

Entropy Change of Fermionic Field of Black Holes of Spin Parameters $a^* = +3/2$ & $-3/2$ in XRBs and AGN

¹Dipo Mahto*, ²Rakesh Paswan, ³Niharika Kumari & ⁴Bijendra Kumar

Author's Affiliations:	¹ Professor & Head, Dept. of Physics, Bhagalpur College of Engineering, Bhagalpur, Bihar 812001, India & ¹ Former Head, Department of Physics, Marwari College, TMBU, Bhagalpur, Bihar 812007, India ² Assistant Professor, Dept. of Mathematics, DSM College Jhajha, Munger University, Bihar 811308, India ³ Lecturer, Department of Physics, Government Polytechnic Khagaria, Govt. of Bihar, Bihar 851212, India ⁴ Professor & Former Head, Univ. Dept. of Mathematics, TMBU Bhagalpur, Bihar 812007, India
*Corresponding author:	Dipo Mahto Professor & Head, Dept. of Physics, Bhagalpur College of Engineering, Bhagalpur, Bihar 812001, India E-mail: dipomahto@hotmail.com

Received on 25.08.2024, Revised on 04.10.2024, Accepted on 25.11.2024

ABSTRACT	The present paper deals the change in entropy of fermionic fields of spinning black holes w.r.t. the mass of spin parameters $a^* = +3/2$ & $-3/2$ and calculates their values in XRBs and AGN for different masses of black holes concluding that the Fermionic field of black holes of co-rotation decreases the entropy and the same black holes of counter rotation increases the entropy.
KEYWORDS	Fermionic fields, Spin Parameters and Entropy

How to cite this article: Mahto D., Paswan R., Kumari N. & Kumar B. (2024). Entropy Change of Fermionic Field of Black Holes of Spin Parameters $a^* = +3/2$ & $-3/2$ in XRBs and AGN. *Bulletin of Pure and Applied Sciences- Physics*, 43D (2), 138-142.

INTRODUCTION

The classical theory of black holes is not able to explain the emission of radiation from black holes. The quantum theory gives the explanation of emission of radiation a black body radiation (Hawking, 1974, 1975) [1, 2]. There are so many discussions have been done regarding the entropy of black holes by Bardeen et al. (1973) [3], Transchen (2000) [4], Wald (2001) [5], Narayan (2005) [6], Dabholkar (2005) [7], Bekenstein (2008) [8], Mahto et al., (2012) [9], Mahto and Kumari (2018) [10], Mahto et al. (2020) [11] and others.

The present paper deals the change in entropy of fermionic fields of black holes w.r.t. the mass of spin parameters $a^* = +3/2$ & $-3/2$ and calculates their values in XRBs and AGN.

THEORETICAL DISCUSSION

The change in entropy with mass of the spinning black holes in terms of the mass (M), angular velocity (Ω) and spin parameter (a^*) is given by the following equation.

$$\delta S / \delta M = 8\pi M(1 - 2\Omega M a^* + a^{*2} / 2 - M\Omega a^{*3}) \quad (1)$$

When the above model is applied for the $+3/2$ and $-3/2$ spin parameters of black holes (Tayal 1990, Yash 2020) [12, 13] of unit angular velocity, the following equation is obtained.

$$\left(\frac{\delta S}{\delta M} \right)_{+3/2} = -\pi M(51M - 17) \quad (2)$$

$$\left(\frac{\delta S}{\delta M} \right)_{-3/2} = \pi M(51M + 17) \quad (3)$$

To obtain the maximum change in entropy, apply the following condition as:

$$\left(\frac{\delta S}{\delta M} \right)_{\pm 3/2} = 0 \quad (4)$$

When this condition is applied on the equations (2) and (3), we get the following equations.

$$M(165M - 33) = 0 \quad (5)$$

$$M(165M + 33) \quad (6)$$

The solution of the equations 5) and (6) gives

$$M=0 \text{ or } M=33/165 \quad (7)$$

$$M=0 \text{ or } M=-33/165. \quad (8)$$

The solutions for the masses are 0, 33/165 and -33/165. This shows the zero, positive and negative masses respectively. The positive mass gives the concept of gravity. The negative mass gives the idea of naked singularity & dark matter. The general relativity and quantum theory gives the support the zero mass. Using the proper data for the change in entropy w.r.t. the mass for XRBs and AGN is listed in the table 1 & 2 respectively.

Table 1: The change in entropy w.r.t. the change in mass of the black holes of spin parameter $+3/2$ and $-3/2$ with $\Omega = 1$ in XRBs.

Sl. No	Mass (M) in (M_\odot)	$\left(\frac{\delta S}{\delta M} \right)_{+3/2} = -\pi M(51M - 17)$ [Joule/Kelvin/kg] $\times 10^{64}$	$\left(\frac{\delta S}{\delta M} \right)_{-3/2} = \pi M(51M + 17)$ [Joule/Kelvin/kg] $\times 10^{64}$
1	5 M_\odot	-1.6014	1.6014
2	6 M_\odot	-2.3060	2.3060
3	7 M_\odot	-3.1387	3.1387
4	8 M_\odot	-4.0996	4.0996
5	9 M_\odot	-5.1885	5.1885
6	10 M_\odot	-6.4056	6.4056
7	11 M_\odot	-7.7508	7.7508
8	12 M_\odot	-9.2241	9.2241
9	13 M_\odot	-10.825	10.825
10	14 M_\odot	-12.555	12.555
11	15 M_\odot	-14.413	14.413
12	16 M_\odot	-16.398	16.398
13	17 M_\odot	-18.512	18.512
14	18 M_\odot	-20.754	20.754
15	19 M_\odot	-23.124	23.124
16	20 M_\odot	-25.622	25.622

Table 2: The change in entropy w.r.t. the change in mass of the black holes of spin parameters +3/2 and -3/2 with $\Omega = 1$ in AGN.

S. No.	Mass of BH_s (M) in solar masses	Mass of BHs in terms of $10^7 M_\odot$	$\left(\frac{\delta S}{\delta M}\right)_{+3/2} = -\pi M(51M - 17)$ [Joule/Kelvin/kg] $\times 10^{77}$	$\left(\frac{\delta S}{\delta M}\right)_{-3/2} = \pi M(51M + 17)$ [Joule/Kelvin/kg] $\times 10^{77}$
1	$1 \times 10^6 M_\odot$.1	-0.006405	0.006405
2	$2 \times 10^6 M_\odot$.2	-0.025624	0.025624
3	$3 \times 10^6 M_\odot$.3	-0.057650	0.057650
4	$4 \times 10^6 M_\odot$.4	-0.01249	0.01249
5	$5 \times 10^6 M_\odot$.5	-0.06014	0.06014
6	$6 \times 10^6 M_\odot$.6	-0.23060	0.23060
7	$7 \times 10^6 M_\odot$.7	-0.313874	0.313874
8	$8 \times 10^6 M_\odot$.8	-0.40995	0.40995
9	$9 \times 10^6 M_\odot$.9	-0.51885	0.51885
10	$1 \times 10^7 M_\odot$	1	-0.64056	0.64056
11	$2 \times 10^7 M_\odot$	2	-2.56224	2.56224
12	$3 \times 10^7 M_\odot$	3	-5.76504	5.76504
13	$4 \times 10^7 M_\odot$	4	-10.249	10.249
14	$5 \times 10^7 M_\odot$	5	-16.014	16.014
15	$6 \times 10^7 M_\odot$	6	-23.0602	23.0602
16	$7 \times 10^7 M_\odot$	7	-31.3874	31.3874
17	$8 \times 10^7 M_\odot$	8	-40.995	40.995
18	$9 \times 10^7 M_\odot$	9	-51.8854	51.8854
19	$1 \times 10^8 M_\odot$	10	-64.056	64.056
20	$2 \times 10^8 M_\odot$	20	-256.224	256.224
21	$3 \times 10^8 M_\odot$	30	-576.504	576.504
22	$4 \times 10^8 M_\odot$	40	-1024.90	1024.90
23	$5 \times 10^8 M_\odot$	50	-1601.40	1601.40
24	$6 \times 10^8 M_\odot$	60	-2306.02	2306.02
25	$7 \times 10^8 M_\odot$	70	-3138.74	3138.74
26	$8 \times 10^8 M_\odot$	80	-4099.58	4099.58
27	$9 \times 10^8 M_\odot$	90	-5188.54	5188.54
28	$1 \times 10^9 M_\odot$	100	-6405.60	6405.60

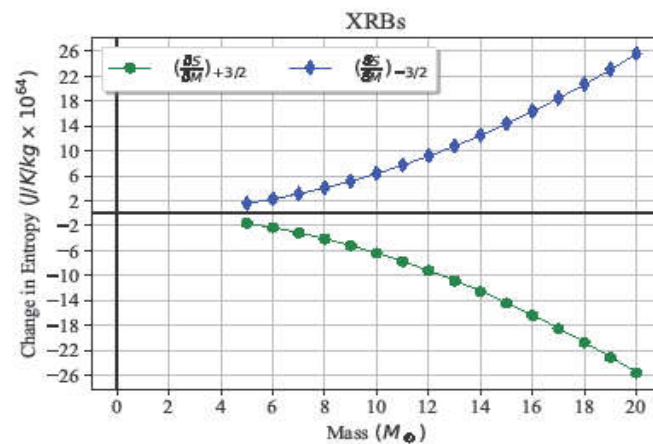


Figure 1: The change in entropy w.r.t. the change in mass of the black holes of spin parameter $+3/2$ and $-3/2$ with $\Omega = 1$ in XRBs.

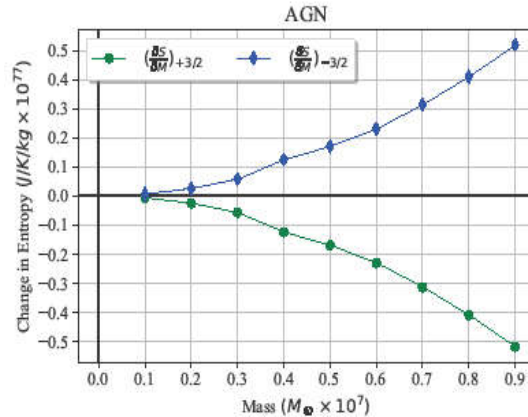


Figure 2(a): shows the change in entropy w.r.t. the change in mass of range 0.1 to 0.9 times 10^6 solar mass of the black holes of spin parameter $+3/2$ and $-3/2$ with unit angular velocity in AGN.

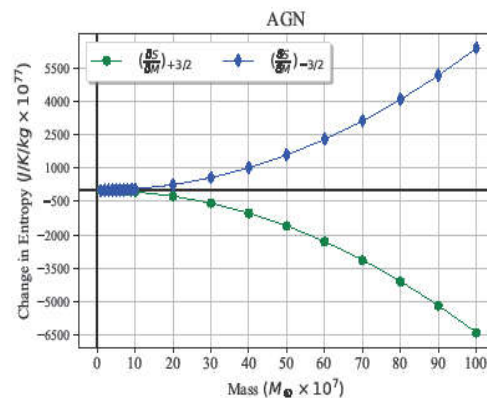


Figure 2(b): The figure 2(b) shows the change in entropy w.r.t. the change in mass of range 1 to 9 times 10^6 solar mass of the black holes of spin parameter $+3/2$ and $-3/2$ with unit angular velocity in AGN.

RESULT AND DISCUSSION:

The change in entropy w.r.t. mass in terms of M , Ω and a^* is obtained by the following equation (Mahto and Kumari 2018) [10].

$$\delta S / \delta M = 8\pi M(1 - 2\Omega M a^* + a^{*2} / 2 - M\Omega a^{*3})$$

From the observation of above equation, it is clear that the change in entropy corresponding change in mass of black holes depends on angular velocity, spin parameters and mass of black holes.

This work is applied for the spin parameters ($a^* = +3/2$ & $-3/2$), we see that the change in

entropy w.r.t mass provides the negative values to show decreasing entropy for $a^* = +3/2$, while provides the positive values to show increasing entropy change for $a^* = -3/2$.

We also have plotted the graph between the mass of spinning black holes and corresponding change in entropy with change in mass in XRBs and AGN with the help of the equations (2) & (3) as shown in the Figs.1 & 2(a) and 2(b) respectively to observe that the variations in nature of entropy change with reference to mass is symmetrical for either types of spin parameters for co-rotation and counter rotations in XRBs and AGN, but differing in their

gradients. This difference is due to their mass, because XRBs black holes are massive of mass range 5 to 20 solar masses, while the AGN black holes are super massive of mass range 10^6 to 10^{10} solar masses.

CONCLUSION

The following conclusions are drawn from the present research work:

1. The change in entropy w.r.t mass provides the negative values for $a^* = +3/2$, to show the decrease in entropy change, while provides the positive values for $a^* = +3/2$ to show the increase in the entropy change.
2. The present model gives the concept of Naked singularity of Fermionic fields of black holes.
3. The positive, negative and zero mass of the Fermionic fields of black holes are justified.
4. Our model for the Fermionic fields of black holes provides the concept of dark matter and dark energy.

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