

## Impedance for Wave Guiding Devices from the Microwave Frequency Regime to Optics and Plasmonics

Abdul Sattar Alam\* & Ashok Kumar

### Author's Affiliations:

<b>Abdul Sattar Alam</b>	Research Scholar, University Department of Physics, B.N. Mandal University, Laloo Nagar, Madhepura, Bihar 852113, India E-mail: <a href="mailto:abdulsattar84058@gmail.com">abdulsattar84058@gmail.com</a>
<b>Ashok Kumar</b>	University Department of Physics, B.N. Mandal University, Laloo Nagar Madhepura, Bihar 852113, India E-mail: <a href="mailto:ashokabnu@yahoo.co.in">ashokabnu@yahoo.co.in</a>
<b>*Corresponding author:</b>	<b>Abdul Sattar Alam</b> , Research Scholar, University Department of Physics, B.N. Mandal University, Laloo Nagar, Madhepura, Bihar 852113, India E-mail: <a href="mailto:abdulsattar84058@gmail.com">abdulsattar84058@gmail.com</a>

Received on 29.03.2019

Accepted on 04.05.2019

### ABSTRACT

We have derived expressions that generalized the impedance concept for wave guiding devices from the microwave frequency regime to optics and plasmonics. Our expressions were based on electromagnetic eigen modes that are excited at the interface of a structure. Impedance in electromagnetic wave theory is the ratio of the electric and magnetic field strength. The ratio between the electric and magnetic field is not constant over the cross section in most devices. This caused several suggestions using averaged or integrated fields which were heuristically proposed for a manifold of photonic structures not only waveguides. The area of photonic crystals were several such heuristic approaches excited until it was proven that the so called Bloch impedance, the ratio of the surface averaged fields was analytically correct solution, provided that the photonic crystal operates only in its fundamental modes. Electromagnetic response closely resembles the solution in the quasistatic limit. Plasmonic metal-insulator-metal waveguide and a waveguide at microwave frequencies facilitated the use of traditional impedance for this kind of structure in plasmonics. For this we required lumped circuit parameters to the radio frequency domain. Impedance definition must correctly describe the reflection that occurs at the boundary between two different structures. We analysed the reflection at an interface of impedance discontinuity i.e. between two different structures. A plasmonic insulator-metal-insulator waveguide characterized by referential impedance; illustrates another photonic structure. We have used Bragg reflector, i.e periodic corrugations in a metal film which intended to describe impedance. In case of circuit theory; we considered as part of a photonic network. Our approach was based on a decomposition of the electromagnetic fields into eigenmodes. We have chosen it because the relatively strong loss in the metal that prevented the use of many radio frequency derivations and the open boundary condition that made it possible to find suitable analogies to voltage and current. We observed that the impedance for the reciprocity based overlap of eigen modes. We found that applicability of simple circuit parameters ends and how the impedance can be interpreted beyond any particular point. The unconjugated reciprocity framework set up to solve for the

interface reflection coefficient of the different electromagnetic eigen modes. This rigorous expression is linked to the relative impedance of the discontinuity between the two structures which yields a very general expression for the modes' impedances. Our obtained results were found in good agreement with previously observed theoretical and experimental works.

**KEYWORDS** Impedance, waveguide, microwave, plasmonics, electromagnetic, eigen modes, heuristically, photonics, radio frequency.

## INTRODUCTION

Heaviside [1] introduced the impedance concept as the ratio of complex voltage and current the evolution of electromagnetic theory accompanied by subsequent generalization of concept for electromagnetic waves. Schilkunoff [2] pointed out that the impedance must be seen as property of the wave in a medium rather than of the medium alone. Collin [3] presented that up to the microwave frequency domain the slow variation of the involved electromagnetic fields allowed to simplify the functionality of a device with lumped circuit quantities such as resistance, capacitance or inductance, which constituted the impedance. Walker [4] showed that for wave guiding devices a confusing situation with mutually contradictory definitions was reached relatively fast. The reason for this was the ratio between the electric and magnetic field was not constant over the cross section in most devices. Buscolo [5], Biswas [6] and Nomeni [7] showed that the area of photonic crystals were several such heuristic until it was proven that the so called Bloch impedance, the ratio of the averaged field was the correct solution. Photonic crystal operates only in its fundamental modes shown by Smigaj [8]. Alu et al. [9] studied the concept of nanostructures by the use of radio frequency. Biagioni et al. [10] presented that as long as the electromagnetic response closely resembled the solution in the quasistatic limit. Greffet et al. [11] made a new idea of impedance for quantum emitters. Huang et al. [12] studied experimentally relevant impedance definition correctly as the reflection that occurs at the boundary between two different structures.

## METHOD

The quantities which are related to impedance are the transmission and reflection at an interface of impedance discontinuity. It is expressed as

$$r = \frac{Z - Z_0}{Z + Z_0}, \quad t = \frac{2Z}{Z + Z_0}$$

Where  $Z$  and  $Z_0$  are the two impedances. The relative impedance ratio  $\hat{Z} = \frac{Z}{Z_0}$  of the two structures

and they obey  $r + 1 = t$ . The individual impedances themselves need to be normalized afterwards in order to be unambiguous. For reflection we have

$$\hat{Z} = \frac{Z}{Z_0} = \frac{1 + r}{1 - r}$$

This reflection is used to derive an expression for impedance for plasmonic wave guides. The  $n^{\text{th}}$  eigen-state of the structure is described by the impedance  $Z$  by an abstract Ket vector  $|\psi_n\rangle$ . The transverse components are  $E_n$  and  $H_n$ . The longitudinal components are determined by Maxwell's equations. The different bound and radiating modes of the plasmonic waveguides are  $\{|\phi_n\rangle\}, \{|\psi_n\rangle\}$  are the Bloch modes of the periodically corrugated metal film. The forward and backward propagating modes are represented by plus or minus superscripts of  $|\phi_1^\pm\rangle$ . Which is

called the fundamental illuminating mode of the plasmonic waveguide. In electromagnetic theory two forms of inner products are derived from the reciprocity using products of  $E \times H^*$ . This is used in waveguide theory. Hence obtained expressions are related to the energy flux by pointing vector

$$\langle S \rangle = \frac{1}{2} \text{Re} [E \times H^*]. \text{ It is valid for loss less structures. The appropriate inner product for modal analysis in plasmonics is given by the unconjugated version of the reciprocity theorem. Then we have}$$

$$\langle \phi_m | \phi_n \rangle = \iint_{\mathbf{R}^2} [E_n \times H_m - E_m \times H_n] dA,$$

where A points into the forward propagation direction z-axis. The orthogonality relations are explicitly given as

$$\langle \phi_m^+ | \phi_n^+ \rangle = 0, \quad \langle \phi_m^- | \phi_n^- \rangle = C_m \delta_{mn},$$

$$\langle \phi_m^- | \phi_n^- \rangle = 0, \quad \langle \phi_m^+ | \phi_n^+ \rangle = -C_m \delta_{mn},$$

Where  $C_m$  is a normalization constant. The reflection at the interface is decomposed into eigen modes of both structures. If  $r_{mn}$  be the reflection into the  $m^{\text{th}}$  backward mode caused by  $n^{\text{th}}$  forward mode similar to  $t_{mn}$ . The electromagnetic fields across the interface is

$$|\phi_n^+\rangle + \sum_m r_{mn} |\phi_m^-\rangle = \sum_m t_{mn} |\Psi_m^+\rangle$$

Illuminating field  $|\phi^+\rangle$  is decomposed in to forward modes as

$$|\phi^+\rangle = \sum_n c_n |\phi_n^+\rangle$$

Where  $c_n$  are the excitation coefficients reflection matrix

$$\hat{r} = -\hat{P}^{-1} \hat{Q}$$

$\hat{P}$  and  $\hat{Q}$  are matrices

$$P_{mn} = \langle \psi_m^+ | \phi_n^- \rangle, \quad Q_{mn} = \langle \psi_m^+ | \phi_n^+ \rangle$$

Thus impedance can be determined.

## RESULTS AND DISCUSSION

Graph (1) shows the results for reflection amplitude using  $r_m = \sum_n c_n \frac{\hat{Z}_{mn} - 1}{\hat{Z}_{mn} + 1}$  together with our

impedance definition in comparison to rigorous results obtained. We considered a rectangular waveguide at microwave frequencies. It consisted a dielectric filling permittivity  $\epsilon_f$  surrounded by a perfect electric conductor of rectangular shape with dimensions  $a$  and  $b$  as shown in graph (1). We found that the reflection and transmission that occurred when a section with dielectric filling  $\epsilon_f \neq 1$  was attached to an air filled section with  $\epsilon_f = 1$ . We have taken into account the periodic symmetry of the photonic crystal, the remaining integrals over  $E^+$  and  $H^+$  was taken into over the periodic unit cell.  $Z$  shrunk to the ratio of the averaged electric and magnetic fields and became identical to the Bloch impedance. The treatment of wave guiding devices in general is more sophisticated since none of the involved modes is a pure plane wave. The inhomogeneous field profiles do not allow to pull contributions out of the integral but modal symmetries may be exploited for special cases. The eigen modes of this waveguide are separable functions in the  $x$ - $y$  direction. The

photonic crystal illuminated by a plane wave is described by Bloch impedance  $Z_B = \frac{\langle E_B \rangle}{\langle H_B \rangle}$  where

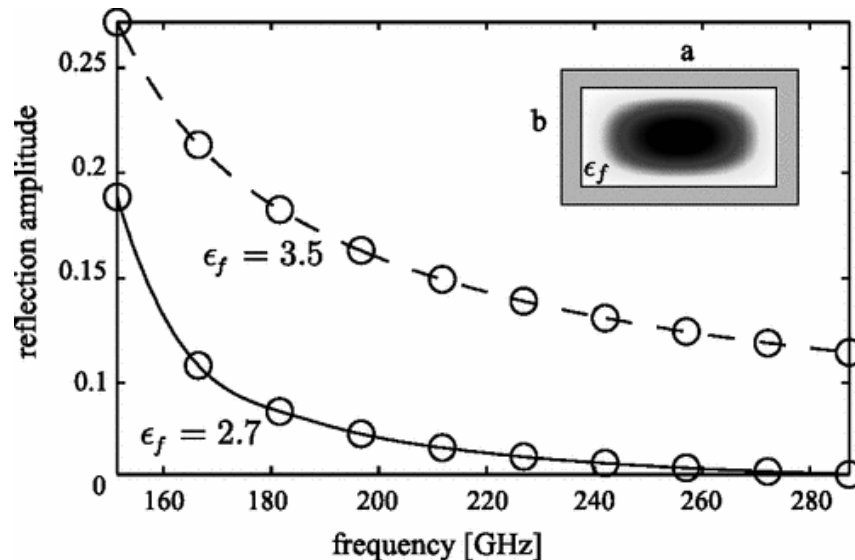
$\langle E_B \rangle$  and  $\langle H_B \rangle$  are the interface averaged fields of the pseudoperiodic Bloch modes. By inserting the plane wave as a reference and exploiting the plane wave symmetry it becomes possible to pull the magnitude  $E_0$  and  $H_0$  out of the integrals. From our results we found that a matrix impedance became necessary to describe the coupling of all modes analytically exact. The elements of such an entity was found by using elementary intermodal reflection given below

$$\hat{Z}_{mn} = \frac{1 - \sum_k (\hat{P} - 1)_{mk} Q_{kn}}{1 + \sum_k (\hat{P} - 1)_{mk} Q_{kn}}$$

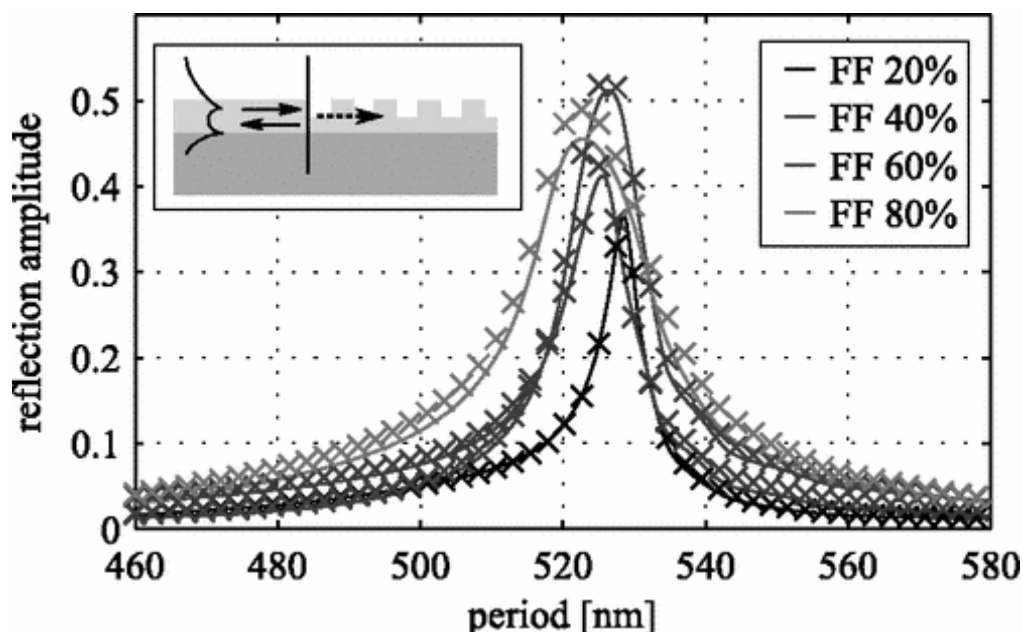
This expression is general form of relative impedance between two eigen modes  $m$  and  $n$  of two different structures and is the main results of our work. The total reflection into the  $m^{\text{th}}$  backward mode is given by  $r_m = \sum_n c_n r_{mn}$ , which is obtained by taking the coefficient  $c_n$  of the illumination mode spectrum and we obtain

$$r_m = \sum_n c_n \frac{\hat{Z}_{mn} - 1}{\hat{Z}_{mn} + 1}$$

The electromagnetic field consists of trigonometric functions that depend solely on the wave guide size, not on the dielectric filling or the propagation constant. We have shown that the impedance known for wave guides in the microwave frequency domain is a special case of our general framework. We found that a relative impedance of the reflector with respect to the plasmonic waveguide is possible. Graph (2) shows the results of the analysis. A reflection peak occurs when the impinging mode is impedance mismatched to the reflector Bloch mode. Based on the two mode, we calculated impedance mismatch and compared the results to rigorous simulations performed by the a periodic Fourier modal method both were found in good agreement.



Graph 1: Reflection of the fundamental mode of a section with  $\epsilon_f = 1$  in a rectangular microwave waveguide ( $a = 1\text{mm}$ ,  $b = 0.5\text{ mm}$ ) at a section of different dielectric filling  $\epsilon_f \neq 1$ .



Graph 2: Rigorous results (lines) obtained by the a periodic Fourier modal method and results of the impedance framework.

## CONCLUSION

We have studied the impedance concept for waveguiding devices from the microwave frequency regime to optics and plasmonics. We have characterized plasmonic insulator-metal-insulator waveguide using referential impedance. The unconjugated reciprocity framework was set to solve the interface reflection coefficients of different electromagnetic eigen modes. This rigorous expression was linked to the relative impedance of the discontinuity between the two structures which yielded general expression for the modes' impedances. We showed that our results were consistent with previous results and it was found that the modal symmetries of the structure are the Key point that allowed absolute impedance in these cases. We have also applied our findings to the case of plasmonic insulator-metal insulator waveguide with a Bragg termination. The obtained results were compared with previously obtained results and were found in good agreement.

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