined as a sum of increasing factorials:

$$K_n = !n = \sum_{i=0}^{n-1} i! = 0! + 1! + \cdots + (n-1)!,$$

for $n \geq 1$.

* Communicated, edited and typeset in Latex by *Lalit Mohan Upadhyaya* (Editor-in-Chief). Received September 10, 2021 / Revised November 12, 2021 / Accepted November 23, 2021. Online First Published on December 30, 2021 at https://www.bpasjournals.com/.
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Let us compute some values of the Kurepa function K_n.

$$K_1 = 0! = 1,$$

$$K_2 = 0! + 1! = 2,$$

$$K_3 = 0! + 1! + 2! = 4,$$

$$K_4 = K_3 + 3! = 4 + 6 = 10,$$

$$K_5 = K_4 + 4! = 10 + 24 = 34,$$

$$K_6 = K_5 + 5! = 34 + 120 = 154,$$

and so on.

Definition 2.2. In the Encyclopedia of Mathematics, the Smarandache-Kurepa (SK) function [5] is defined as follows: "Given the left factorial function:

$$\sum(n) = \sum_{k=1}^{n} k!.$$

$SK(p)$ for p prime is the smallest integer n such that $p|\{1 + \sum(n-1)\}$, i.e., p divides $1 + \sum(n-1)$.
The first few known values of $SK(p)$ are $2, 4, 6, 6, 5, 7, 7, 12, 22, 16, 55, 54, 42, 24, \ldots$ for $p = 2, 5, 7, 11, 17, 19, 23, 31, 37, 41, 61, 71, 73, 89, \ldots$.
The function $SK(p)$ does not exist for $p = 3, 13, 29, 43, 47, 53, 67, 79, 83, \ldots$."

Definition 2.3. Let us also present the Smarandache (S) function [3, 4], used in the construction of the SK function, which is defined as below: $S(n)$ is the smallest integer n such that $n|S(n)!$, i.e., n divides $S(n)!$.

3 Comparison of the K and SK functions

From the above three definitions, we clearly see that the SK function is a combination of the S function ("the smallest integer n such that p divides …"), and the K function ("the expression that has to be divisible by p is the Kurepa left factorial) - where its name the SK function comes from.
However, the two functions, K and SK are analytically different as it can be seen easily.
Neither their values are the same:
The first values of the K_n computed above are: $1, 2, 4, 10, 34, 154, \ldots$.
While, the first few known values of $SK(p)$ are $2, 4, 6, 6, 5, 7, 7, 12, 22, 16, 55, 54, 42, 24, \ldots$ for $p = 2, 5, 7, 11, 17, 19, 23, 31, 37, 41, 61, 71, 73, 89, \ldots$ (see, Weisstein [5]).
Thus, the values of K and SK functions are also different.
The SK function was introduced by M.R. Mudge [6, 7] in 1996, an English mathematician, not by Ashbacher as asserted by Mijajlović [1].
Mijajlović does not say anything about the *Wagstaff's left factorial* [6–8], which looks more intuitive than Kurepa's, and is defined as:

$$B_n = !(n+1) - 1 = 1! + 2! + 3! + \ldots + n!.$$

When this author found out about the above said paper of Mijalović, he sent e-mails [9, 10] to Mijajlović and to the Editor of this Journal [1] - Gradimir V. Milovanović, inviting the author Mijajlović to update his paper [1], since it has a wrong section, or else to publish this author's response in this reference, but they both declined this author's request.

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