Quantum Pumping of Charge and Spin Currents in Graphene Nano Ribbons

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Abstract

We have studied the case of nonmagnetic leads in which only part of the graphene nano ribbons is subjected to an ac gate voltage to break the left-right spatial symmetry. It was shown that peaks of the negative (positive) charge pumping current appear at the Fermi energies around the energy maximums (minimums) of the sub-bands in the ac field free region of the graphene nano ribbon. In the case of ferromagnetic leads with the lead magnetizations being antiparallel in order to break the left-right symmetry. It was found that a pure spin current can be achieved when the whole graphene nanoribbon was under an ac gate voltage. It was also found that peaks in the negative spin pumping current at the Fermi energies around the energy maximums of the majority spin sub-bands in the ferro-magnetic leads. We have shown that the pumping current increases monotonically with the ac field strength due to the enhanced photon-assisted transmissions. We found that at low ac field frequency, the pumping currents exhibited non-monotomic frequency dependence. This is due to the competition of the weakened photon assisted transmissions and the increasing energy space of the states contributing to the pumping current. AT high frequency, the pumping current decreased monotonically with increasing frequency because the ac field became in effective in existing photons. Our results showed that the Fermi energies corresponding to peaks are influenced by the energy level shifting in the ac field free region, but independent of that in the ac field applied region.

Keywords: Nonmagnetic, graphene nanoribbons, gate voltage, spatial symmetry, charge pumping, Fermi energy, ferromagnetic, spin pumping, photon.

1. Introduction

Julicher et al. [1] and Ivchenko et al. [2] studied the quantum pumping is highly related to the ratchet effect and photo galvanic effect presented by Ganichev [3]. Thouless [4] and Altshuler et al. [5] described the generation of direct current at zero bias in a spatially asymmetric system under ac fields. Agrawal et al. [6] and Romeo et al [7] theoretically investigated the quantum pumping of charge and spin currents and were observed experimentally by Blumenthal et al. [8] and Fujiwara et al. [9] in semiconductor quantum dots, quantum wires. Foatorres et al. [10] and Wu [11] studied about quantum charge and spin pumping in graphene nanoribbons. Grichuk et al. [12] and Liu et al. [13] also focused on the pumping involving more than one time dependent parameters, partially because the single parameter pumping is only possible beyond the adiabatic approximations and thus needs a more complex theoretical tools suggested by Brower [14]. The single parameter pumping is more favorable to the application, since the reduction in the number of necessary contacts makes the

scalable and low dissipative device more proming as suggested by San-Jose et al. [15]. Graphene and its lower dimensional cousins, graphene nanoribbons and carbon nanotubes, exhibit abundant new physics and potential applications and hence have attracted much interest shown by Abergel et al. [16] and Das Sarma et al. [17]. The effect of an ac field on the electrical and optical properties in these materials is on by the main focuses of attention. Many interesting phenomena have been reported such as the photon assisted transport, the laser induced quasi energy gap and photovoltaic Hall effect, the dynmical Franz Keldysh effect and the quantum pumping.

2. Method

We have setup the tight-binding Hamiltonian and the formula of the pumping current. The tight binding Hamiltonian can be written as

$$H = H_L + H_g + H_R + H_T$$

In which H_L represents the Hamiltonian of left and H_R represents the Hamiltonian of right leads, H_g represents the Hamiltonian of graphene nanoribbon and H_T describes the hopping between the graphene nanoribbon and the leads. We have used the nonequilibrium Green's function method, then the time averaged current can be written as the current flowing from left to right.

$$\begin{split} \overline{I} &= \frac{e}{\hbar T_0} \int_0^{T_0} dt \int_{-\infty}^{\infty} \frac{d\varepsilon \, d\varepsilon' \, d\varepsilon_1}{2\pi \, 2\pi \, 2\pi} e^{i(\varepsilon' - \varepsilon)t} \\ &\times Tr \Big\{ \hat{G}^a \left(\varepsilon, \varepsilon_1 \right) \hat{T}_R \left(\varepsilon_1 \right) \hat{G}^r \left(\varepsilon_1, \varepsilon' \right). \\ &\hat{T}_L \left(\varepsilon' \right) f_L \left(\varepsilon' \right) - \hat{G}^a \left(\varepsilon, \varepsilon_1 \right) \hat{T}_L \left(\varepsilon_1 \right). \\ &\hat{G}^r \left(\varepsilon_1, \varepsilon' \right) \hat{T}_R \left(\varepsilon' \right) f_R \left(\varepsilon' \right) \Big\}. \end{split}$$

Where $(^{\wedge})$ represent the matrics in the lattice space, T_r denotes the trace operations over the spin and lattice space, $T_0 = \frac{2\pi}{\Omega}$, $G^{r(a,<)}(\varepsilon,\varepsilon')$ is the retarded Green's function in the field applied region of the graphene nanoribbon, $f_a(\varepsilon)$ is the Fermi distribution, $\hat{T}_a(\varepsilon) = i \Big[\hat{\Sigma}_a(\varepsilon) - \hat{\Sigma}_a(\varepsilon) \Big]$, with $\Sigma_L^{r(a)}(\varepsilon)$ and $\Sigma_R^{r(a)}(\varepsilon)$ representing the retarded self energies from the left lead and both the right lead and the ac field free region of the graphene nanoribbon. The self energy has the form

$$\hat{\Sigma}_{\alpha}^{\square} = \hat{R}_{\alpha} egin{pmatrix} \hat{\Sigma}_{\alpha+}^{\square} & 0 \ 0 & \hat{\Sigma}_{\alpha-}^{\square} \end{pmatrix} \hat{R}_{\alpha}^{\dagger},$$

Where $\Box = r, a$, the rotational matrix \hat{R}_a is

$$\hat{R}_{\alpha} = \begin{pmatrix} \cos\frac{\theta_{\alpha}}{2} & -\sin\frac{\theta_{\alpha}}{2} \\ \sin\frac{\theta_{\alpha}}{2} & \cos\frac{\theta_{\alpha}}{2} \end{pmatrix}$$
with $\theta_{L} = 0$ and $\theta_{R} = \theta$.

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The eigen states of the isolated graphene nano ribbon with and without an ac gate voltage labeled as

$$\Phi(t) \text{ and } \Phi_0(t) \Phi_0(t) \text{ satisfy the relation } \Phi(t) = \Phi_0(t) \exp\left\{-\frac{1}{\hbar} \int_0^t V_{ac} \cos\left(\Omega t\right) d\tau\right\}. \text{ Thus }$$

the Green's function of the isolated ac field applied graphene nanoribbon $\hat{g}^r(\mathcal{E},\mathcal{E}')$ has the form

$$\hat{g}^{r}\left(\varepsilon_{r}+n\Omega,\varepsilon_{r}^{'}+m\Omega\right)=2\pi\delta\left(\varepsilon_{r}-\varepsilon_{r}^{'}\right)\overline{g}^{r}\left(\varepsilon_{r},n,m\right),$$

$$\overline{g}^{r}\left(\varepsilon_{r}, n, m\right) = \sum_{N} J_{n-N}\left(\frac{V_{ac}}{\Omega}\right) J_{m-N}\left(\frac{V_{ac}}{\Omega}\right) \hat{g}_{0}^{r}\left(\varepsilon_{r} + N\Omega\right)$$

In these equations $\mathcal{E}_r \in \left[-\frac{\Omega}{2}, \frac{\Omega}{2}\right], \hat{g}_0^r(\mathcal{E})$ is the corresponding Green's function of this graphene nanoribbon without the ac gate voltage, which can be obtained by the recursive method. Form the Dyson equation

$$\hat{G}^{r}(\varepsilon,\varepsilon') = \hat{g}^{r}(\varepsilon,\varepsilon') + \int \frac{d\varepsilon_{1}}{2\pi} \hat{g}^{r}(\varepsilon,\varepsilon^{1}) \hat{\Sigma}^{r}(\varepsilon_{1}) \hat{G}^{r}(\varepsilon_{1},\varepsilon')$$

one obtains

$$\hat{G}^{r}\left(\varepsilon_{r}+n\Omega,\varepsilon_{r}^{'}+m\Omega\right)=2\pi\delta\left(\varepsilon_{r}-\varepsilon_{r}^{'}\right)\overline{G}^{r}\left(\varepsilon_{r},n,m\right)$$

where $\overline{G}^r(\varepsilon_r n, m)$ is determined by

$$\overline{G}^{r}(\varepsilon_{r},n,m) = \overline{g}^{r}(\varepsilon_{r},n,m) + \sum_{n_{t}} \overline{g}^{r}(\varepsilon_{r},n,n_{1})$$

$$\times \hat{\Sigma}^r \left(\varepsilon_r + n_1 \Omega \right) \overline{G}^r \left(\varepsilon_r, n_1, m \right).$$

The pumping current with spin σ is given by

$$I_{pump}^{\sigma} = \frac{e}{h} \sum_{n} \int_{-\infty}^{E_{F}} d\varepsilon \left[T_{LR\sigma}^{n}(\varepsilon) - T_{RL\sigma}^{n}(\varepsilon) \right]$$

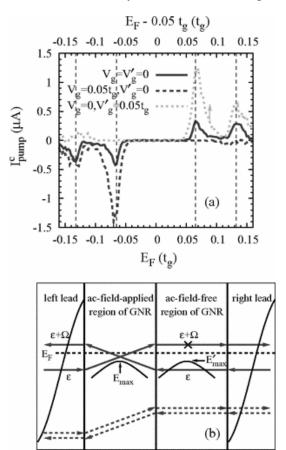
Where E_F denotes the Fermi energy. The charge and spin pumping currents are defined as $I^c_{pump}=I^+_{pump}+I^-_{pump}$ and $I^S_{pump}=I^+_{pump}-I^-_{pump}$ when H(t)=h(-t) one has $T^n_{LR\sigma}(\varepsilon)=T^-_{RL\sigma}(\varepsilon+n\Omega)$. Thus the photon assisted transmission with initial and final energies both below the Fermi energy is canceled by the corresponding one in the opposite direction and hence cannot contribute to the pump current, consequently I^σ_{pump} can be expressed as

$$I_{pump}^{\sigma} = \frac{e}{h} \sum_{n>0} \int_{E_{F}-n\Omega}^{E_{r}} d\varepsilon \Big[T_{LR\sigma}^{n}(\varepsilon) - T_{RL\sigma}^{n}(\varepsilon) \Big].$$

3. Results and Discussion

Graph (1)(a) shows the plot of charge pumping current I^c pump against the Fermi energy E_F for $V_g = V_g^{'} = 0$. The energy zero point was chosen to be the Dirac point of the pristine graphene nano ribbon. It was shown that peaks of negative or positive current appeared arount the energy maximums or minimums of the sub bands in graphene nano ribbon indicated by vertical dashed lines. These peaks originate from the pronounced symmetry breaking in the transmissions with energies around the sub bands edges in the ac field free region. Graph (1) (b) present the relevant sub

bands in the ac field applied and ac field free regions of the graphene nano ribbon, whose energy maximums are labeled by $E_{
m max}$ and $E_{
m max}^{'}$. The relevant sub bands in the left and right leads are shown in which propagating modes exist in the whole energy range. It is seen that the initial or final energies of these transmissions are lower or higher than $E_{
m max}$. Both transmissions are allowed in the ac field applied region due to the side bands effect $T^1_{LR\sigma}(\varepsilon)$ is forbidden in the sub band in the ac field free region owing to the lack of the propagating modes above $E_{\rm max}^{'}$. This makes the symmetry in these transmissions become notable. For $E_{F}-\Omega<arepsilon< E_{F}$, the single photon assisted transmissions can contribute to the pumping current. As results a large negative pumping current appeared when $E_{\rm max}^{'} - \Omega < E_{\rm F} < E_{\rm max}^{'} + \Omega$ and reached its peak value at the Fermi energy around $E_{\rm max}^{'}$ similar symmetry breaking can also be found in the multiphoton assisted transmissions. The presence of the peaks in the positive pumping current is based on the similar way. When the Fermi energy is around the minimum of one sub band in the ac field free region, this subband only contribute to the transmissions from left to right. This symmetry breaking leaded to the peak of positive current around this minimum. This is only valid in the long ribbons where the energy level spacing is much smaller than the photon energy and hence the energy spectrum can be considered quasi continuous but invalid in the short ribbons. Their energy level spacing is larger than photon energy, thus the behavior of the pumping current is dominated by the resonant tunneling.



Graph 1: Armchair GNR connected with nonmagnetic leads. $N_b=100$. (a) Charge pumping current as function of the Fermi energy E_F for $V_g=V_g^{'}=0$, $V_g=0.05t_g$, $V_g^{'}=0$ and $V_g=0$, $V_g^{'}=0.05t_g$.

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4. Conclusion

We have presented the quantum charge and spin pumping currents in the armchair graphene nano ribbon connected with naonmagnetic and ferromagnetic leads. The parameters were chosen as $N_W=41$, $N_{ac}=400$, $V_{ac}=\Omega=0.011t_g$ and $V_g=V_g^{'}=0$. Since the width satisfied $N_W=3M+2$ with M being an integer number, this armchair graphene nano ribbon was metallic. We found that in the case of nonmagnetic leads only part of the nano ribbon was subjected to ac gate voltage to break the left –right spatial symmetry. It was found that the peaks of the charge pumping current appeared at the Fermi energies around the sub band edges in the ac field free region of the nanoribbon. In ferromagnetic leads with the lead magnetizations being antiparallel to break the left-right symmetry. Similar peaks appeared in the spin pumping current when the Femi energies were around the edges of the majority spin sub bands in the ferromagnetic leads. The obtained results were compared with previously obtained results of theoretical and experimental values and found in good agreement.

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