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Solution of some multi-objective nonlinear programming problems *

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Abstract In this paper a method for solving bi-objective nonlinear programming problem is derived. Here, the bi-objective nonlinear programming problem is converted into a crisp problem using Zimmermann's fuzzy programming technique with linear membership function and after that the crisp problem is solved by separable programming technique to find the optimal compromise solution. A numerical example is given for the sake of illustration.

Key words Nonlinear Programming Problem, Separable Programming, Linear Membership Function, Fuzzy Programming Technique, Multi-objective Nonlinear Programming Problem.

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1 Introduction

The situation of simultaneous optimization having several objective functions occurs in various problems of science and technology. Generally these objective functions are measured in different units and are often competing and conflicting. Multi-objective optimization having such conflicting objective functions gives rise to a set of optimal solutions, instead of one optimal solution because no solution can be considered to be better than any other with respect to all objectives. These optimal solutions are known as Pareto-optimal solutions. Nonlinear Programming Problem (NLPP) is the problem of optimization in which either the objective function is nonlinear, or one or more constraints have a nonlinear relationship or both. A number of very successful techniques have been reported to solve the multi-objective linear programming problems in [1, 4, 7–12]. Further many successful applications of these formulations and techniques have also been reported. The concept of single objective nonlinear programming problems is found extensively in the literature, see, for example, [2, 3] and [6]. But, the multi-objective nonlinear programming problems provide a better approximation to the real world problems than the single objective nonlinear programming formulations. In this paper, our intention is to develop a method of solving Bi-objective nonlinear programming problems in which we will use fuzzy programming (Zimmerman's technique) [5] to obtain a compromise solution from these Pareto-optimal solutions.

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A general Bi-objective nonlinear programming problem is as follows:
Maximize/Minimize $f_i(x_1, x_2, \ldots, x_n), i = 1, 2$ subject to:

$$g^1(x_1, x_2, \ldots, x_n)\{\leq, \geq, \text{or}, =\}\, b_1$$
$$g^2(x_1, x_2, \ldots, x_n)\{\leq, \geq, \text{or}, =\}\, b_2$$
$$\cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots$$
$$g^m(x_1, x_2, \ldots, x_n)\{\leq, \geq, \text{or}, =\}\, b_m$$

where g^i's are real valued functions of n variables $x_1, x_2, \ldots, x_n$, $x_j \geq 0, j = 1, 2, \ldots, n$ with some $f_i(x_1, x_2, \ldots, x_n), i = 1, 2$ or, some $g^i(x_1, x_2, \ldots, x_n), i = 1, 2, \ldots, m$ or, all are nonlinear.

2 Algorithm for solution of bi-objective non-linear programming problem

Algorithm 2.1.
Step 1
Solve the bi-objective nonlinear programming problem by considering the first objective at a time and ignoring the second objective. Repeat the process for the second objective function. We use Wolfe's modified simplex method to solve the nonlinear programming problem.
Step 2
Using all the above two ideal solutions in Step-1 construct a pay-off matrix of size 2×2. Then from the pay-off matrix find the lower bound (L_r) and the upper bound (U_r) for the objective function:

$${Z_r}' \text{ as } L_r \leq {Z_r}' \leq U_r,\ r = 1, 2.$$

Step 3
Define fuzzy linear membership function $\mu_{Z_r}(x)$ for the r^{th} objective function ${Z_r}', r = 1, 2$ as below:

$$\mu_{Z_r}(x) = \begin{cases} 0, \text{ if } Z_r \leq L_r, \\ \frac{{Z_r}' - L_r}{U_r - L_r}, \text{ if } L_r \leq Z_r \leq U_r, \\ 1, \text{ if } U_r \leq Z_r. \end{cases}$$

Step 4
Using the above membership function we formulate a crisp model by introducing an augmented variable λ as:

$$\text{Min. } \lambda$$

subject to:

$$\sum {C_{rj}}' x_j + (U_r - L_r)\lambda \geq U_r, r = 1, 2,$$
$$\sum {a_{ij}}' x_j \leq {b_i}', i = 1, 2, \ldots, m,$$
$$\lambda \geq 0, x_j \geq 0, j = 1, 2, \ldots, n.$$

Step 5
Then we solve this crisp model by separable programming technique to find the optimal compromise solution.

3 Solution of a numerical problem of bi-objective NLPP

Problem 3.1. Maximize

$$Z_1 = 2x_1 + 3x_2 - 2{x_1}^2, \tag{3.1}$$
$$Z_2 = 2x_1 + 3x_2 - 4{x_1}^2, \tag{3.2}$$

subject to:

$$\begin{aligned} &x_1 + 4x_2 \leq 4, \\ &x_1 + x_2 \leq 2, \\ &x_1 \geq 0, x_2 \geq 0. \end{aligned} \tag{3.3}$$

Solution. We solve the problem in the following manner:
Step 1 Solving (3.1) and (3.3)

$$\text{Maximize } Z_1 = 2x_1 + 3x_2 - 2{x_1}^2$$

subject to:

$$x_1 + 4x_2 \leq 4,$$
$$x_1 + x_2 \leq 2,$$
$$x_1 \geq 0, x_2 \geq 0,$$

by Wolfe's modified simplex method, we get the optimum solution as
$x_1 = \frac{5}{16} = 0.3125$, $x_2 = \frac{59}{64} = 0.9218$ and Max. $Z_1 = 3.183$,
and solving (3.2) and (3.3)

$$\text{Maximize } Z_1 = 2x_1 + 3x_2 - 4{x_1}^2$$

subject to:

$$x_1 + 4x_2 \leq 4,$$
$$x_1 + x_2 \leq 2,$$
$$x_1 \geq 0, x_2 \geq 0,$$

by Wolfe's modified simplex method, we get the optimum solution as
$x_1 = \frac{5}{32} = 0.15$, $x_2 = \frac{123}{128} = 0.960$ and Max. $Z_2 = 3.11$.
Step 2 A pay-off matrix is formulated

	Z_1	Z_2
$x^{(1)}$	3.183	3
$x^{(2)}$	3.14	3.11

Step 3 Define fuzzy linear membership function $\mu_{Z_r}(x)$ for the r^{th} objective function ${Z_r}'$, $r = 1, 2$ as

$$\mu_{Z_r}(x) = \begin{cases} 0, \text{ if } Z_r \leq L_r, \\ \frac{{Z_r}' - L_r}{U_r - L_r}, \text{ if } L_r \leq Z_r \leq U_r, \\ 1, \text{ if } U_r \leq Z_r. \end{cases}$$

Step 4 Using the membership function as defined in Algorithm 2.1 (Step 3) and introducing an augmented variable λ, the crisp model is formulated as

$$\text{Minimize } \lambda \tag{3.4}$$

subject to:

$$\begin{aligned} &2x_1 + 3x_2 - 2{x_1}^2 + 0.1\lambda \geqslant 3.183, \\ &2x_1 + 3x_2 - 4x_1^2 + 0.11\lambda \geqslant 3.11, \\ &x_1 + 4x_2 \leqslant 4, \\ &x_1 + x_2 \leqslant 2, \\ &x_1, x_2 \geqslant 0, \lambda \geqslant 0. \end{aligned} \tag{3.5}$$

Step 5 We solve this crisp model (3.4) and (3.5) by using separable programming
Let $f(x_i) = {x_1}^2 = \alpha_{10} f(x_{10}) + \alpha_{11} f(x_{11}) + \alpha_{12} f(x_{12}) + \alpha_{13} f(x_{13}) + \alpha_{14} f(x_{14}), \quad 0 \leq x_i \leq 4$

$$x_{10} = 0, \quad x_{11} = 1, \quad x_{12} = 2, \quad x_{13} = 3, \quad x_{14} = 4$$

with $\alpha_{10} + \alpha_{11} + \alpha_{12} + \alpha_{13} + \alpha_{14} = 1$

$${x_1}^2 = 0^2 * \alpha_{10} + 1^2 * \alpha_{11} + 2^2 * \alpha_{12} + 3^2 * \alpha_{13} + 4^2 * \alpha_{14}$$

$$= \alpha_{11} + 4 * \alpha_{12} + 9 * \alpha_{13} + 16 * \alpha_{14}$$

Hence we have the crisp model as

$$\text{Minimize } \lambda \tag{3.6}$$

subject to:

$$\begin{aligned}
&2x_1 + 3x_2 - 2\alpha_{11} - 8\alpha_{12} - 18\alpha_{13} - 32\alpha_{14} + 0.1\lambda \geqslant 3.183,\\
&2x_1 + 3x_2 - 4\alpha_{11} - 16\alpha_{12} - 36\alpha_{13} - 64\alpha_{14} + 0.11\lambda \geqslant 3.11,\\
&x_1 + 4x_2 \leqslant 4,\\
&x_1 + x_2 \leqslant 2,\\
&\alpha_{10} + \alpha_{11} + \alpha_{12} + \alpha_{13} + \alpha_{14} = 1,\\
&x_1, x_2 \geqslant 0; \lambda \geqslant 0; \alpha_{10}, \alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{14} \geqslant 0.
\end{aligned} \tag{3.7}$$

Solving (3.6) and (3.7), we get

$$x_1 = \frac{128}{625} = 0.2047, x_2 = \frac{593}{625} = 0.9488$$

and

$$Z_1 = 3.1719, \quad Z_2 = 3.08819.$$

4 Conclusion

From the results obtained by solving the above Bi-objective nonlinear separable type of problem, we see that the compromise solution satisfies the theoretical aspects of Zimmerman's fuzzy technique, which means that the compromise solution is well within the bounds and is close to the Pareto-optimal solutions. Thus the idea of linearization introduced to solve the crisp model in the final step of Zimmerman's fuzzy technique is an ideal way of tackling the bi-objective nonlinear separable problems. This technique can be extended to solve the non-linear programming problem having more than two objective functions.

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