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Shielded Composite Dielectric Spherical Shell Resonator with a Concentric Metal Sphere at the Centre

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Abstract

Shielded Composite Dielectric Spherical Shell Resonator with a Concentric Metal Sphere at the Centre the characteristic equation for the TE and TM modes has been derived. Effect of the Presence of the inner metal sphere on the resonant frequency and quality factor is studied. In this paper it is clear that by the concentric metal sphere at the centre a dielectric spherical resonator, is a more effective controlling parameter of the resonant frequency.

Keywords: Spherical shell resonator, Shielded Composite Dielectric

INTRODUCTION

Resonators are important components in microwave communication system. These create filter and select frequencies in oscillator's amplifiers, and turners. Fields inside a resonator store energy at the resonant frequency where equal storage of electric and magnetic energies occurs. Since the reactance is proportional to the difference between electric and magnetic field energy storage the input impedance at resonance is purely real. Quality factor Q is on important figure of merit for a resonant circuit. The Q factor relates a resonant circuit's capacity for electromagnetic energy storage with its energy dissipation through heat. Microwave Q factors can be as high as 10,000. At lower frequencies the Q factor is usually between 50 and 500. Resonator band width is inversely proportional to Q factor. Thus high Q factor resonators have narrow band widths. Shielded composite dielectric spherical shell resonator with a concentric metal sphere at centre expression for the field components and characteristic equation and the quality factors has been derived.

THEORY

Shielded composite dielectric spherical shell resonator with a metallic sphere of radius *a* at the centre and enclosed in a metallic spherical shell of larger radius c studied presently is shown in figure (1).

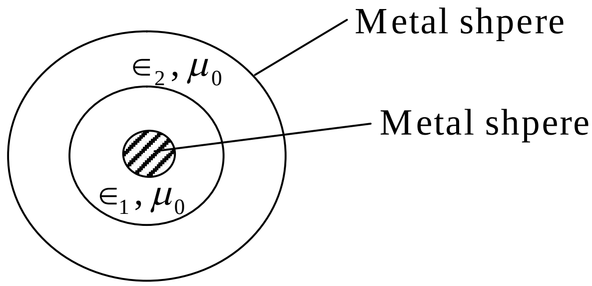

Fig. 1:

An inner perfectly conducting metallic sphere of radius *b* and permittivity ε_1 which in turn is embedded in another concentric dielectric sphere of radius c and permittivity ε_n which is shielded by a perfectly conducting metal cave of radius c. The materials of the two dielectric spheres are non magnetic i.e. $\mu_1 = \mu_0 = \mu_2$.

Following the procedure outlined in preceding work the solution of the radial part of the wave equation in the two dielectric spheres is given by

$$X(r) = AJ_{n+\frac{1}{2}}(K_1 r) + BY_{n+\frac{1}{2}}(K_1 r) \text{ for } a \le r \le b \tag{1}$$

and

$$X(r) = CJ_{n+\frac{1}{2}}(K_1 r) + DY_{n+\frac{1}{2}}(K_1 r) \text{ for } b \le r \le c \tag{2}$$

Hence, the field expressions for the TE_{nml} and TM_{nml} modes both contain $J_{n+\frac{1}{2}}(K_1 r)$ and $Y_{n+\frac{1}{2}}(K_1 r)$ where $i=1$ for $a \le r \le b$ and $i=2$ for $b \le r \le c$.

With the help of field expressions for TE and TM modes region $a \le r \le b,\ b \le r \le c$ derive the characteristic equations for the TE_{nml} modes boundary conditions that the tangential components of E (E_θ and E_ϕ) must vanish for all θ and ϕ on metal surfaces i.e. at $r = a$ and $r = c$ and the tangential components of $\vec{E}$ and $\vec{H}$ are continuous at $r = b$ are used.

Applying these conditions we get the characteristic equation for the TE_{nml} modes is

$$\begin{aligned}
& y_{n+\frac{1}{2}}(K_1 a)\, JJ_{n+\frac{1}{2}}(K_1 b)\left\{ J_{n+\frac{1}{2}}(K_2 b)\, y_{n+\frac{1}{2}}(K_2 c) - J_{n+\frac{1}{2}}(K_2 c)\, y_{n+\frac{1}{2}}(K_2, b) \right\} \\
& + J_{n+\frac{1}{2}}(K_1 a)\, YY_{n+\frac{1}{2}}(K_1 b)\left\{ J_{n+\frac{1}{2}}(K_2 c)\, Y_{n+\frac{1}{2}}(K_2 b) - J_{n+\frac{1}{2}}(K_2 b)\, Y_{n+\frac{1}{2}}(K_2 c) \right\} \\
& + J_{n+\frac{1}{2}}(K_1 b)\, y_{n+\frac{1}{2}}(K_1 a)\left\{ J_{n+\frac{1}{2}}(K_2 c)\, yy_{n+\frac{1}{2}}(K_2 b) - y_{n+\frac{1}{2}}(K_2 c)\, JJ_{n+\frac{1}{2}}(K_2 b) \right\} \\
& + J_{n+\frac{1}{2}}(K_1 a)\, y_{n+\frac{1}{2}}(K_1 b)\left\{ y_{n+\frac{1}{2}}(K_2 c)\, JJ_{n+\frac{1}{2}}(K_2 b) \right. \\
& \qquad \left. - J_{n+\frac{1}{2}}(K_2 c)\, yy_{n+\frac{1}{2}}(K_2 c)\, yy_{n+\frac{1}{2}}(K_2 b) \right\}
\end{aligned} \tag{3}$$

and for the TE_{nml} modes as

$$yy_{n+\frac{1}{2}}(K_1a)\,yy_{n+\frac{1}{2}}(K_2c)\left\{E_2^r J_{n+\frac{1}{2}}(K_2b)\,JJ_{n+\frac{1}{2}}(K_1b) \in_1^r J_{n+\frac{1}{2}}(K_1b)\,JJ_{n+\frac{1}{2}}(K_2b)\right\}$$

$$+JJ_{n+\frac{1}{2}}(K_1a)\,JJ_{n+\frac{1}{2}}(K_2c)\left\{\in_2^r y_{n+\frac{1}{2}}(K_2b)\,yy_{n+\frac{1}{2}}(K_1b)-\in_1^r y_{n+\frac{1}{2}}(K_1b)\,yy_{n+\frac{1}{2}}(K_2b)\right\}$$

$$+JJ_{n+\frac{1}{2}}(K_1a)\,yy_{n+\frac{1}{2}}(K_2c)\left\{\in_1^r y_{n+\frac{1}{2}}(K_1b)\,JJ_{n+\frac{1}{2}}(K_2b)-\in_2^r J_{n+\frac{1}{2}}(K_2b)\,yy_{n+\frac{1}{2}}(K_1b)\right\}$$

$$+JJ_{n+\frac{1}{2}}(K_2c)\,yy_{n+\frac{1}{2}}(K_1a)\left\{\in_1^r J_{n+\frac{1}{2}}(K_1b)\,yy_{n+\frac{1}{2}}(K_2b)\right.$$

$$\left.-\in_2^r y_{n+\frac{1}{2}}(K_2b)\,JJ_{n+\frac{1}{2}}(K_1b)\right\}=0 \qquad (4)$$

Energy, Losses and quality factors: The energy W stored in the dielectric sphere is given by

$$W=\frac{1}{2}\in\iiint_V \vec{E}\cdot\vec{E}^*dV=\frac{1}{2}\mu\iiint_V \vec{H}\cdot\vec{H}^*dV \qquad (5)$$

is the sum of the energy w_1, stored inside the inner dielectric sphere $(0\le r\le a)$ and the energy w_2 stored within the outer dielectric spherical shell $(a\le r\le b)$.

RESULT AND DISCUSSION

Computations of the Resonant Frequencies and quality factors

The characteristic equations of shielded composite dielectric spherical shell resonator with concentric metal sphere at the centre. For the TE and TM modes, numerically use has been made of spherical Bessel functions $f_n(x)$ and $y_n(x)$. In this case modes have five roots $(l=1-5)$ for each of equations (3) and (4) have been determined and the resonant frequencies have been calculated for the TE_{nml} and TM_{nml} modes $(n=1-3, l=1-5)$ using $E_1^r=4.0$ and $E_2^r=3.78$, $a=0.1-0.9\,\mu m$, $b=1.0-9.5\,\mu m$ and $c=1.5-10.0\,\mu m$. In this case the value of a is limited by the value of b (radius of the inner dielectric sphere). Resonant frequency has been computed as a function of one of the three radii a, b and c keeping the other two constant α.

Table 1 (a-c) and 2(a-c) show variation of the Resonant frequency with the radius a of the concentric metallic sphere for the TE_{nml} and TM_{nml} modes, it is observed that when the concentric metallic sphere radius a is increased from $0.1\mu m$ to $0.9\,\mu m$ with $b=1.0\,\mu m$ and $c=1.5\,\mu m$ with resonant frequency increases monotonically. However, increase is slow due to the fact that the inner radius can not exceed $1\mu m$ (the value of b) and the outer radius c is $1.5\,\mu m$.

The Table 3 (a-c) and table 4 (a-c) shows the variation of the resonant frequency with b for TE_{nml} and TM_{nml} modes ($n=1-3$). By changing the radius b of the inner dielectric sphere no appreciable change in frequency is observed due to small difference in the permittivities of the materials of the inner and the outer dielectric spheres. Table 5 (a-c) and table 6 (a-c) show the variation of the resonant frequency with c the radius of the outer dielectric sphere for the TE_{nml} and TM_{nml} modes ($n=1-3$). As expected on increasing the radius c of the outer dielectric sphere the resonant frequency decreases monotonically. Table 6 (a) and table 6 (b) show variation of the quality factor with the radius c for the TE_{10l} and TM_{10l} modes. From the analysis of data value on increasing the outer radius the quality factor increases. Effect of the radius (a) of the inner metal surface on the quality factor has also been studied. By increasing the size of the inner concentric metal sphere the quality factor is reduced significantly, it is found that when the radius a of the inner metal sphere is increased from $0.1\mu m$ to $0.5\mu m$ with $b=1.0\,\mu m,\ c=1.5\,\mu m$ the quality factor drops from 138 to 28 for the TE_{iol} mode.

Table 1 (a): Variation of Resonant Frequency $\left(v^{+}_{1ml}\right)$ with a for TE_{10l} Modes

l a (μm)	1	2	3	4	5
0.1	0.721	1.246	1.766	2.286	2.816
0.3	0.749	1.351	1.952	2.580	3.198
0.5	0.840	1.565	2.314	3.064	3.823
0.7	1.007	1.933	2.883	3.828	4.778
0.9	1.317	2.582	3.847	5.126	6.396

+ Frequencies are given by the numbers under the columns multiplied by $10^{14}\,Hz$. The value of $b=1.0\,\mu m,\ c=1.5\,\mu m,\ \in^r_1=4.0$ and $\in^r_2=3.78$.

Table 1 (b): Variation of Resonant Frequency $\left(v^{+}_{1ml}\right)$ with a for TE_{30l}

l a (μm)	1	2	3	4	5
0.1	0.926	1.460	1.976	2.496	3.007
0.3	0.934	1.499	2.074	2.673	3.276
0.5	0.980	1.656	2.380	3.117	3.856
0.7	1.107	1.986	2.921	3.856	4.801
0.9	1.317	2.615	3.875	5.140	6.410

+ Frequencies are given by the numbers under the columns multiplied by $10^{14}\,Hz$. The value of $b=1.0\,\mu m,\ c=1.5\,\mu m,\ \in^r_1=4.0$ and $\in^r_2=3.78$.

Table 1 (c): Variation of Resonant Frequency $\left(v^{+}_{3ml}\right)$ with a for TE_{10l} modes

l a (μm)	1	2	3	4	5
0.1	0.126	1.675	2.195	2.720	3.231
0.3	0.130	1.685	2.238	2.810	3.389
0.5	0.146	1.780	2.470	3.188	3.923
0.7	1.327	2.310	3.360	4.443	5.522
0.9	1.470	2.663	3.909	5.169	6.434

+ Frequencies are given by the numbers under the columns multiplied by $10^{14}\,Hz$. The value of $b=1.0\,\mu m,\ c=1.5\,\mu m,\ \in^r_1=4.0$ and $\in^r_2=3.78$.

Table 2 (a): Variation of Resonant Frequency $\left(v_{1ml}^{+}\right)$ with a for TM_{10l} Modes

l a (μm)	1	2	3	4	5
0.1	0.267	0.568	0.816	1.055	1.279
0.3	0.277	0.592	0.864	1.084	1.327
0.5	0.310	0.635	0.879	1.127	1.421
0.7	0.363	0.721	0.927	1.412	1.761
0.9	0.374	0.821	1.420	1.921	2.027

+ Frequencies are given by the numbers under the columns multiplied by $10^{14}\,Hz$. The value of $b = 1.0\,\mu m,\ c = 1.5\,\mu m,\ \in_1^r = 4.0$ and $\in_2^r = 3.78$.

Table 2 (b): Variation of Resonant Frequency $\left(v_{2ml}^{+}\right)$ with a for TM_{20l} Modes

l a (μm)	1	2	3	4	5
0.1	0.449	0.797	1.069	1.317	1.546
0.3	0.453	0.797	1.079	1.341	1.589
0.5	0.463	0.835	1.172	1.391	1.641
0.7	0.515	0.901	1.371	1.526	1.932
0.9	0.568	1.462	1.706	1.917	2.311

+ Frequencies are given by the numbers under the columns multiplied by $10^{14}\,Hz$. The value of $b = 1.0\,\mu m,\ c = 1.5\,\mu m,\ \in_1^r = 4.0$ and $\in_2^r = 3.78$.

Table 2 (c): Variation of Resonant Frequency $\left(v_{1ml}^{+}\right)$ with a for TM_{30l} Modes

l a (μm)	1	2	3	4	5
0.1	0.635	1.017	1.312	1.570	1.814
0.3	0.635	1.017	1.317	1.580	1.828
0.5	0.640	1.036	1.351	1.623	1.934
0.7	0.678	1.137	1.421	1.936	1.721
0.9	0.756	1.420	1.882	2.371	3.412

+ Frequencies are given by the numbers under the columns multiplied by $10^{14}\,Hz$. The value of $b = 1.0\,\mu m,\ c = 1.5\,\mu m,\ \in_1^r = 4.0$ and $\in_2^r = 3.78$.

Table 3 (a): Variation of Resonant Frequency $\left(v_{1ml}^{+}\right)$ with b for TE_{10l} Modes

l b (μm)	1	2	3	4	5
0.1	0.110	0.186	0.267	0.344	0.425
0.3	0.110	0.186	0.262	0.339	0.415
0.5	0.105	0.186	0.262	0.339	0.415
0.7	0.105	0.185	0.259	0.336	0.412
0.9	0.105	0.181	1.258	0.334	0.410

+ Frequencies are given by the numbers under the columns multiplied by $10^{14}\,Hz$. The value of $b = 1.0\,\mu m,\ c = 1.5\,\mu m,\ \in_1^r = 4.0$ and $\in_2^r = 3.78$.

Table 3 (b): Variation of Resonant Frequency $\left(v^{+}_{2ml}\right)$ with b for TE_{20l} Modes

l b (μm)	1	2	3	4	5
0.1	0.138	0.220	0.301	0.377	0.457
0.3	0.138	0.220	0.296	0.377	0.453
0.5	0.138	0.220	0.296	0.377	0.453
0.7	0.134	0.216	0292	0.369	0.447
0.9	0.134	0.215	0.291	0.367	0.444

+ Frequencies are given by the numbers under the columns multiplied by $10^{14}\,Hz$. The value of $b = 1.0\,\mu m,\ c = 1.5\,\mu m,\ \in^{r}_{1} = 4.0$ and $\in^{r}_{2} = 3.78$.

Table 3 (c): Variation of Resonant Frequency $\left(v^{+}_{3ml}\right)$ with b for TM_{3ml} Modes

l b (μm)	1	2	3	4	5
0.1	0.167	0.253	0.334	0.414	0.491
0.3	0.177	0.253	0.329	0.410	0.487
0.5	0.167	0.248	0.329	0.410	0.487
0.7	0.162	0.248	0.326	0.404	0.478
0.9	0.162	0.248	1.325	0.401	0.477

+ Frequencies are given by the numbers under the columns multiplied by $10^{14}\,Hz$. The value of $b = 1.0\,\mu m,\ c = 1.5\,\mu m,\ \in^{r}_{1} = 4.0$ and $\in^{r}_{2} = 3.78$.

Table 4 (a): Variation of Resonant Frequency $\left(v^{+}_{1ml}\right)$ with b for TM_{10l} Modes

l b (μm)	1	2	3	4	5
0.1	0.062	0.143	0.215	0.282	0.353
0.3	0.052	0.115	0.172	0.241	0.277
0.5	0.048	0.095	0.138	0.181	0.220
0.7	0.038	0.081	0.119	0.153	0.181
0.9	0.029	0.067	0.100	0.129	0.157

+ Frequencies are given by the numbers under the columns multiplied by $10^{14}\,Hz$. The value of $b = 1.0\,\mu m,\ c = 1.5\,\mu m,\ \in^{r}_{1} = 4.0$ and $\in^{r}_{2} = 3.78$.

Table 4 (b): Variation of Resonant Frequency $\left(v^{+}_{2ml}\right)$ with b for TM_{20l} Modes

l b (μm)	1	2	3	4	5
0.1	0.091	0.181	0.258	0.334	0.406
0.3	0.086	0.157	0.220	0.277	0.301
0.5	0.076	0.134	0.181	0.229	0.267
0.7	0.062	0.115	0.153	0.191	0.224
0.9	0.054	0.100	0.134	0.162	0.191

+ Frequencies are given by the numbers under the columns multiplied by $10^{14}\,Hz$. The value of $b = 1.0\,\mu m,\ c = 1.5\,\mu m,\ \in^{r}_{1} = 4.0$ and $\in^{r}_{2} = 3.78$.

Table 4 (c): Variation of Resonant Frequency $\left(v_{3ml}^{+}\right)$ with b for TM_{30l} Modes

l b (μm)	1	2	3	4	5
0.1	0.119	0.210	0.291	0.372	0.449
0.3	0.119	0.200	0.262	0325	0.382
0.5	0.105	0.172	0.224	0.272	0.315
0.7	0.091	0.148	0.191	0.229	0.262
0.9	0.072	0.129	0.162	0.196	0.224

+ Frequencies are given by the numbers under the columns multiplied by $10^{14}\,Hz$. The value of $b = 1.0\,\mu m,\ c = 1.5\,\mu m,\ \in_1^r = 4.0$ and $\in_2^r = 3.78$.

Table 5 (a): Variation of Resonant Frequency $\left(v_{1ml}^{+}\right)$ with c for TE_{10l} Modes

l c (μm)	1	2	3	4	5
0.1	0.544	0.935	1.325	1.709	2.105
0.3	0.272	0.472	0.663	0.854	1.050
0.5	0.181	0.315	0.447	0.573	0.702
0.7	0.134	0.234	0.334	0.430	0.525
0.9	0.110	0.186	0.267	0.344	0.420

+ Frequencies are given by the numbers under the columns multiplied by $10^{14}\,Hz$. The value of $b = 1.0\,\mu m,\ c = 1.5\,\mu m,\ \in_1^r = 4.0$ and $\in_2^r = 3.78$.

Table 5 (b): Variation of Resonant Frequency $\left(v_{1ml}^{+}\right)$ with b for TM_{10ml} Modes

l c (μm)	1	2	3	4	5
2.0	0.702	1.098	1.495	1.876	2.262
4.0	0.353	0.554	0.754	0.445	1.136
6.0	0.234	0.370	0.501	0.630	0.764
8.0	0.177	0.277	0.377	0.472	0.573
10.0	0.138	0.220	0.301	0.377	0.458

+ Frequencies are given by the numbers under the columns multiplied by $10^{14}\,Hz$. The value of $b = 1.0\,\mu m,\ c = 1.5\,\mu m,\ \in_1^r = 4.0$ and $\in_2^r = 3.78$.

Table 5 (c): Variation of Resonant Frequency $\left(v_{3ml}^{+}\right)$ with c for TE_{30ml} Modes

l c (μm)	1	2	3	4	5
2.0	0.854	1.264	1.657	2.048	2.434
4.0	0.425	0.635	0.838	1.031	1.227
6.0	0.284	0.425	0.558	0.692	0.821
8.0	0.210	0.315	0.420	0.515	0.616
10.0	0.167	0.253	0.334	0.415	0.492

+ Frequencies are given by the numbers under the columns multiplied by $10^{14}\,Hz$. The value of $b = 1.0\,\mu m,\ c = 1.5\,\mu m,\ \in_1^r = 4.0$ and $\in_2^r = 3.78$.

Table 6 (a): Variation of Resonant Frequency $\left(v_{1ml}^{+}\right)$ with b for TM_{10ml} Modes

l / c (μm)	1	2	3	4	5
2.0	0.239	0.492	0.711	0.916	1.117
4.0	0.148	0.310	0.458	0.597	0.597
6.0	0.105	0.224	0.334	0.439	0.539
8.0	0.081	0.177	0.262	0.344	0.430
10.0	0.062	0.143	0.215	0.282	0.353

+ Frequencies are given by the numbers under the columns multiplied by $10^{14}\,Hz$. The value of $b = 1.0\,\mu m,\ c = 1.5\,\mu m,\ \in_1^r = 4.0$ and $\in_2^r = 3.78$.

Table 6 (b): Variation of Resonant Frequency $\left(v_{2ml}^{+}\right)$ with c for TM_{20l} Modes

l / c (μm)	1	2	3	4	5
2.0	0.391	0.682	0.921	1.145	1.351
4.0	0.229	0.420	0.577	0.730	0.873
6.0	0.157	0.296	0.415	0.525	0.635
8.0	0.115	0.224	0.320	0.410	0.496
10.0	0.091	0.181	0.267	0.334	0.406

+ Frequencies are given by the numbers under the columns multiplied by $10^{14}\,Hz$. The value of $b = 1.0\,\mu m,\ c = 1.5\,\mu m,\ \in_1^r = 4.0$ and $\in_2^r = 3.78$.

Table 6 (c): Variation of Resonant Frequency $\left(v_{3ml}^{+}\right)$ with c for TM_{30l} Modes

l / c (μm)	1	2	3	4	5
2.0	0.544	0.864	1.122	1.360	1.580
4.0	0.301	0.515	0.687	0.850	1.002
6.0	0.200	0.353	0.482	0.601	0.721
8.0	0.148	0.267	0.367	0.463	0.554
10.0	0.119	0.210	0.291	0.372	0.449

+ Frequencies are given by the numbers under the columns multiplied by $10^{14}\,Hz$. The value of $b = 1.0\,\mu m,\ c = 1.5\,\mu m,\ \in_1^r = 4.0$ and $\in_2^r = 3.78$.

Table 7: Variation of Q Factor with c for TE_{10l} Modes

l / c (μm)	1	2	3	4	5
2.0	217.09	269.34	281.57	310.53	373.66
4.0	339.92	413.27	470.51	571.32	543.66
6.0	415.42	527.61	581.80	623.81	687.72
8.0	481.06	607.05	691.19	758.49	803.72
10.0	493.63	672.43	763.92	854.38	924.60

+ Frequencies are given by the numbers under the columns multiplied by $10^{14}\,Hz$. The value of $b = 1.0\,\mu m,\ c = 1.5\,\mu m,\ \in_1^r = 4.0$ and $\in_2^r = 3.78$.

Table 8: Variation of Q Factor with c for TM_{10l} Modes

l c (μm)	1	2	3	4	5
2.0	17.78	73.93	121.97	258.20	269.20
4.0	26.60	119.37	267.03	301.37	379.72
6.0	37.45	169.72	326.05	411.59	453.93
8.0	59.61	201.67	391.07	478.17	497.31
10.0	79.31	317.12	438.37	516.07	546.73

$a = 0.1\,\mu m,\ b = 1.0\,\mu m,\ \in_1^r = 4.0$ and $\in_2^r = 3.78$.

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