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Nonperturbative Contributions to the Massive Propagator in a Class of Strong Interactions

Simon Davis*

Author's Affiliations:
Research Foundation of Southern California
8861 Villa La Jolla Drive #13595
La Jolla, CA 92037

Corresponding author:
Simon Davis
Research Foundation of Southern California
8861 Villa La Jolla Drive #13595
La Jolla, CA 92037

E-mail:
sbdavis@resfdnsca.org

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Abstract

The nonperturbative contribution to the correlation of gluon field strengths is computed within the field theory limit of a string diagrammatic expansion.

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1. Introduction

The mass of the η' meson may be found through nonperturbative effects in the theory of hadrons. It has been noted that the spontaneous symmetry breaking of the an octet axial symmetry would generate massless Goldstone bosons which then could generate the masses of the the lightest mesons. The η' meson similarly might result from the breaking of a singlet axial symmetry, although it would be considered to heavy for this effect, which could be induced by instantons and requires a sum of quark-antiquark annihilation diagrams in the $\frac{1}{N}$ expansion [1].

The variable $\left(\frac{d^2E}{d\theta^2}\right)_{\theta=0} = U(k)$ is the expectation value $\langle 0 | F\tilde{F}F\tilde{F} | 0 \rangle$, where $F_{\mu\nu}$ is the field strength of the gluon. Each term in the perturbative expansion would vanish in the zero-momentum limit when the masses of the quarks tend to zero. The value of this second derivative at $\theta = 0$ has been equated to the sum of terms from glueballs and mesons. Given the different dependences on N, the number of colours in the nonabelian gauge theory, the existence of a massive meson canceling the first term, would follow. Furthermore, evaluating the expectation values, the mass of this meson may be estimated. It is evident that the sum of the series can have a nontrivial value. When $\lim_{k\to 0} U(k) \neq 0$, a cancellation may not occur between the sums over the glueballs and the mesons. The mechanism for predicting the mass of a singlet meson then must be modified or replaced by another model.

New technique in the evaluation of the four gluon amplitudes will be developed in the evaluation of this expectation value. The momentum dependence shall be evaluated, and it will be proven in §2 that a non-zero value results in the zero-momentum limit. The effect on the prediction for the mass of the meson is discussed.

This role of this nonperturbative effect may be clarified by considering the diagrammatic expansion of the scattering matrix to be a field theory limit of a string amplitude. The four-point amplitude may be evaluated at each genus in a superstring theory. By the nonrenormalization theorems, this amplitude would be the first to be nonvanishing amongst the $N-$point functions. The momentum dependence of each term will indicate the uniform decrease of each of the terms in the zero-momentum limit. Nonperturbative effects also may be investigated in this formalism.

2. Nonperturbative Form of the Four Gluon Amplitude

The addition of the topological term to the action for quantum chromodynamics gives

$$I=\int d^4x\,\mathrm{Tr}\left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+\frac{\theta g^2}{16\pi^2N}F^{\mu\nu}\tilde{F}_{\mu\nu}\right] \qquad 2.1$$

and the expectation of the energy in the system is

$$\langle E\rangle=-\frac{1}{\beta}\frac{\partial\ln Z}{\partial\beta}=-\frac{1}{\beta}\frac{\partial}{\partial\beta}\ln\int\left[DA^\rho\right]\exp\left(-\int d^4x\,\mathrm{Tr}\left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+\frac{\theta g^2}{16\pi^2N}F^{\mu\nu}\tilde{F}_{\mu\nu}\right]\right) \qquad 2.2$$

in a space-time with the time coordinate identified in the imaginary direction with period β. Then, in the limit $\Delta t\to\infty$, where Δt is the length of the time interval, the partition function of the theory tends to $e^{-E\Delta t}$, and $\langle E\rangle=-\frac{1}{\beta}\frac{\partial\ln Z}{\partial\Delta t}$, without requiring the replacement of the action by the Hamiltonian [2]. Differentiating with respect to this time interval yields

$$\begin{aligned}\langle E\rangle&=\frac{1}{Z}\int\left[DA^\rho\right]\int d^3x\,\mathrm{Tr}\left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+\frac{\theta g^2}{16\pi^2N}F^{\mu\nu}\tilde{F}_{\mu\nu}\right]\\&\qquad e^{-\int d^4x\,\mathrm{Tr}\left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+\frac{\theta g^2}{16\pi^2N}F^{\mu\nu}\tilde{F}_{\mu\nu}\right]}\\&=\left\langle\int d^3x\,\mathrm{Tr}\left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+\frac{\theta g^2}{16\pi^2N}F^{\mu\nu}\tilde{F}_{\mu\nu}\right]\right\rangle\end{aligned} \qquad 2.3$$

Then the first derivative $\left(\frac{d^2E}{d\theta^2}\right)_{\theta=0}$ equals

$$\frac{d}{d\theta}\left\langle\int d^3x\,\mathrm{Tr}\left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+\frac{\theta g^2}{16\pi^2N}F^{\mu\nu}\tilde{F}_{\mu\nu}\right]\right\rangle_{\theta=0}=\frac{g^2}{16\pi^2N}\left\langle\int d^3x\,\mathrm{Tr}\left(F^{\mu\nu}\tilde{F}_{\mu\nu}(x)\right)\right\rangle_{\theta=0} \qquad 2.4$$

The derivative of this integral with respect to θ vanishes. The path integral factor generates another term through

$$\begin{aligned}&\frac{d}{d\theta}\frac{1}{Z}\int\left[DA^\rho\right]\frac{g^2}{16\pi^2N}\int d^3x\,\mathrm{Tr}\left[F^{\mu\nu}\tilde{F}_{\mu\nu}(x)\right]e^{-\int d^4x\,\mathrm{Tr}\left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+\frac{\theta g^2}{16\pi^2N}F^{\mu\nu}\tilde{F}_{\mu\nu}\right]}\\&=-\frac{1}{Z^2}\frac{dZ}{d\theta}\int\left[DA^\rho\right]\frac{g^2}{16\pi^2N}\int d^3x\,\mathrm{Tr}\left[F^{\mu\nu}\tilde{F}_{\mu\nu}(x)\right]e^{-\int d^4x\,\mathrm{Tr}\left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+\frac{\theta g^2}{16\pi^2N}F^{\mu\nu}\tilde{F}_{\mu\nu}\right]}\\&\qquad+\frac{g^2}{16\pi^2N}\frac{1}{Z}\int\left[DA^\rho\right]\int d^3x\,\mathrm{Tr}\left[F^{\mu\nu}\tilde{F}_{\mu\nu}(x)\right]\\&\qquad-\frac{g^2}{16\pi^2N}\int d^4x'\,F^{\mu\nu}\tilde{F}_{\mu\nu}(x')e^{-\int d^4x\,\mathrm{Tr}\left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+\frac{\theta g^2}{16\pi^2N}F^{\mu\nu}\tilde{F}_{\mu\nu}\right]}\end{aligned}$$

$$=\int dt\frac{g^4}{256\pi^4N^2}\left\langle \mathrm{Tr}\left[\int d^3xF^{\mu\nu}\tilde{F}_{\mu\nu}(x)\right]\right\rangle\left\langle \mathrm{Tr}\left[\int d^3x'F^{\mu\nu}\tilde{F}_{\mu\nu}(x')\right]\right\rangle$$
$$-\int dt\frac{g^4}{256\pi^4N^2}\left\langle \mathrm{Tr}\left[\int d^3xF^{\mu\nu}\tilde{F}_{\mu\nu}(x)\right]\mathrm{Tr}\left[\int d^3x'F^{\mu\nu}\tilde{F}_{\mu\nu}(x')\right]\right\rangle \qquad 2.5$$

Then

$$=\frac{g^4}{256\pi^4N^2}\int dt\left\{\left\langle \mathrm{Tr}\left[\int d^3xF^{\mu\nu}\tilde{F}_{\mu\nu}(x)\right]\right\rangle\left\langle \mathrm{Tr}\left[\int d^3x'F^{\mu\nu}\tilde{F}_{\mu\nu}(x')\right]\right\rangle\right.$$
$$\left.-\left\langle \mathrm{Tr}\left[\int d^3xF^{\mu\nu}\tilde{F}_{\mu\nu}(x)\right]\mathrm{Tr}\left[\int d^3x'F^{\mu\nu}\tilde{F}_{\mu\nu}(x')\right]\right\rangle\right\} \qquad 2.6$$

Differentiation with respect to t yields

$$\left(\frac{\partial^3E}{\partial\theta^2\partial t}\right)_{\theta=0}=\frac{g^4}{256\pi^4N^2}\left\{\left[\int d^3x\left\langle T\left(F\tilde{F}(x)\right)\right\rangle\right]^2-\int d^3xd^3x'\left\langle T\left(F\tilde{F}(x)F\tilde{F}(x')\right)\right\rangle\right\} \qquad 2.7$$

Consequently there is a product of two-point functions and a four-point function.

The values of x and x' will be defined on a hypersurface of constant time. The limit of vanishing momentum of the Fourier transform is

$$\frac{g^4}{256\pi^4N^2}\lim_{k,k'\to 0}\int d^3x\int d^3x'e^{ik.x}e^{ik'.x'}\left[\left\langle T\left(F\tilde{F}\right)(x)\right\rangle\left\langle T\left(F\tilde{F}\right)(x')\right\rangle\right.$$
$$\left.-\left\langle T\left(F\tilde{F}\right)(x)T\left(F\tilde{F}\right)(x')\right\rangle\right] \qquad 2.8$$

$$=\frac{g^4}{256\pi^4N^2}\lim_{k,k'\to 0}\mathrm{U}(k,k')$$
$$\mathrm{U}(k,k')=\mathrm{U}_1(k)\mathrm{U}_1(k')-\mathrm{U}_2(k,k')$$
$$\mathrm{U}_1(k)=\int d^3xe^{ik.x}\left\langle T\left(F\tilde{F}(x)\right)\right\rangle \qquad 2.9$$
$$\mathrm{U}_2(k,k')=\int d^3x\int d^3x'e^{ik.x}e^{ik'.x'}\left\langle T\left(F\tilde{F}(x)\right)T\left(F\tilde{F}(x')\right)\right\rangle.$$

Integration over k' gives

$$\int dk'\mathrm{U}(k,k')=\mathrm{U}_1(k)\int dk'\int d^3x'e^{ik'.x'}\left\langle T\left(F\tilde{F}(x')\right)\right\rangle$$
$$-\int dk'\int d^3x\int d^3x'e^{ik.x}e^{ik'.x'}\left\langle T\left(F\tilde{F}(x)\right)T\left(F\tilde{F}\right)(x')\right\rangle$$
$$=(2\pi)^3\mathrm{U}_1(k)e^{ik_0't}\left\langle T\left(F\tilde{F}\right)(0)\right\rangle-(2\pi)^3e^{ik_0't}\int d^3xe^{ik.x}\left\langle T\left(F\tilde{F}(x)\right)T\left(F\tilde{F}\right)(0)\right\rangle$$
$$=(2\pi)^3\mathrm{U}_1(k)e^{ik_0't}\left\langle T\left(F\tilde{F}\right)(0)\right\rangle-(2\pi)^3e^{ik_0't}\mathrm{U}_2(k) \qquad 2.1$$

Setting $k_0'=0$ and integrating over time gives

$$(2\pi)^3\left\langle T\left(F\tilde{F}\right)(0)\right\rangle\int d^4xe^{ik.x}\left\langle T\left(F\tilde{F}\right)(x)\right\rangle-(2\pi)^3\int d^4xe^{ik.x}\left\langle T\left(F\tilde{F}\right)(x)T\left(F\tilde{F}\right)(0)\right\rangle \qquad 2.11$$

The perturbative expansion of the second integral in the large N limit has been given

$$U(k)=\int d^4xe^{ik.x}\left\langle T\left(F\tilde{F}\right)(x)T\left(F\tilde{F}\right)(0)\right\rangle=N^2\left[a_1k^2+a_2g^2k^2\left(\ln\left(k^2\right)\right)+a_3g^4k^2\left(\ln\left(k^2\right)\right)^2+\cdots\right] \qquad 2.12$$

The limit of vanishing momentum of each term would be zero, even though the sum could be non-zero.

The value of the sum has occurred in the estimation of the masses of certain mesons because the four-point function may be factorized in the $\frac{1}{N}$ expansion such that $U(k)=\sum_{glueballs}\frac{N^2a_n^2}{k^2-M_n^2}+\sum_{mesons}\frac{N^2c_n^2}{k^2-m_n^2}$ where M_n is the mass of n^{th} glueball and m_n is the mass of the n^{th} meson. Consequently, the limit of k tending to zero would require cancellation of the sums over the two sets of states for the vanishing of $U(k)$. If $U(k)$ is nonvanishing, however, a cancellation of the two sums is not necessary, and the masses of any mesons generating a gauge-invariant term would not satisfy the condition for cancellation.

A nonperturbative shift in the mass in the propagator has been noted in a model of pions, σ fields and a nonabelian gauge potential. The momentum transform of the quadratic pion term in the effective Lagrangian had been demonstrated to be equal to

$\frac{N}{8\pi}k^2F\left(k^2\right)$, where

$$F\left(k^2\right)=\int_0^1 d\alpha\frac{1}{M^2-\left(\alpha-\alpha^2\right)k^2}=-\frac{1}{k\sqrt{\frac{k^2}{4}-M^2}}\left[\log\left(1+\frac{\frac{1}{2}k}{\sqrt{\frac{k^2}{4}-M^2}}\right)-\log\left(1-\frac{\frac{1}{2}k}{\sqrt{\frac{k^2}{4}-M^2}}\right)\right]$$ and

$$\lim_{k\to 0}\frac{N}{8\pi}k^2F\left(k^2\right)=iM\lim_{k\to 0}k\left[\log\left(1+\frac{\frac{1}{2}k}{\sqrt{\frac{k^2}{4}-M^2}}\right)-\log\left(1-\frac{\frac{1}{2}k}{\sqrt{\frac{k^2}{4}-M^2}}\right)\right]=0,$$

and after the mixing with the gauge potential with

$N\sqrt{2}\frac{M}{4\pi}F\left(k^2\right)\varepsilon_{\mu\nu}$ for $A^\mu\pi$ and $\frac{N}{4\pi}F\left(k^2\right)\left(-g_{\mu\nu}k^2+k_\mu k_\nu\right)$ for $A^\mu A_\mu$. The diagonalization of the quadratic momentum space matrix yields $\langle\pi(k)\pi(-k)\rangle=\frac{8\pi}{N}\frac{1}{k^2-4M^2}\frac{1}{F\left(k^2\right)}$ such that the mass in the π propagator is shifted to $2M$ [3].

It has been demonstrated that the four gluon amplitude may be evaluated nonperturbatively through the representation in terms of actions on anti-de Sitter space. It is found that $\mathrm{A}=\exp\left[(i\mathrm{S})_{\mathrm{div}}+\frac{\sqrt{\lambda}}{8\pi}\left(\log\frac{s}{t}\right)^2+\tilde{C}\right]$, where $\mathrm{S}_{\mathrm{div}}$ is a divergent part of the action, s and t are the Mandelstam variables and $\tilde{C}=\frac{\sqrt{\lambda}}{4\pi}\left(\frac{\pi^2}{3}+2\log 2-\left(\log 2\right)^2\right)$ [4]. The corresponding formula for $U(k)$ would have a momentum-dependent term

$$s\exp\left(\frac{\sqrt{\lambda}}{8\pi}\left(\log\frac{s}{t}\right)^2\right)=s\left(e^{\left(\log\frac{s}{t}\right)^2}\right)^{\frac{\sqrt{\lambda}}{8\pi}} \qquad 2.13$$

Since $s\left(\frac{s}{t}\right)^{\log\left(\frac{s}{t}\right)}=t\left(\frac{s}{t}\right)^{\log\left(\frac{s}{t}\right)+1}$, it follows from $\lim_{x\to 0}x^{\log x}$. There are two different values depending on the form of the logarithm of this expression. For example, $\lim_{x\to 0}\log\left(x^{\log x}\right)=\lim_{x\to 0}\left(\log x\right)^2=\infty$. L'Hopital's rule may be used to establish that

$\lim_{x\to 1^+}\left(\frac{1}{x-1}\right)^{\ln x}=\lim_{x\to 1^+}\exp\left(\frac{-\ln(x-1)}{\frac{1}{\ln x}}\right)=\lim_{x\to 1^+}\exp\left(\frac{-x(\ln x)^2}{x-1}\right)=1$. By contrast,

$\exp\left(\lim_{x\to 0}\frac{\ln x}{\frac{1}{\ln x}}\right)=\exp\left(\lim_{x\to 0}\frac{\frac{1}{x}}{-\frac{1}{(\ln x)^2}\frac{1}{x}}\right)=\exp\left(\lim_{x\to 0}-\left(\ln x\right)^2\right)$. Based on these considerations, the first limit may be chosen to give an infinite value.

A regularization of this infinity would yield a non-zero nonperturbative value. The cancellation of the two sums over states is not required to be valid, and the mechanism for determining the mass of the meson transforming as a singlet state would be affected. Given that there are no other contributions from the mesons, the zero-momentum limit of the equality would be

$$U_0(k) - \sum_{mesons} \frac{Nc_n^2}{m_n^2} = U(0)_{\text{reg.}} \qquad 2.14$$

The mass of the meson state transforming under the trivial representation will be denoted by m_s. Then

$$\frac{Nc_s^2}{m_s^2} = U_0(0) - U(0)_{\text{reg.}} \qquad 2.15$$

Given that

$$\sqrt{N_f} m_s^2 f_s = \langle 0 | \partial_\mu J^\mu | s \rangle = \frac{g^2}{16\pi^2 N} 2N_f \langle 0 | F\tilde{F} | \rangle = \frac{g^2}{16\pi^2 N} 2N_f \sqrt{N} c_s, \qquad 2.16$$

with f_s being the equivalent of the Goldberger-Treiman constant for the singlet state $|s\rangle$, it follows that

$$N_f m_s^2 f_s^2 = \frac{g^4}{64\pi^4 N^2} N_f^2 \left(U_0(0) - U(0)_{\text{reg.}} \right). \qquad 2.17$$

By (2.11),

$$\begin{aligned} &-\frac{1}{(2\pi)^3} \lim_{k\to 0} \int dt \lim_{k'\to 0} \int dk'' e^{ik''.x} \delta(k''-k) \int dk' e^{ik'.x'} \left(\frac{\partial^3 E}{\partial\theta^2 \partial t} \right)_{\theta=0}^{no\ quarks} \\ &+ \langle T(F\tilde{F})(0) \rangle \int d^4x e^{ik.x} \langle T(F\tilde{F})(x) \rangle \qquad 2.18 \\ &= \frac{g^4}{256\pi^4 N^2} U_0(0) = \frac{1}{4N_f} m_s^2 f_s^2 + U(0)_{\text{reg.}} \end{aligned}$$

The relation differs from that derived without a nonvanishing sum of terms for $U(k)$ in the limit of vanishing momentum. This formula must be considered with a value of f_s rather than f_π. It has been found that these two constants are not equal [5].

Nevertheless, the method is verified by the contribution of additional terms in the Ward identities. The dependence on N can be restored consistently only if the summation of intermediate states in the quark diagrams includes a single state yielding a factor $i\sqrt{\frac{N_f}{N}} \lambda_s q_\mu$, and summing the perturbation series yields a shift in the mass in the integral of the product of divergences of currents, such that, for

$$\frac{N_f}{N_c} \lambda_\eta^2 \gg m_{NS}^2$$

where m_{NS} is the octet mass, a formula for m_s follows [6]. It does not yield the mass m_s directly, which may be found from the diagonalization of a mass matrix related to current algebra [7].

3. String Theory Expansion and the Nonperturbative Propagator

The four gluon amplitude is the field theory limit of string amplitude. Six gluon amplitudes [8] have been evaluated from open superstring theory. It has been established that the couplings for $t_8 tr(F^4)$ and $t_8 tr(F^2)^2$ terms are fixed at one loop, and although there are divergences upon coincidences of the vertex operators which reduce the number of derivatives of the terms in the effective action higher derivatives are required from second order, and it is necessary to include planar surfaces with handles attached to represent the coupling of closed strings [9]. The presence of derivatives at higher order does not preclude a non-zero four gluon amplitude, and the degree of divergence with the term $g_s^{L-1} \partial^2 tr t_8 F^4$ would be $(D-4)L-6$ and ultraviolet finite for $D < 4 + \frac{6}{L}$, which a conventional result for super-Yang-Mills theory [10], while the degree of divergence of

$g_s^{L-1}\partial^{2\frac{L}{2}}t_8\left(tr\left(F^2\right)\right)^2$ is $(D-4)L-2\frac{L}{2}-4$ with an ultraviolet finite series for $D<4+\left(4+2\frac{L}{2}\right)\Big/L$ [8]. When the topological action $\frac{\theta g^2}{16\pi^2 Nn}\int d^4xF\tilde{F}$ is included, the four-point function is $\varepsilon_4\varepsilon_4\left\langle tr\left(F^4\right)\right\rangle$. The combinations $\varepsilon_4\varepsilon_4 tr\left(F^4\right)$ and $\varepsilon_4\varepsilon_4\left(tr\left(F^2\right)\right)^2$ would be similar to H^2W^3 terms that have not been derived from a superfield, and an additional topological term or a modification of the connection in the covariant derivatives may be necessary [11].

The sum of the terms in $U(k)$ therefore would remain unchanged even if there are nonrenormalization theorems for the couplings of $tr\left(F^4\right)$ terms in the effective action. Since each of the terms in the series expansion of $U(k)$ vanishes in the limit of zero momentum, and the sum is non-zero, the momentum dependence in the series and the infinite-genus amplitude could be considered in connection with its regularized value.

The contribution of infinite-genus surfaces to the scattering amplitudes has been determined and found to have only physical divergences. The four-point amplitude of bosonic strings in the ground state has been calculated at infinite order for a class of surfaces, which are conformally equivalent to spheres with an infinite number of handles, which can be uniformized by an extension of a Schottky group. The rate of the decrease of the size of the handles must be chosen such that the sums of the squares of the radii of the corresponding isometric circles in the covering surface converge [12], and the finite series representation of the Green function in the integral of the four-point correlation function generates an amplitude with poles at the string resonances. The evaluation of superstring amplitudes at infinite genus would follow a similar method. These integrals have been calculated for several vertex operators [13]. With a chiral vertex operator $V=\int dzd\bar{z}\omega(z)\bar{\omega}(z)d\bar{\theta}\varepsilon_{IJ\mu}\Lambda^I\Lambda^J\bar{D}X^\mu e^{ip.X}$, where $\omega(z)$ is a holomorphic one-form, X^μ is a scalar superfield, Λ^I are 32 left moving scalar gauge fermions, $\varepsilon_{IJ\mu}=f_If_J\bar{\varepsilon}_\mu$ is polarization tensor, $p^\mu\bar{\varepsilon}^\mu=f_I\tilde{f}^I=p^2=0$, the scattering amplitude of four gauge bosons in heterotic string theory has been given as

$$\begin{aligned}A_{4;g}^{gauge\ bos.}&=\int_{s\bar{M}_g}d\mu_g\int_{\Sigma_g}\prod_{k=1}^{4}dz_id\bar{z}_i\omega(z_i)\bar{\omega}(z_i)d\bar{\theta}_id\bar{\sigma}_id\eta_id\tilde{\eta}_i\\&\prod_{i<j}^{4}\exp\left[-iP_iP_j\frac{\Delta(1,2)}{4}-E_iE_j\frac{\Delta_F(z_1,z_2)}{4}\right]\\iP^\mu&=ip^\mu+\bar{\sigma}\,\bar{\varepsilon}^\mu\bar{D}\\E^I&=\eta f^I+\tilde{\eta}\tilde{f}^I\end{aligned}\qquad 3.1$$

where $\bar{\sigma},\eta$ and $\tilde{\eta}$ are Grassmann variables [13]. The exponential form of the integrand will be preserved for surfaces of finite genus. No additional momentum dependence is introduced at infinite genus. Consequently, the nonperturbative four gluon amplitude will be determined by the by the field theory limit of the summation over the genus.

The nonperturbative form of the quark propagator includes a term from the gluon condensate $S_{non\ pert.}=N_q\left(p^2\right)\left(-\frac{i\delta^{ab}}{p+M\left(p^2\right)}\right)$ and

$$N_q\left(p^2\right)=\left(1+\frac{1}{864}g^2\frac{1}{\left(p^2+m^2\right)^3}\left(2p^2+3m^2\right)\left\langle 0|F^{\mu\nu}F_{\mu\nu}|0\right\rangle\right)^{-1}$$

and

$$M\left(p^2\right)=\left(m+\frac{4}{9}g^2\frac{1}{p^2}\left\langle 0|\bar{\psi}\psi|0\right\rangle+\frac{1}{864}g^2\frac{1}{\left(p^2+m^2\right)^3}\left(3mp^2+4m^3\right)\left\langle 0|F^{\mu\nu}F_{\mu\nu}|0\right\rangle\right)N_q\left(p^2\right)$$

with $\langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle = 1.3\langle 0|\bar{s}s|0\rangle = -(0.25GeV)^3$, $\langle 0|\bar{c}c|0\rangle = \langle 0|\bar{b}b|0\rangle = \langle 0|\bar{t}t|0\rangle = 0$ and $\left\langle 0|\frac{\alpha_s}{\pi}F^{\mu\nu}F_{\mu\nu}|0\right\rangle = 0.012GeV^4$ [14].

The range of validity is $\sqrt{-p^2} \geq 0.8GeV$, and if p^2 is set equal to $-1GeV^2$, and the squared average mass of the lightest quarks is $0.2GeV^2$,

$$\Delta M(p^2) \equiv M(p^2) - m$$
$$= \frac{4}{9}\frac{4\pi}{-(1GeV)^2}\left[-2(0.25GeV)^3 - 0.76923(0.25GeV)^3\right]$$
$$+\frac{4\pi}{864}\frac{\left(4(0.089442)GeV^3 + 3(0.44721GeV)(-1GeV^2)\right)(0.03769911184GeV^4)}{\left((-1GeV^2)+(0.2GeV^2)\right)^3} \quad 3.2$$
$$-\frac{4\pi}{864}\frac{\left(3(0.2GeV^2 + 2(-1GeV^2))\right)(0.03769911184GeV^4)}{\left((-1GeV^2)+(0.2GeV^2)\right)^3}M_0$$
$$= 0.24079196689GeV$$

The effect of the $\bar{q}q$ annihilation loops would be the sum of the two mass shifts equaling $0.48158394GeV$, which is sufficient to yield the highest mass in the pseudoscalar multiplet.

With $m_\mu = 0.305GeV, m_d = 0.308GeV$, and $m_s = 0.487GeV$ and $A = 0.06GeV^3$, the pseudoscalar multiplet formula for the η' meson,

$$m_{\eta'}^{pseudoscalar} + \Delta M(p^2)_{q\bar{q}} = \frac{1}{3}\left(2m_u - \frac{3A}{4m_u^2}\right) + \frac{1}{3}\left(2m_d - \frac{3A}{4m_d^2}\right) + \frac{1}{3}\left(m_s - \frac{3A}{4m_s^2}\right) + \Delta M(p^2)_{q\bar{q}}$$
$$= 0.3494976GeV + 0.48158394GeV$$
$$= 0.83108154GeV \quad 3.3$$

which can be combined with the octet mass in quadrature to give a nonperturbative value of the mass of the η' meson [15]. Similarly, the nonperturbative gluon propagator would occur in the evaluation of the regularized value of $U(k)$. Derivation of the mass of the η' meson. The remaining mass may be evaluated in another model of the structure of the η' meson. The relation between $U_0(0)$ and the sums over glueball states and meson states is suggestive of a dual representation of the η' state in terms of a glueball state or a bound state of a quark and anti-quark, which can have angular momentum one, with a perturbative mass given by a spin coupling formula that is closer to the experimental value [16]-[21]. With an extra nonperturbative contribution from the gluon self-energy diagrams, with a matching condition for the propagators at a given energy scale, the entire mass may be theoretically predicted.

4. Conclusion

The shift in the mass of the lightest mesons through quark-antiquark annihilation diagrams is feasible for singlet states. Together with the occurrence of Goldstone bosons through the introduction of instantons in quantum chromodynamics, a relation between the mass of the meson and a functional of the second derivative of the energy with respect to the theta angle results. This functional includes another term consisting of the product of two-point functions $\langle tr(F\tilde{F})\rangle$ after differentiating with respect to the time, the limit of vanishing momenta and integration with respect to time. The series representing the Fourier transform of the four-point function $\langle tr(F\tilde{F})tr(F\tilde{F})\rangle$ is an infinite sum consisting of terms that may be deduced from the nonperturbative four gluon amplitude. The form of this amplitude has been found through the duality between $N=4$ super-Yang-Mills theory and string theory on $(AdS)_5 \times S^5$. The Fourier transform of the four-point function yields two powers of the momenta through integration by parts and the function $U(k)$ can have the form se $se^{\frac{\sqrt{\lambda}}{8\pi}\log\left(\frac{s}{t}\right)^2}$. Given that the limit for vanishing momentum is infinite and the regularized value is non-zero, an extra term would result in the relation for the mass of the meson transforming under the trivial representation of the nonabelian gauge group.

The four gluon amplitude also may be derived from the field theory limit of supersting theory. Nonrenormalization theorems will ensure that the corrections to the couplings with $\left(tr\left(F^2\right)\right)^2$ and $tr\left(F^4\right)$ in the string effective action do not occur beyond one loop, while derivative terms begin at higher orders in perturbation theory. Nevertheless, the gluon amplitudes in the limit of vanishing string size are non-zero to all orders and it follows from the ultraviolet finiteness in the $N=4$ super-Yang-Mills theory that the form of the summed amplitude will not be altered by the renormalization procedure. The string perturbation series may be examined with regard to momentum dependence and it can be verified it is consistent with the form of the function $U(k)$. Nonperturbative effects have been derived from the summation of the series. The formula for the four gluon amplitude through a classical action of a configuration representing the interaction of four vertex operators in anti-de Sitter space is based on a conventional transformation from $N=4$ super-Yang-Mills variables to string theory on $(AdS)_5 \times S^5$ without introducing an instanton state. The nonperturbative quark propagator in quantum chromodynamics is sufficient to establish the shift in the mass of the meson through quark-antiquark annihilation diagrams. It is evident that both gluon condensates and quark-antiquark pairs comprise the model of this meson.

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