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# Study of Covid-19 Growth and Decay Cases through Mathematical Models

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## ABSTRACT

The main motto of the article is to provide various results of epidemic model. The results of linear growth model and effects of immigration of models ensures and make us aware to follow standard operating procedure (SOP) measures provided by Government in the view of Covid-19. The study of model provides better understanding of the challenges faced and curb the disease.

**Keywords:** Covid-19, Differential equation of first order, Mathematical model.

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## INTRODUCTION

Models are representations of specific objects, concepts, or states; they are abstractions of reality. Simplified representations of real-world entities are known as mathematical models. It could be computer code or equations. A conceptual model is created initially, followed by a quantitative model, in order to create a mathematical model.

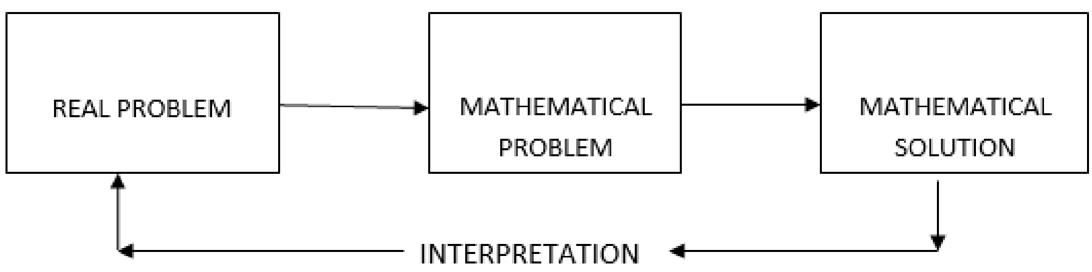
Our conception of the system's operation is reflected in a conceptual model. A model diagram with boxes and arrows is used to represent it. After that, equations are created for each process's rates and then integrated to create a quantitative model with dynamic equations for every state variable. The numerical solution of these dynamic equations can be obtained by studying them mathematically or by converting them into computer code. A hypothesis regarding the studied system is embodied in the model, and the data and the hypothesis are compared. Computer simulations and mathematical models are helpful experimental resources.

A good model should be fit for purpose i.e. it should be as simple as possible, but sufficiently complex to adequately represent the real system without obstructing understanding, there should be balance between accuracy, transparency and flexibility. Many mathematical models are developed according to the usability such as :
(1) Empirical models –These models are used for determining patterns and relationships between data.
(2) Mechanistic models- These models are used for underlining the mechanisms of this phenomenon.
(3) Hybrid Models- These models are used to show progress.
(4) Simulation Models- Simulation models are not specific types of mathematical models. These type models simulate the process of data generation assuming the model was true.
For example- Simulate an epidemic.

These models are helpful in understanding how an illness behaves once it enters a community and determining the circumstances that will allow it to persist. Researchers, governments, and everyone else are currently very interested in COVID-19 due to the high pace of infection transmission and the huge number of deaths that have occurred. A recently identified corona virus is the cause of the infectious disease corona virus, which was originally identified in Wuhan, China. When an infected individual coughs, sneezes, or exhales, droplet are created that are the primary means of spreading the Covid-19 virus. We may inhale respiratory droplets from close touch with one another, which is how most of the transmission occurs (Ming et al., 2020). In Ming et al., 2020 study the actual number of infected cases is projected using a modified SIR epidemic model. An SIR (susceptible, infected, and recovered) epidemic model was created by Nesteruk, 2020. Who also talked about possible antiviral and vaccine candidates that are being developed in several nations. The estimation of the corona virus epidemic's ultimate size was examined by Batistia, 2020. Numerous scholars examined dynamical behavior and created various models of COVID-19. It was determined that human contact may be the root cause of COVID-19 epidemics. In this sequel, we worked on linear model and tried to find out various results of epidemic model, which provide better understanding of the challenges faced and curb the disease.

## PRELIMINARIES

**Definitions Mathematical Modelling -** A representation of a system that enables analysis of its characteristics and, in some situations, forecasting of future events is known as mathematical modeling.
**The technique of mathematical modeling -** Actual-world issues are converted into mathematical ones using mathematical modeling, which then solves the mathematical issues and interprets the results in terms of the actual world.



This is figuratively stated as follows: we grasp the real-world problem with our teeth, plunge into the mathematical ocean, and swim for a while, and emerge with the answer to the real-world problem. In other word, we may say that we take the problem and fly high into the mathematical atmosphere, stay there for a while, and then return to Earth with the solution.

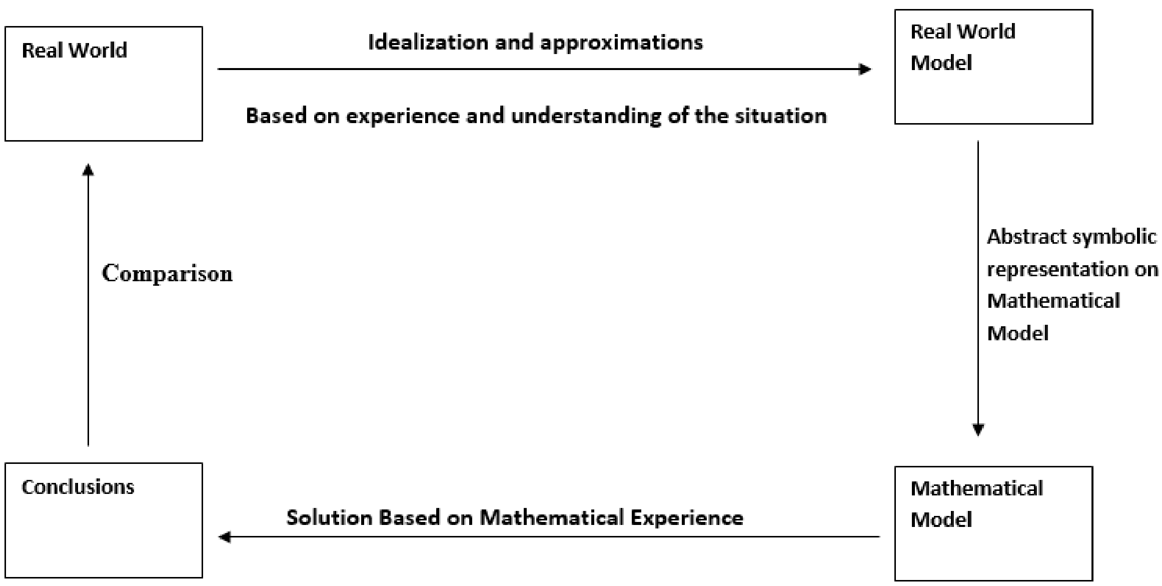
In general, real-world problems are rarely able to be converted into mathematical ones, and even when they can, the resulting mathematical problem might not be amenable to solution. Therefore, it is frequently necessary to "idealize" or "simplify" the issue or approximate it using a different problem that is theoretically solvable and fairly similar to the original problem.

We attempt to preserve all that is necessary or pertinent to the issue we are looking into in this idealization.

The idealization assumptions can appear extremely extreme at times. Therefore, we can disregard the planets' sizes and structures and treat the Sun and planets as point masses while analyzing their observations. In a similar vein, we can think of a fluid's motion as a continuous medium and disregard its molecular structure's distinct nature.

The closeness of the agreement between data and mathematical model projections is frequently used to justify such assumptions.

The above model be modify as:




**Differential Equation of first order linear -** The equation of the form $\frac{dq}{dp} + a(p)q = b(p)$ is called ' first order linear' differential equation. The 'first-order' means only first derivatives appear in the equation.

**MAIN RESULTS**

**A simple pandemic model-** Let P(t) and Q(t) be the number of susceptibles (i.e. those who can get a Covid-19 diseases) and infected persons (i.e. those who are covid +ve ).

Now initially let n be the no. of susceptible and one infected person in Indore so that
P(t) + Q(t) = n + 1
When t = 0, then
P(0) = n and Q(0) = 1 ……………………(1)

We know that the number of susceptible people falls at the same rate as the number of infected people, and that the number of infected people increases proportionately to the product of susceptible and infected people. This gives us the system of differential equations.

$$\frac{dQ}{dt}\ \alpha\ PQ \ \ and \ \ \frac{dP}{dt}\ \alpha - PQ$$

$$\frac{\mathrm{dQ}}{\mathrm{dt}} = \mathrm{kPQ} \quad \text{and} \quad \frac{dP}{dt} = -kPQ \qquad ..................(2)$$

On adding we get

$$\frac{dP}{dt} + \frac{dQ}{dt} = 0$$

On integrating both the sides

$$\int \frac{dP}{dt}dt + \int \frac{dQ}{dt}dt = C$$

$$\Rightarrow P(t) + Q(t) = C$$

$$\text{Put} \quad t = 0, \quad \text{we get} \quad P(0) + Q(0) = C$$

$$\therefore n + 1 = C$$

$$\text{Hence} \quad P(t) + Q(t) = n + 1 \qquad ...........(3)$$

$$\because \frac{dP}{dt} = -kPQ$$

$$\Rightarrow \frac{dP}{dt} = -kP(n + 1 - P) \qquad .......... \ ........(4)$$

Now,

$$\frac{dP}{dt} = kP(n + 1 - I)$$

$$\int \frac{dP}{P(n + 1 - P)} = \int -kdt + \log C$$

$$\frac{1}{n + 1}[\int \frac{1}{P} + \int \frac{1}{n + 1 - P}]dt = -kt + \log C$$

$$\frac{1}{n+1}[\log P - \log(n+1-P)] = -kt + \log C$$

$$\frac{1}{n+1}[\log \frac{P}{n+1-P}] - \log C = -kt$$

$$\log[\frac{P}{n+1-P}]^{\frac{1}{n+1}} = -kt$$

We get

$$P(t) = \frac{n(n+1)}{n+e^{(n+1)kt}} \quad \text{and} \quad Q(t) = \frac{(n+1)e^{(n+1)kt}}{n+e^{(n+1)kt}} \qquad \ldots\ldots\ldots\ldots(5)$$

When $t = 0$

$$P(0) = \frac{n(n+1)}{n+e^{(n+1)k.0}} \quad \text{and} \quad Q(0) = \frac{(n+1)e^{(n+1)k.0}}{n+e^{(n+1)k.0}}$$

$$P(0) = \frac{n(n+1)}{n+1} \quad \text{and} \quad Q(0) = \frac{n+1}{n+1}$$

i.e. P(0) = n and Q(0) = 1, which is verified but we know that time is running so that

$$\lim_{t\to\infty} P(t) = \lim_{t\to\infty} \frac{n(n+1)}{n+e^{(n+1)kt}} = \frac{n(n+1)}{n+e^{\infty})} = 0$$

$$\lim_{t\to\infty} Q(t) = \lim_{t\to\infty} \frac{(n+1)e^{(n+1)kt}}{n+e^{(n+1)kt}} = \frac{n(n+1)}{\infty(\frac{n}{\infty}+1)} = n+1$$

It means, we get the result $\lim_{t\to\infty} P(t) = 0$ and $\lim_{t\to\infty} Q(t) = n+1$. The results show that if we will not take care of SOP measure provided by the Government, then all the susceptible i.e. prone to disease become infected to Covid +ve.

**Linear growth and decay model -**

Let p(t) be the population size at time t in Indore and $p^*(t)$ be the total population of covid infected persons (i.e. those who are Covid + ve). Let b be the new Covid infected person i.e. new Covid + ve infected person and let d be the death rates i.e. number of individuals Covid + ve or dying with Covid disease t be the unit time.

Then in time interval (t, Δt), the number of Covid + ve (birth) and death would be

$$bp^*\Delta t + o(\Delta t) \text{ and } dp^*\Delta t + o(\Delta t)$$

Where o(Δt) is very small quantity approaches to zero, so that

$$\frac{p^*(t+\Delta t) - p^*(t)}{\Delta t} = \frac{(bp^*(t) - dp^*(t))\Delta t}{\Delta t} + \frac{o(\Delta t)}{\Delta t}.$$

Taking the lim Δt→0.

$$\lim_{\Delta t\to 0} \frac{p^*(t+\Delta t) - p^*(t)}{\Delta t} = (b-d)p^*(t).$$

$$\Rightarrow \frac{dp^*}{dt} = (b-d)p^*.$$

$$\Rightarrow \frac{dp^*}{p^*} = adt, \qquad \text{where} \qquad b - d = a$$

On integrating both sides, we get

$$\int \frac{dp^*}{p^*} = a\int dt + \log c$$

$$\Rightarrow \log p^* = at + \log c$$

$$\Rightarrow \log(\frac{p^*}{c}) = at$$

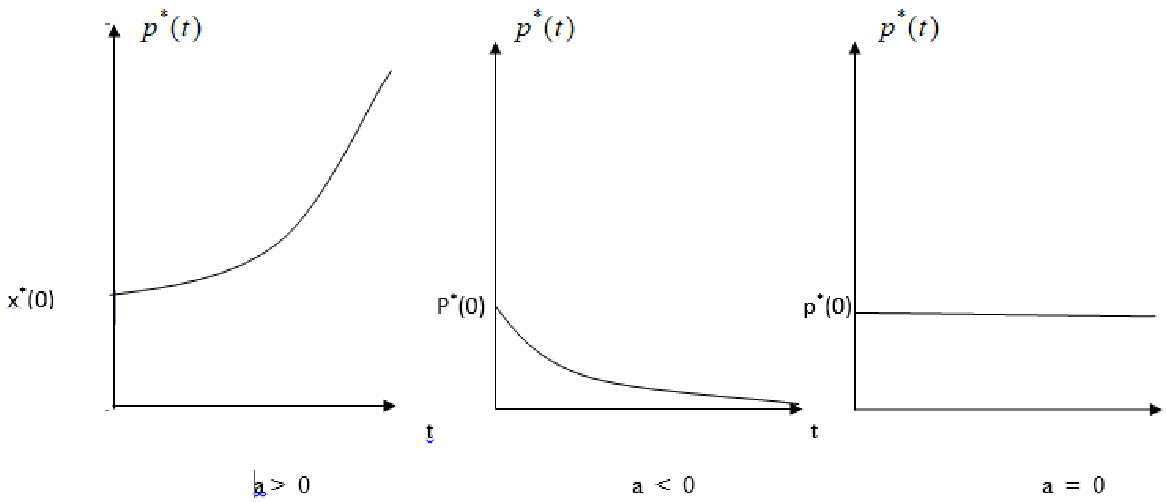
$$\Rightarrow \frac{p^*}{c} = \exp(at)$$

$$\Rightarrow p^* = c\exp(at)$$

Hence

$$p^*(t) = ce^{at} \quad \text{or} \quad p^*(t) = p^*(0)\exp(at)$$

This result shows that population of Covid patients grows exponentially if a > 0 i.e. (b-d) > 0 and decays exponentially if a < 0 i.e. (b-d) < 0. This can be demonstrated by the following graphs.




If a > 0, the population of Covid patient will become double of it's present size at time T, then we have

$$2p^*(0) = p^*(0)\exp(aT) \quad \text{or} \exp(aT) = 2$$

Taking log of both sides, we get

$$aT = \log_e 2 \Rightarrow T = \frac{1}{a}\log_e 2 = a^{-1}(0.69314118)$$

T is referred to as the Covid patients' doubling period, and it is clear that it is unaffected by p(0).

It only depends on a, and because T and "a" are inversely proportional, the smaller the doubling time, the larger the value of a (i.e., the greater the disparity between birth and mortality rates).

(i) If a < 0, the population of Covid patient will become half of it's present size in time T. Then

$$\frac{1}{2}p^*(0) = p^*(0)\exp(aT')$$

Or

$$\exp(\mathrm{aT}') = \frac{1}{2}$$

Taking log of both sides, we get

$$\mathrm{aT}' = \log(\frac{1}{2}) = \log 1 - \log 2 = -\log 2$$

$$\Rightarrow T' = -\frac{1}{2}\log 2 = -a^{-1}(0.69314118)$$

**Effects of immigration in population size of Covid patients –**

If there is immigration into the population size from outside and if we will follow SOP measures provided by government, then the rate is direct proportional to the population of Covid infected per case. Let p(t) be the population size at time t in Indore city anf $p^*(t)$ be the total population of Covid infected persons and d be the death rates in Indore city. Then in time interval (t, t+Δt), the number of Covid +ve (birth) and death would be

$$bp^*\Delta t + o(\Delta t) \text{ and } \mathrm{dp}^*\Delta t + o(\Delta t)........................(1)$$

So the increment is given by

$$p^*(t+\Delta t) - p^*(t) = (bp^*(t) - dp^*(t))\Delta t + o(\Delta t)$$

On dividing both sides by Δt, we get

$$\frac{p^*(t+\Delta t) - p^*(t)}{\Delta t} = \frac{(b-d)p^*(t))\Delta t}{\Delta t} + \frac{o(\Delta t)}{\Delta t}$$

Taking lim $\Delta t \to 0$, we get

$$\lim_{\Delta t \to 0} \frac{p^*(t+\Delta t) - p^*(t)}{\Delta t} = \frac{(b-d)p^*(t))\Delta t}{\Delta t}$$

If immigration take place at cinstant rate k, then

$$\frac{dp}{dt} = ap + k \quad (\text{where} \quad b - d = 0)$$

On integrating both sides

$$\therefore \int \frac{dp}{ap + k} = \int dt + c$$

$$\Rightarrow \frac{1}{a} \log(ap + k) = t + c$$

when t = 0, p(t) = 0

$$\therefore \frac{1}{a} \log k = c$$

$$\Rightarrow \frac{1}{a} \log(ap + k) = t + \frac{1}{a} \log k$$

$$\Rightarrow \frac{1}{a} [\log(ap + k) - \frac{1}{a} \log k] = t$$

$$\Rightarrow \frac{1}{a} [\frac{\log(ap + k)}{\log k}] = at$$

$$\Rightarrow \frac{ap + k}{k} = \exp(at)$$

$$\Rightarrow p(t) = \frac{k}{a} (p_0 \exp at - 1)$$

Hence if we will not follow SOP measure in immigration population of Covid + ve patients will increase exponentially.

## CONCLUSION

In this article a studied is performed on different mathematical models i.e. Pandemic model, Linear growth model, Effect of immigration a conclusion is drown that an exponential growth is seen in Covid + ve cases which is also evident from the present Covid cases of today's society. To curb the exponential growth it is must that standard operating procedure (SOP) measures provided by Government and WHO must be followed i.e. wear a mask, use hand sanitizer frequently and keep at least one meter distance from others. This will break the chain and may bring the normalcy and good health back to the society.

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