Original Article
Bulletin of Pure and Applied Sciences.
Mathematics & Statistics Vol.44 E, No.1.
January-June 2025: P.34-40

Print version ISSN 0970 6577
Online version ISSN 2320 3226
Available online at www.bpasjournals.com

Corona Domination Number of Some Ladder Graphs

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ABSTRACT

Corona domination was initiated by G. Mahadevan, *et.al*. in 2021 [4]. A subset S of a vertex set V is called a dominating set of a graph if every vertex of $V - S$ is adjacent with at least one vertex (point) in S. The dominating set S is called a corona dominating set (CD-set), if every vertex in $< S >$ is either a pendant vertex or a support vertex. The cardinality which is minimum among all CD-sets is known as corona domination number and is represented by $\gamma_{CD}(G)$. This paper investigates the corona domination number of some ladder graphs.

Keywords: Corona domination, support vertex, diamond ladder, triangle ladder, slanting ladder, diagonal ladder

AMS Subject Classification Number: 05C69, 05C76

How to cite this article: Sajiya Merlin Mahizl A, and Lincy Elizabeth E A (2025). Corona Domination Number of Some Ladder Graphs. *Bulletin of Pure and Applied Sciences- Math & Stat.*, 44E (1), 34-40.

Received on 29.01.2025, Revised on 14.04.2025, Accepted on 12.05.2025

1. INTRODUCTION

Every graph $G = (V, E)$ taken here are undirected, connected and contain no isolated vertex and loops. A total dominating set is a dominating set S with the added requirement that the induced subgraph $< S >$ contains no isolated vertices. The total domination number $\gamma_t(G)$ of G represents the minimum cardinality of any such total dominating sets of G [1, 2].

A dominating set S of a graph G is said to be a CD-set, if every vertex in $< S >$ is either a pendant vertex or a support vertex (If a vertex is adjacent to a pendant vertex, that vertex is known to be support vertex). The minimum cardinality of a CD-set is called as CD-number and it is denoted by $\gamma_{CD}(G)$. In Fig.1. $S = \{1, 3, 8\}$ is a CD-set of G and it is minimum, hence $\gamma_{CD}(G) = 3$. Since all CD-sets of G is a dominating set as well as a total dominating set, we have $\gamma(G) \leq \gamma_t(G) \leq \gamma_{CD}(G)$. Various researchers have studied related domination concepts such as paired domination, total domination, tadpole domination and so on. Mahadevan, *et.al*. Initiated a concept of corona domination and it was named so as the structure of $< S >$ resembles the arrangement of corona cells. They have found the corona domination number of path, cycle and central graph, middle graph, splitting graph of some graphs [4]. In 2023, Praveenkumar *e. al.* obtained the corona domination number of $P_2 \times C_r$, diamond snake graph, middle graph of friendship graph and tadpole graph [5].

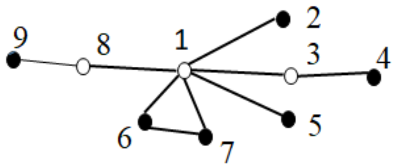

Figure 1: Graph *G*

The ladder graph is named for its visual similarity to a ladder consisting of two rails and n rungs between them. The n −ladder graph is defined as $L_n = P_2 \times P_n$ where P_n is a path graph. It is therefore same as the $2 \times n$ grid graph. It plays an important role in many applications like electronics, digital to analog conversions, wireless and electrical communication such as Wi Fi, cellular phones and so on. The main goal of this article is to obtain the CD-number of certain ladder graphs.

2. PRELIMINARIES

Definition 2.1.
A ladder graph L_n, for $n \geq 2$ is a graph of order $2n$, constructed from $P_2 \times P_n$ with $V(L_n) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(L_n) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_i : 1 \leq i \leq n\}$[6].

Definition 2.2.
Diamond ladder Dl_n, is a graph with $V(Dl_n) = \{x_i y_i z_j ;\ 1 \leq i \leq n, 1 \leq j \leq 2n\}$ and set $E(Dl_n) = \{x_i x_{i+1}, y_i y_{i+1} ;\ 1 \leq i \leq n-1\} \cup \{x_i y_i ;\ 1 \leq i \leq n\} \cup \{z_j z_{j+1}; 2 \leq j \leq 2n-2 \text{ for } j \text{ even}\} \cup \{x_i z_{2i-1}, x_i z_{2i}, y_i z_{2i-1}, y_i z_{2i}; 1 \leq i \leq n\}$ [3].

Definition 2.3.
Triangular ladderTL_n, is derived from L_n with $n \geq 2$ by adding the edges $E(G) = \{u_{i+1} v_i ;\ 1 \leq i \leq n-1\}$ [6].

Definition 2.4.
A slanting ladderSL_n, is the graph derived from two paths u_i and v_j by joining each u_i with v_{j+1}, $1 \leq i \leq n-1, 1 \leq j \leq n-1$[6].

Definition 2.5.
Diagonal ladder DL_n, is a derived from L_n by adding the edges $E(G) = \{u_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_{i+1} v_i : 1 \leq i \leq n-1\}$ for all $n \geq 2$ [6].

3. MAIN RESULTS

Theorem 3.1. Let G be a diamond ladder, then $\gamma_{CD}(G) = n$.

Proof. We denote the vertices of top, middle and bottom part of a diamond ladder by x_i, z_j, y_i, $1 \leq i \leq n, 1 \leq j \leq 2n$ respectively as shown in Fig. 2. There are $4n$ vertices and $8n-3$ edges in diamond ladder.

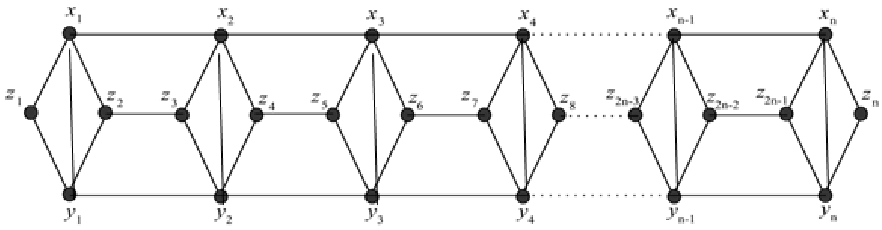

Fig. 2: Diamond ladder graph Dl_n

Consider the sets,

$S_1 = \{x_i x_{i+1}, i \equiv 1 \; mod\ (4), i \leq n-1\}$, $S_2 = \{y_i y_{i+1}, i \equiv 3 \; mod\ (4), i \leq n-3\}$
$S_3 = \{x_i x_{i+1}, i \equiv 1 \; mod\ (4), i \leq n-2\}$, $S_4 = \{y_i y_{i+1}, i \equiv 3 \; mod\ (4), i \leq n-4\}$
$S_5 = \{x_i x_{i+1}, i \equiv 1 \; mod\ (4), i \leq n-3\}$, $S_6 = \{y_i y_{i+1}, i \equiv 3 \; mod\ (4), i \leq n-1\}$
$S_7 = \{x_i x_{i+1}, i \equiv 1 \; mod\ (4), i \leq n-4\}$, $S_8 = \{y_i y_{i+1}, i \equiv 3 \; mod\ (4), i \leq n-2\}$

Now, let

$$S = \begin{cases} S_1 \cup S_2 & \text{if } n \equiv 2 \; mod\ (4) \\ S_3 \cup S_4 \cup \{x_n\}, & \text{if } n \equiv 3 \; mod(4) \\ S_5 \cup S_6 & \text{if } n \equiv 0 \; mod(4) \\ S_7 \cup S_8 \cup \{y_n\}, & \text{if } n \equiv 1 \; mod(4) \end{cases}$$

Then S is CD- set & hence $\gamma_{CD}(G) \leq |S| = n$. Since $\gamma_t(G) \leq \gamma_{CD}(G)$ and $\gamma_t(Dl_n) = n$, we have $\gamma_{CD}(G) \geq n$. Thus we get the corona domination number of diamond ladder graph is n.

We illustrate the above theorem in the Fig. 3. The minimum CD- set is $\{(x_1, x_2), (y_3, y_4), (x_5, x_6)\}$. Hence, $\gamma_{CD}(Dl_6) = 6$.

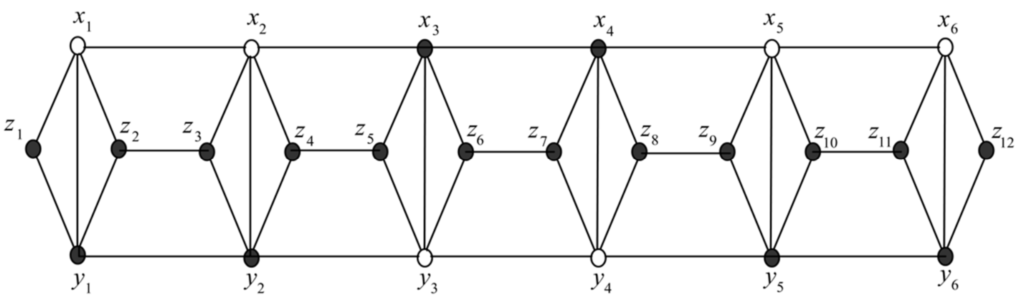

Figure 3: Diamond ladder graph Dl_6

Theorem 3.2. For the triangular ladder TL_n,

$$\gamma_{CD}(G) = \begin{cases} 4\left\lceil \frac{n}{7} \right\rceil, if\ n \equiv\ 0,5,6\ mod(7) \\ 4\left\lfloor \frac{n}{7} \right\rfloor + m\ , if\ n \equiv m\ mod(7)\ where\ m \in \{1,2\} \\ 4\left\lfloor \frac{n}{6} \right\rfloor + 2\ , if\ n \equiv 3\ mod(7) \\ 4\left\lfloor \frac{n}{6} \right\rfloor + 3\ , if\ n \equiv 4\ mod(7) \end{cases}$$

Proof. We denote the vertices of path P_n in the top and bottom part of triangular ladder by u_i and $v_i, 1 \leq i \leq n$ and it is shown in Fig. 4(a). There are $2n$ vertices and $4n-3$ edges in a triangular ladder.

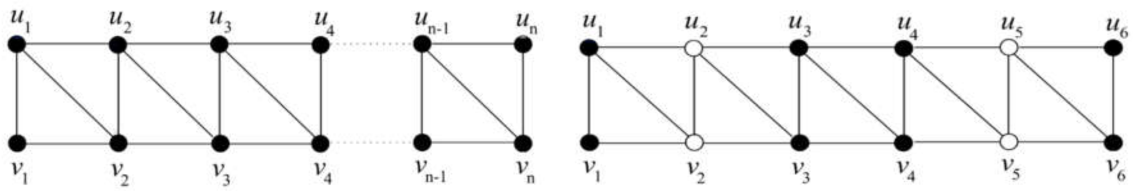

Figure 4: (a) Triangular ladder graph TL_n **(b) Triangular ladder graph TL_6**

Consider the sets
$S_1 = \{u_i, u_{i+1};\ i \equiv 5\ mod\ (7),\ i \neq n, i < n\}\ \ and\ S_2 = \{v_i, v_{i+1};\ i \equiv 2\ mod\ (7), i < n\}$
Now, let

$$S = \begin{cases} S_1 \cup S_2, if\ n \equiv 0{,}3\ mod(7) \\ S_1 \cup S_2 \cup \{u_{n-1}u_n\}, if\ n \equiv 5{,}6\ mod(7) \\ S_1 \cup S_2 \cup \{u_{n-1}\}, if\ n \equiv 1\ mod(7) \\ S_1 \cup S_2 \cup \{u_{n,}v_n\}, if\ n \equiv 5{,}6\ mod(7) \\ S_1 \cup S_2 \cup \{u_{n-1}\}, if\ n \equiv 4\ mod(7) \end{cases}$$

Clearly S is a CD-set & hence

$$\gamma_{CD}(G) \leq |S| = \begin{cases} 4\left\lceil\frac{n}{7}\right\rceil, if\ n \equiv\ 0{,}5{,}6\ mod(7) \\ 4\left\lfloor\frac{n}{7}\right\rfloor + m\ , if\ n \equiv m\ mod(7)\ where\ m \in \{1{,}2\} \\ 4\left\lfloor\frac{n}{7}\right\rfloor + 2\ , if\ n \equiv 3\ mod(7) \\ 4\left\lfloor\frac{n}{7}\right\rfloor + 3\ , if\ n \equiv 4\ \ mod(7) \end{cases}$$

Since $\gamma_t(G) \leq \gamma_{CD}(G)$ and

$$\gamma_t(TL_n) = \begin{cases} 4\left\lceil\frac{n}{7}\right\rceil, if\ n \equiv\ 0{,}5{,}6\ mod(7) \\ 4\left\lfloor\frac{n}{7}\right\rfloor + m\ , if\ n \equiv m\ mod(7)\ where\ m \in \{1{,}2\} \\ 4\left\lfloor\frac{n}{7}\right\rfloor + 2\ , if\ n \equiv 3\ mod(7) \\ 4\left\lfloor\frac{n}{7}\right\rfloor + 3\ , if\ n \equiv 4\ \ mod(7) \end{cases},$$

we have

$$\gamma_{CD}(G) \geq \begin{cases} 4\left\lceil\frac{n}{7}\right\rceil, if\ n \equiv\ 0{,}5{,}6\ mod(7) \\ 4\left\lfloor\frac{n}{7}\right\rfloor + m\ , if\ n \equiv m\ mod(7)\ where\ m \in \{1{,}2\} \\ 4\left\lfloor\frac{n}{7}\right\rfloor + 2\ , if\ n \equiv 3\ mod(7) \\ 4\left\lfloor\frac{n}{7}\right\rfloor + 3\ , if\ n \equiv 4\ \ mod(7) \end{cases}.$$

Therefore,

$$\gamma_{CD}(G) = \begin{cases} 4\left\lceil \frac{n}{7} \right\rceil, if\ n \equiv\ 0,5,6\ mod(7) \\ 4\left\lfloor \frac{n}{7} \right\rfloor + m\ , if\ n \equiv m\ mod(7)\ where\ m \in \{1,2\} \\ 4\left\lfloor \frac{n}{7} \right\rfloor + 2\ , if\ n \equiv 3\ mod(7) \\ 4\left\lfloor \frac{n}{7} \right\rfloor + 3\ , if\ n \equiv 4\ \ mod(7) \end{cases}$$

The minimum CD- set of a graph in Fig. 4 (b) is $\{(u_2, v_2),(u_5, v_5)\}$. Hence $\gamma_{CD}(TL_6) =\ 4$.

Theorem 3.3. For the slanting ladder SL_n,

$$\gamma_{CD}(G) = \begin{cases} 2\left\lceil \frac{n}{3} \right\rceil, if\ n \equiv 0,4,5, mod(6) \\ 4\left\lfloor \frac{n}{6} \right\rfloor + k\ , if\ n \equiv k\ mod(6)\ \ and\ k \in \{1,2,3\} \end{cases}$$

Proof. We denote the vertices of path P_n in the top and bottom part of slanting ladder by u_i and v_i, $1 \leq i \leq n$ and it is shown in Fig. 5(a). There are $2n$ vertices and $3n - 3$ edges in G.

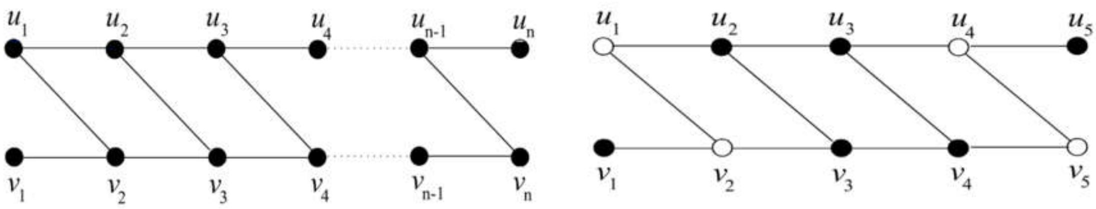

Fig. 5: (a) Slanting ladder graph SL_n (*b*) Slanting ladder graph SL_5

Consider the sets

$S_1\ = \{u_i, u_{i+1};\ i \equiv 4\ mod\ (6),\ i \neq n, i < n\}\ \ and\ S_2\ = \{v_i, v_{i+1};\ i \equiv 2\ mod\ (6), i < n\}$

Now, let,

$$S = \begin{cases} S_1 \cup S_2, if\ n \equiv 0\ mod(6) \\ S_1 \cup S_2 \cup \{u_{n-1}, u_n\}, if\ n \equiv 2, 4,5\ mod(6) \\ S_1 \cup S_2 \cup \{u_{n-1}\}, if\ n \equiv 1\ mod(6) \\ S_1 \cup S_2 \cup \{u_{n-1}\}, if\ n \equiv 3\ mod(6) \end{cases}$$

Clearly, S is a CD-set of G & hence

$$\gamma_{CD}(G)\ \leq\ |S| = \begin{cases} 2\left\lceil \frac{n}{3} \right\rceil, if\ n \equiv 0,4,5, mod(6) \\ 4\left\lfloor \frac{n}{6} \right\rfloor + k\ , if\ n \equiv k\ mod(6)\ \ and\ k \in \{1,2,3\} \end{cases}$$

Since $\gamma_t(G) \leq \gamma_{CD}(G)$ and $\gamma_t(SL_n) = \begin{cases} 2\left\lceil \frac{n}{3} \right\rceil, if\ n \equiv 0,4,5, mod(6) \\ 4\left\lfloor \frac{n}{6} \right\rfloor + k\ , if\ n \equiv k\ mod(6)\ \ and\ k \in \{1,2,3\} \end{cases}$,

we have $\gamma_{CD}(G) \geq \begin{cases} 2\left\lceil \frac{n}{3} \right\rceil, if\ n \equiv 0,4,5, mod(6) \\ 4\left\lfloor \frac{n}{6} \right\rfloor + k\ , if\ n \equiv k\ mod(6)\ \ and\ k \in \{1,2,3\} \end{cases}$.

Therefore,

$$\gamma_{CD}(G) = \begin{cases} 2\left\lceil \frac{n}{3} \right\rceil, if\ n \equiv 0,4,5, mod(6) \\ 4\left\lfloor \frac{n}{6} \right\rfloor + k\ , if\ n \equiv k\ mod(6)\ \ and\ k \in \{1,2,3\} \end{cases}$$

The minimum CD-set of a graph in Fig. 5 (b) is $\{(u_1, v_2),(u_4, v_5)\}$.

Hence, $\gamma_{CD}(SL_5) = 4$.

Theorem 3.4. For the diagonal ladder DL_n,

$$\gamma_{CD}(G) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil, if\ n \equiv 0,1,3\ mod(4) \\ \frac{n}{2} + 1, if\ n \equiv 2\ mod(4) \end{cases}$$

Proof. Consider $V(G) = \{u_i, v_i : 1 \le i \le n\}$ and $E(G) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \le i \le n-1\} \cup \{u_i v_i : 1 \le i \le n\} \cup \{u_i v_{i+1} : 1 \le i \le n-1\} \cup \{u_{i+1} v_i : 1 \le i \le n-1\}\ for\ all\ n \ge 2$. There are $2n$ vertices and $3n-3$ edges in G.

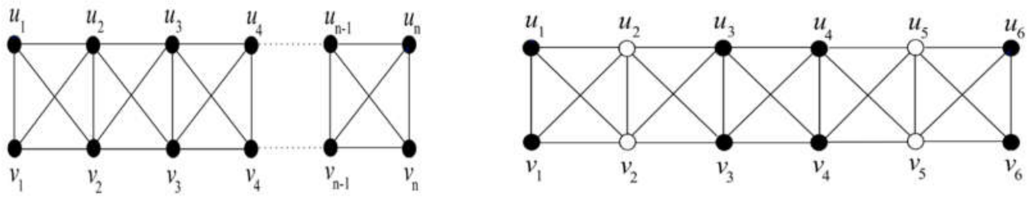

Figure 6: (a) Diagonal ladder graph DL_n (b) Diagonal ladder graph DL_6

Consider the set $S_1 = \{u_i v_{i+1};\ i \equiv 2\ mod\ (4)\}$

Now let,
$$S = \begin{cases} S_1, if\ n \equiv 0,3\ mod\ (4) \\ S_1 \cup \{u_{n-1}\}, if\ n \equiv 1\ mod(4) \\ S_1 \cup \{u_n, v_n\}, if\ n \equiv 2\ mod(4) \end{cases}$$

Then S is a CD-set & hence

$$\gamma_{CD}(G) \le |S| = \begin{cases} \left\lceil \frac{n}{2} \right\rceil, if\ n \equiv 0,1,3\ mod(4) \\ \frac{n}{2} + 1, if\ n \equiv 2\ mod(4) \end{cases}$$

Since $\gamma_t(G) \le \gamma_{CD}(G)$ and $\gamma_t(DL_n) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil, if\ n \equiv 0,1,3\ mod(4) \\ \frac{n}{2} + 1, if\ n \equiv 2\ mod(4) \end{cases}$,

we have
$$\gamma_{CD}(G) \ge \begin{cases} \left\lceil \frac{n}{2} \right\rceil, if\ n \equiv 0,1,3\ mod(4) \\ \frac{n}{2} + 1, if\ n \equiv 2\ mod(4) \end{cases}.$$

Therefore,

$$\gamma_{CD}(G) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil, if\ n \equiv 0,1,3\ mod(4) \\ \frac{n}{2} + 1, if\ n \equiv 2\ mod(4) \end{cases}$$

The minimum CD-set of a graph in Fig. 6 (b) is $\{(u_2, v_2), (u_5, v_5)\}$. Hence, $\gamma_{CD}(DL_6) = 4$.

4. CONCLUSION

In this article, we have got the corona domination number of ladder graphs such as diamond ladder, triangle ladder, slanting ladder and diagonal ladder. Finding the corona domination number helps in determining the minimum cardinality of monitoring points needed for complete surveillance, ensuring that key locations (vertices) have adjacent monitors.

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