

## Theory of Harmonic Oscillations: A Gross Error in Physics

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<b>ABSTRACT</b>	<p>The critical analysis of the foundations of the standard theory of harmonic oscillations is proposed. The unity of formal logic and rational dialectics is methodological basis of the analysis. The analysis leads to the conclusion that this theory represents gross error. The substantiation (validation) of this statement is the following main results. I. In the case of the material point suspended on the elastic spring, the linear differential equation of harmonic oscillations is the equation (condition) of balance of Newton's force (Newton's second law) and "Hooke's force" ("Hooke's law" as pseudolaw). This equation contains the following gross methodological errors: (a) the differential equation of motion of the material point does not satisfy the dialectical principle of the unity of the qualitative and quantitative determinacy of physical quantities (i.e., Newton's force and Hooke's force). In other words, the left and right sides of the differential equation (i.e., the equation of balance of the forces) have no identical qualitative determinacy: the left side of the the equation of balance of the forces represents Newton's force, and the right side of the the equation of balance of the forces represents the "Hooke's force" (as pseudolaw); (b) the sum of Newton's force and the "Hooke force" (as pseudolaw) in the the equation of balance of the forces is equal to zero. This means that the sum of the numerical values of Newton's force and "Hooke's force" (as pseudolaw) is equal to zero. Consequently, the numerical values of Newton's force and "Hooke's force" (as pseudolaw) are equal to zero in the region of neutral real numbers. This means that the equation of balance of the forces is incorrect; (c) "Hooke's force" (as pseudolaw) in the equation of balance of the forces represents the product of the spring constant (coefficient of stiffness of the spring) and the coordinate of the material point. In this case, "Hooke's force" (as pseudolaw) does not represent Hooke's law. "Hooke's force" (as pseudolaw) contradicts to Hooke's law because the coordinate of the material point does not determine the spring constant (coefficient of stiffness of the spring). "Hooke's force" (as pseudolaw) has the dimension of Newton's force. But, as the practice of measurement of Hooke's force with the help of a dynamometer shows, the dynamometer readings are real neutral numbers with the dimension "kilogram-force"; (d) the mathematical operation of division of the equation of balance of the forces by the mass of the material point leads to the linear equation of balance of</p>
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the accelerations of the material point. In this case, the mathematical operation gives rise to the term “frequency”: (spring stiffness coefficient)-to-(mass) ratio is “squared frequency”. But the spring stiffness coefficient is the constant that does not define the concept of frequency. Therefore, the quantity of the acceleration of the material point does not define the concept of the frequency of periodic motion; (e) the solution of the linear differential equation of balance of the accelerations of the material point has imaginary roots. This leads to the following contradiction: the coordinate of the material point is both an exponential function and a trigonometric function. II. In the case of oscillations of the mathematical pendulum, the linear differential equation of harmonic oscillations of the material point suspended on the inextensible thread represents a mathematical description of the angular displacement of the inextensible thread in the Cartesian coordinate system. This equation is a mathematical consequence of the standard differential equation of the rotational motion dynamics and contains the following gross methodological errors: (a) the differential equation of motion of the material point suspended on the inextensible thread does not satisfy the dialectical principle of the unity of the qualitative and quantitative determinacy of physical quantities (i.e., the physical quantity of rate of change of the angular momentum (moment of momentum) and the physical quantity of moment of the acting force). This equation expresses the condition of balance of the rate of change of the angular momentum (moment of momentum) and the moment of the acting force. Gross error is that the left and right sides of the balance equation have no identical qualitative determinacy: the left side of the balance equation is the rate of change of the angular momentum (moment of momentum), and the right side of the balance equation is the moment of the acting force; (b) the sum of the rate of change of the angular momentum and the moment of the acting force is equal to zero in the balance equation. This means that the sum of the numerical values of the rate of change of the angular momentum and the moment of the acting force is equal to zero. Consequently, the numerical values of the rate of change of the angular momentum and the moment of the acting force are equal to zero in the region of neutral real numbers. This means that the balance equation is incorrect; (c) the mathematical operation of division of the equation of balance of the rate of change of the angular momentum and the moment of the acting force by the mass of the material point and the square of the thread length results in the equation of balance of the angular accelerations. In this case, the mathematical operation results in the term “frequency” (“squared frequency”). But the quantity of the angular acceleration does not determine the frequency of the periodic motion; (d) the linear differential equation of balance of the angular accelerations is analogous to the linear differential equation of balance of the accelerations of the material point suspended on the spring. Therefore, the solution of the linear differential equation of balance of the angular accelerations has imaginary roots and leads to the following contradiction: the angle of displacement of the pendulum from the equilibrium position is both an exponential function and a trigonometric function.

# KEYWORDS

General physics, Classical Mechanics, Formalisms in Classical Mechanics, Newtonian Mechanics, Harmonic Oscillations, Engineering, Technology, General Mathematics, Methodology of Mathematics, General Applied Mathematics, Philosophy of Mathematics, Education, Philosophy of Science, Logic in the Philosophy of Science, History of Science.

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## INTRODUCTION

As is known, the theory of harmonic oscillations is an important part of the foundations of physics and mathematics [1-10]. This theory was created by well-known scientists and is presented (reproduced) in textbooks and monographs [11-16]. Therefore, the theory looks convincing. But the validity of the theory has not been questioned so far.

In my opinion, the validity of a theory can only be researched (studied, analyzed) within the framework of the correct methodological basis: the unity of formal logic and rational dialectics. The unity of formal logic and rational dialectics represents the only correct criterion of truth. Formal logic is the general science of the laws of correct thinking. Rational dialectics is the general doctrine of universal connection and change in the world. The most important concepts of rational dialectics are the concepts of quality, quantity and measure.

Quality (i.e., the qualitative determinacy) is the unity of properties, essential features of an object. The qualitative determinacy of an object represents the essence (as a set of essential features) of an object. Quantity is the quantitative determinacy of the essence of an object. The quantitative determinacy belongs to the qualitative determinacy of the object (i.e., the quantitative determinacy belongs to the essential feature of the object) and is expressed by dimensional neutral numbers.

The measure of an object is the unity of the qualitative and quantitative determinacy of an object. From this point of view, a correct mathematical relationship represents the unity of qualitative and quantitative determinacy: the left and right sides of the mathematical relationship denote (designate, indicate) an identical qualitative determinacy (i.e., an identical meaning). Therefore, a mathematical relationship is incorrect if the left and right sides of the relationship belong to different qualitative determinacy.

Well-known scientists did not find the correct criterion of truth for mathematical and physical

theories. As is known, Albert Einstein and Henri Poincaré were probably the last scientists who tried to find the correct criterion of truth. After Albert Einstein, the problem of truth in science was not studied (researched) by mathematicians and physicists. Scientists worshiped (adored, venerated) the works of the classics of sciences. Scientists believed in the inviolability (firmness, stability) of the foundations of sciences. Therefore, scientists did not question the standard theories. That is why theoretical physics and mathematics contain gross methodological errors and enter into the greatest crisis.

The purpose of the present work is to propose the critical analysis of the standard theory of harmonic oscillations. The methodological basis for analysis is the unity of formal logic and rational dialectics. This way of analysis gives an opportunity to understand the erroneous essence of the theory of harmonic oscillations.

### 1. Elements of correct kinematics and dynamics [17-20]

As is known, the starting point of correct kinematics is based on the following statements.

1) "The quantity  $l^{(M)}(t) - l^{(M)}(t_0)$  is called increment of the length of the path of the material point  $M$  over time  $\Delta t_0 \equiv t - t_0$  where  $\Delta t_0 \neq 0$ ,  $t_0$  is the initial point of time. The length of the path and the increment of the length of the path are not vector quantities. The quantity

$$\frac{l^{(M)}(t) - l^{(M)}(t_0)}{\Delta t_0} \equiv v^{(M)}(\Delta t_0)$$

is rate of change in the quantity  $l^{(M)}$ . In other words, the speed of motion of the point  $M$  is rate of change in quantity  $l^{(M)}(t)$ . (Movement is change in general). Therefore, the speed is not a vector quantity. By definition, the speed of the motion of the point  $M$  is the average speed over time  $\Delta t_0$ . There is no "instantaneous speed" (i.e., speed at point of time  $t$ ). The speed of the

motion is the essential feature (property, characteristic) of motion: speed is the rate of the change in quantity. The rate of the change in the quantity  $l^{(M)}(t)$  has no a graphical representation in system  $XOY$  because the quantity of the rate has no the dimension of "meter (m)". The rate of the change in the quantity  $l^M(t)$  is not defined and is not characterized by any direction because the quantity  $l^{(M)}(t)$  is not defined and is not characterized by a direction of the motion of the point  $M$  in the system  $XOY$ . Thus, the rate of the change in the path length is independent of a direction of the motion of the point  $M$ .

2) "The variable quantity  $v^{(M)}(\Delta t_0)$  takes on the values  $v_1^{(M)}(\Delta t_{10})$ ,  $v_2^{(M)}(\Delta t_{20})$ ,  $v_3^{(M)}(\Delta t_{30})$  under  $\Delta t_{10}$ ,  $\Delta t_{20}$ ,  $\Delta t_{30}$ , respectively. If the interval (duration) of time is the variable quantity  $\Delta t_0 \equiv t - t_0$ , then the quantity  $v^{(M)}(\Delta t_0)$  of the speed is a function of the argument  $\Delta t_0 \equiv t - t_0$ . The conventional concept of speed at point of time (at instant of time)  $t$  (or at point of plane  $XOY$ ) has no scientific and practical sense because the speed of the motion is determined by two (different) positions of the moving point  $M$  on plane  $XOY$  and by two (different) points of time: movement is change in general; but there is no change in position at point of time  $t$  (or at point of plane  $XOY$ )".

3) "If the speed of the motion of the point  $M$  depends on time, then the quantity

$$\frac{v^{(M)}(\Delta t_0) - v_1^{(M)}(\Delta t_0)}{\Delta t_0} \equiv a^{(M)}$$

is called acceleration of the point  $M$  on the path length  $l^{(M)}(t) - l^{(M)}(t_0)$  where  $v_1^{(M)}(\Delta t_0)$  is certain value of speed, which is experimentally determined. Acceleration characterizes the motion of the point  $M$ : acceleration is the essential feature (property, characteristic) of the motion of point  $M$ . The quantity of the

acceleration of the point  $M$  has no graphical representation in the system  $XOY$  because the quantity of the acceleration has no dimension of "meter (m)". The quantities  $l^{(M)}(t)$  and  $a^{(M)}$  are connected by the following relationship:

$$l^{(M)}(t) - l^{(M)}(t_0) = a^{(M)} \cdot (\Delta t_0)^2.$$

If

$$l^{(M)}(t) - l^{(M)}(t_0) = \Delta l^{(M)}(\Delta t_0),$$

$$a^{(M)} = g,$$

$$\text{then } \Delta t_0 = \sqrt{\frac{\Delta l^{(M)}}{g}}$$

where  $\Delta t_0$  is the free fall time of the material point  $M$  in the gravitational field;  $g$  is the gravitational acceleration. This expression does not depend on the mass of the material point  $M$  and represents a reliable experimental result.

4) The product of mass and speed of the moving object  $M$  represents the essential physical property (essential feature) of the moving material object:

$$p^M(\Delta t_0) \equiv m^M \times v^M(\Delta t_0)$$

where the physical quantity  $p^M(\Delta t_0)$  is called momentum of object  $M$ . The dimension of the quantity of the momentum is "kg m s<sup>-1</sup>". This definition of the momentum satisfies the formal-logical law of identity:

$$\begin{aligned} &(\text{property of the moving object "M"}) = \\ &= (\text{property of the moving object "M"}). \end{aligned}$$

In addition, the definition of the momentum satisfies the formal-logical law of lack (absence) of contradiction:

$$\begin{aligned} &(\text{property of the moving object "M"}) \neq \\ &\neq (\text{property of the moving object "non-M"}) \end{aligned}$$

5) The rate of change in the momentum of the moving object  $M$  represents the essential

physical property (essential feature) of the motion of the material object  $M$ . The rate of change in the momentum of the moving object  $M$  is defined as follows:

$$\frac{p^M(\Delta t_0) - p_1^M(\Delta t_0)}{\Delta t_0} = m^M \times \frac{v^M(\Delta t_0) - v_1^M(\Delta t_0)}{\Delta t_0},$$

$$\frac{p^M(\Delta t_0) - p_1^M(\Delta t_0)}{\Delta t_0} = m^M \times a^M$$

where  $p_1^M(\Delta t_0)$  is a certain value of the momentum, which is experimentally determined. The dimension of the quantity of the rate of change in the momentum is " $\text{kg m s}^{-2}$ ". The dimension " $\text{kg m s}^{-2}$ " characterizes the qualitative determinacy of the quantity of rate of change in the momentum. The definition of the rate of change in the momentum of the moving object satisfies the formal-logical law of identity:

$$\begin{aligned} &(\text{property of the moving object "M"}) = \\ &= (\text{property of the moving object "M"}). \end{aligned}$$

In addition, the definition of the rate of change in the momentum satisfies the formal-logical law of lack (absence) of contradiction:

$$\begin{aligned} &(\text{property of the moving object "M"}) \neq \\ &\neq (\text{property of the moving object "non-M"}). \end{aligned}$$

6) As is known, the starting point of the correct dynamics of the material body  $S$  is the following formulation of Hooke's law.

(a) In the case of the spring  $S$  stretching, Hooke's law is the following correct expression:

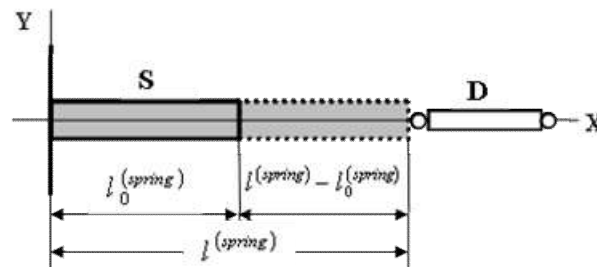
$$\frac{F^D - F_1^D}{F_1^D} = \frac{l^{(spring)} - l_1^{(spring)}}{l_1^{(spring)}},$$

$$F^D = \left( \frac{F_1^D}{l_1^{(spring)}} \right) l^{(spring)},$$

$$F^D \neq m \ddot{l}^{(spring)}, \quad l^{(spring)} \neq 0$$

where  $l^{(spring)} - l_1^{(spring)}$  is the increment of the length of the spring  $S$ ; the dimension of the force  $F^D$  is " $\text{kgf}$ ", i.e.  $[F] = \text{kgf}$ ;  $F_1^D$  is a certain value of variable quantity  $F^D$  which is the reading of the dynamometer  $D$ ;  $k^{(spring)} \equiv \left( \frac{F_1^D}{l_1^{(spring)}} \right)$  is coefficient of stiffness of the spring  $S$  having the dimension " $\text{kgf/m}$ ". (The coefficient  $k^{(spring)}$  does not depend on time  $t$ ).

Let one, using the dynamometer  $D$ , stretch out the spring  $S$  to a certain length  $l^{(spring)}$  (Figure 1).



**Figure 1:** Change of the length of the spring.  $S$ .  $l_0^{(spring)}$  is initial value of the spring length;  $l^{(spring)} - l_0^{(spring)}$  is the finite value of the spring length.  $D$  is a dynamometer, the readings of which are neutral numbers with the dimension " $\text{kgf}$ ".

As practice shows, the dynamometer  $D$  readings are real neutral numbers of the dimension “ $\text{kgf}$ ”, but not numbers of the dimension “ $\text{kg m s}^{-2}$ ”. If one disconnects the dynamometer  $D$  from the spring  $S$ , the spring  $S$  returns to its original (neutral) state during the relaxation time:  $l^{(spring)}(t) \rightarrow l_0^{(spring)}$ .

(b) In the case of compression of the spring  $S$ , Hooke's law is the following correct expression:

$$\frac{F^D - F_1^D}{F_1^D} = \frac{1/l^{(spring)} - 1/l_1^{(spring)}}{1/l_1^{(spring)}},$$

$$F^D = \left( \frac{F_1^D}{1/l_1^{(spring)}} \right) \frac{1}{l^{(spring)}}, \quad l^{(spring)} \neq 0,$$

7) Mathematical pendulum. In the case of a cycloidal periodic motion of a material circle (solid body), Huygens' experimental result is the following statement: “Ratio of the time of one small oscillation of the circular pendulum to the time of falling on the double length of the pendulum is as ratio of the circumference of the circle to the diameter” (Huygens). In the case of the mathematical pendulum, the Huygens kinematic relationship for the oscillation period has the following form:

$$T^{(pendulum)} = \frac{l^{(circle)}}{r^{(circle)}} \sqrt{\frac{l^{(pendulum)}}{g}}$$

where  $l^{(pendulum)}$  is the length of the mathematical pendulum;  $l^{(pendulum)} \equiv r^{(circle)}$ ;  $g$  is the gravitational acceleration. The oscillation period in the Huygens formula does not depend on the mass of the pendulum. This result is a reliable experimental fact. Comparison of the expressions

$$\Delta t_0 = \sqrt{\frac{\Delta l^{(M)}}{g}}$$

and

$$T^{(pendulum)} = \frac{l^{(circle)}}{r^{(circle)}} \sqrt{\frac{l^{(pendulum)}}{g}}$$

shows that the kinematics of the mathematical pendulum and the kinematics of free fall of the material point  $M$  are qualitatively identical. The quantitative difference between these expressions is determined by the design features of the mathematical pendulum. For example, the dimensionless coefficient

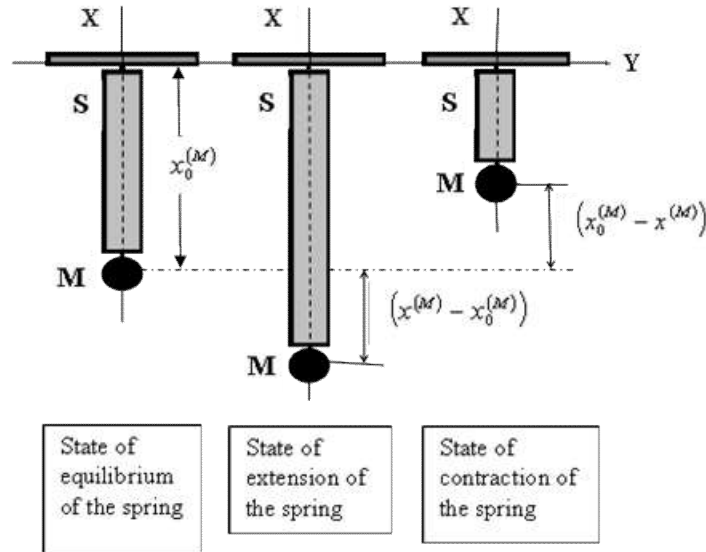
$$\frac{l^{(circle)}}{r^{(circle)}}$$

is determined by the design features of the mathematical pendulum. Essential meaning of this coefficient is that this coefficient determines the amplitude of oscillations

If the fall (dropping) and lifting of the material point  $M$  is repeated continuously several times by some material device, then the quantity  $\omega \equiv 1/\Delta t_0$  is the frequency of the periodic movement. The frequency has the dimension “hertz” (i.e., cycle per second). The frequency is measured with a material counter. But if the fall (dropping) and lifting of the material point  $M$  is not repeated continuously several times, then the quantity  $\omega \equiv 1/\Delta t_0$  has no physical meaning.

#### STANDARD DIFFERENTIAL EQUATION OF HARMONIC OSCILLATIONS IN THE CASE OF A MATERIAL POINT SUSPENDED ON AN ELASTIC SPRING

In the case of the material point  $M$  suspended on the elastic spring  $S$ , the displacement of the material point  $M$  is motion in the vertical straight line segment – the coordinate ruler  $X$  (Figure 2).



**Figure 2:** The coordinate  $x^{(M)}$  of the oscillating material point  $M$  suspended on the elastic spring  $S$ .

As is known, the standard equation of harmonic oscillations of the material point  $M$  is the following linear differential equation of the second order:

$$\ddot{x}^{(M)} + \omega_0^2 x^{(M)} = 0$$

where  $x^{(M)}$  is the displacement of the material point  $M$ , having the dimension “m”;  $\omega_0^2 = k/m^{(M)}$  is the coefficient (frequency) having the dimension “1/s<sup>2</sup>”; the mass  $m^{(M)}$  of the material point  $M$  is the coefficient having the dimension “kg”; the spring stiffness coefficient  $k$  is the coefficient having the dimension “kg s<sup>-2</sup>”. This equation of motion of the material point  $M$  is a mathematical consequence of the following expressions:

- (a) Lagrange function:
- $$L^{(M, \text{spring})} = \frac{m^{(M)} (\dot{x}^{(M)})^2}{2} - \frac{k (x^{(M)})^2}{2} \quad \text{for}$$
- the “material point  $M$  + spring” system;

- (b) “Hook’s law” (as pseudolaw):  $F^{(M)} = -kx^{(M)}$ . This expression is not an equation of motion of the material point  $M$ ;

- (c) Newton’s second law for the material point  $M$ . This expression is the equation of motion of the material point  $M$ ;

- (d) dynamic equation of balance (connection) of forces:  $m^{(M)} \ddot{x}^{(M)} = -kx^{(M)}$ .

The only meaning of the standard equation is that it is an equation (condition) for the balance of accelerations of the material point  $M$ .

## OBJECTIONS TO THE STANDARD DIFFERENTIAL EQUATION OF HARMONIC OSCILLATIONS

In the case of the material point  $M$  suspended on the elastic spring  $S$ , the displacement of the material point  $M$  is motion in the vertical straight line segment – the coordinate ruler  $X$  (Figure 2). Physical, mathematical and formal-logical objections to the standard theoretical description of the harmonic oscillations of the material point  $M$  are as follows.

(a) The first gross error is as follows. The quantities  $\ddot{x}^{(M)}$  and  $\omega_0^2 x^{(M)}$  in the standard equation  $\ddot{x}^{(M)} + \omega_0^2 x^{(M)} = 0$  represent accelerations and have the dimension " $m/s^2$ ". These quantities designate numbers. If these quantities take on numerical values and the sum of these quantities (numbers) is equal to zero, then  $\ddot{x}^{(M)} \equiv 0$  and  $\omega_0^2 x^{(M)} \equiv 0$  in the region of neutral real numbers. Consequently, the standard equation is incorrect;

(b) the second gross error is that the coefficient  $\omega_0^2 = k/m^{(M)}$  in the standard equation has the dimension " $1/s^2$ ". From the point of view of formal logic, the following statement is true: the correct dimension of the quantity  $\omega_0^2$  is " $kgf/mkg$ ". Really, in the correct relationship  $\omega_0^2 = k^{(spring)}/m^{(M)}$ , the quantities  $k^{(spring)}$  and  $m^{(M)}$  have the following meanings:  $m^{(M)}$  is mass of the material point  $M$ , which does not depend on the characteristics of the spring  $S$  and time  $t$ ;  $k^{(spring)} \equiv \left( \frac{F_1^{(spring)}}{l_1^{(spring)}} \right)$  is the

coefficient of spring  $S$  stiffness, which does not characterize the material point  $M$  and does not depend on time  $t$ ; the coefficient  $k^{(spring)} \equiv \left( \frac{F_1^{(spring)}}{l_1^{(spring)}} \right)$  has the dimension

" $kgf/m$ ". Therefore, the correct dimension of the quantity  $\omega_0^2$  is not " $1/s^2$ ". This means that the coefficient  $\omega_0^2$  cannot be contained in the equation of motion of the material point  $M$ ;

(c) the third gross error is that the quantity  $\omega_0$  in the standard relationship  $\omega_0^2 = k/m^{(M)}$  is called the frequency of oscillations. The dimension of the quantity  $\omega_0^2$  is  $1/s^2$  because one uses the formula  $F = ma$ . But frequency

(as the number of iterations of oscillation per unit time) cannot be contained in the equation of motion because the quantity  $\omega_0^2$  is

$$\omega_0^2 \equiv \frac{(\text{coefficient of stiffness of spring } S)}{(\text{mass of the material point } M)} = (\text{square of frequency}) = (\text{absurdity}).$$

In other words, the quantity

$$\frac{(\text{coefficient of stiffness of spring } S)}{(\text{mass of the material point } M)}$$

cannot depend on time  $t$ . This means that the formula  $F = ma$  is incorrect, and the quantity  $\omega_0$  is not a frequency;

(d) the fourth gross error is that the expression  $F^{(M)} = -kx^{(M)}$  (as pseudolaw) contradicts to the following correct formulation of Hooke's law:

$$F^{(Hooker's)} \equiv F^D = \left( \frac{F_1^D}{l_1^{(spring)}} \right) l^{(spring)},$$

$$F^{(Hooker's)} \equiv F^{(spring)} = k^{(spring)} l^{(spring)}.$$

Really, the spring  $S$  and the material point  $M$  are different (non-identical) objects. This is expressed by the formal-logical law of the lack of contradiction:

$$(\text{spring } S) \neq (\text{material point } M).$$

Violation of the formal-logical law of the lack of contradiction is that one replaces the concept "spring  $S$ " (which is not characterized by the term "coordinate") by the non-identical concept "material point  $M$ " (which is characterized by the terms "coordinate" and "mass"). In other words, the mistake is that one identifies non-identical concepts, i.e.

$$(\text{spring } S) = (\text{material point } M).$$



But the coordinate  $x^{(M)}$  of the material point  $M$  is not the length of the spring  $S$ . Consequently, the term  $kx^{(M)}$  cannot be contained in the equation

$$m^{(M)}\ddot{x}^{(M)} = -kx^{(M)}.$$

(e) The fifth gross error is that the equation  $\ddot{x}^{(M)} + \omega_0^2 x^{(M)} = 0$  is not a logical consequence of the equation  $m^{(M)}\ddot{x}^{(M)} + kx^{(M)} = 0$ . Really, from the point of view of formal logic, the concepts  $\ddot{x}^{(M)}$  and  $\omega_0^2 x^{(M)}$  are not identical to the concepts  $m^{(M)}\ddot{x}^{(M)}$  and  $kx^{(M)}$ , respectively.

(f) The sixth gross error is the following expression:  $F^{(Newton's)} = F^{(Hooker's)}$ . But, from the point of view of formal logic and rational dialectics,  $F^{(Newton's)} \neq F^{(Hooker's)}$  because the left and right sides of the quantitative relationship  $F^{(Newton's)} = F^{(Hooker's)}$  have different qualitative determinacy, different meanings (i.e., the left and right sides of this relationship do not represent the same physical law). In other words, the gross error is that the expression  $F^{(Newton's)} = F^{(Hooker's)}$  violates the formal-logical law of the lack of contradiction:

"(Newton's second law)  $\neq$  (Hooke's law)".

(g) The seventh gross error is the following formula:  $F^{(Newton's)} \equiv F^{(M)} = m^{(M)}\ddot{x}^{(M)}$ . But

$F^{(M)} \neq m^{(M)}\ddot{x}^{(M)}$  because Newton's formula  $F = ma$  is not a definition of force.

(h) The eighth gross error reveals in the next operation. The operation is based on the correct definition of acceleration. If one represents (interprets) the standard differential equation in the form of the algebraic equation

$$\frac{x^{(M)} - x_0^{(M)}}{(t - t_0)^2} + \omega_0^2 (x^{(M)} - x_0^{(M)}) = 0$$

(where  $x^{(M)} - x_0^{(M)} \neq 0$  is an increment;

$\frac{x^{(M)} - x_0^{(M)}}{(t - t_0)^2}$  is a definition of the acceleration

of the material point  $M$ ), then the standard differential equation will take the following algebraic form:

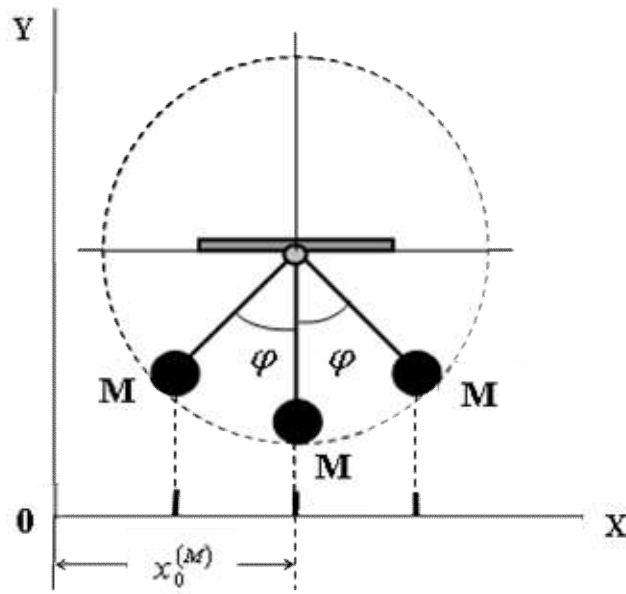
$$1 = -\omega_0^2 (t - t_0)^2.$$

This algebraic expression is the proof that the standard differential equation is nonsense.

Thus, the standard differential equation of harmonic oscillations represents a gross error.

## MATHEMATICAL PENDULUM

In the case of a mathematical pendulum, the displacement of the material point  $M$  suspended on an inextensible thread is the angular displacement  $\varphi$  of the inextensible thread (Figure 3).



**Figure 3:** The positions of the oscillating material point  $M$  suspended on the inextensible thread in the “mathematical pendulum + coordinate system  $XOY$ ” system. The quantity  $x_0^{(M)}$  is the coordinate of the equilibrium position of the point  $M$ . The quantities  $(x_0^{(M)} - x^{(M)})$  and  $(x^{(M)} - x_0^{(M)})$  characterize the displaced positions of the material point  $M$ .  $\varphi$  is the quantity of the angle of displacement of the pendulum thread from the equilibrium value  $\varphi_0 = 270^\circ$ . The trajectory of the oscillating point  $M$  is the arc of the circle.

The trajectory of the material point  $M$  is the arc of the circle. The correct relationship between the length of the arc of the circle and the quantity of the central angle is as follows:

$$\begin{aligned}
 l^{(circular\ arc)} &= \frac{l^{(circle)}}{360^\circ} \varphi^{(central\ angel)}, \\
 l^{(circle)} &= \left( \frac{l_1^{(circle)}}{r_1^{(circle)}} \right) r^{(circle)} \equiv q^{(circle)} r^{(circle)}, \\
 q^{(circle)} &= \left( \frac{l_1^{(circle)}}{r_1^{(circle)}} \right), \\
 l^{(circular\ arc)} &= \frac{q^{(circle)}}{360^\circ} r^{(circle)} \varphi^{(central\ angel)}, \\
 l^{(circular\ arc)} &\neq l^{(pendulum)} \varphi^{(central\ angel)}, \\
 r^{(circle)} &\equiv l^{(pendulum)} = const, \\
 l^{(circular\ arc)} &\equiv l^{(M)},
 \end{aligned}$$

$$\begin{aligned}
 \dot{l}^{(M)} &= \frac{q^{(circle)}}{360^\circ} r^{(circle)} \dot{\varphi}^{(central\ angel)}, \\
 p^{(M)} &\equiv m^{(M)} \dot{l}^{(M)}
 \end{aligned}$$

where  $\varphi^{(central\ angel)}$  is the quantity of the central angle leaning (resting) on the arc of the circle;  $l^{(circular\ arc)}$  is the length of the arc of the circle;  $r^{(circle)}$  is the radius of the circle;  $l^{(circular\ arc)} \equiv l^{(M)}$  is the length of the path traveled by the material point  $M$ ;  $l^{(pendulum)}$  is the length of the pendulum thread;  $p^{(M)}$  is the momentum of the material point  $M$ .

The quantitative relationships between the circumference  $l^{(circle)}$  and the radius  $r^{(circle)}$  has form of the following proportion:

$$\left( \frac{l^{(circle)} - l_1^{(circle)}}{l_1^{(circle)}} \right) = \left( \frac{r^{(circle)} - r_1^{(circle)}}{r_1^{(circle)}} \right),$$

$$l^{(circle)} = \left( \frac{l_1^{(circle)}}{r_1^{(circle)}} \right) r^{(circle)}$$

where the dimensionless coefficient  $\left( \frac{l_1^{(circle)}}{r_1^{(circle)}} \right)$  is experimentally determined.

### STANDARD DIFFERENTIAL EQUATION OF HARMONIC OSCILLATIONS OF THE MATHEMATICAL PENDULUM

As is known, the standard linear differential equation of harmonic oscillations of the mathematical pendulum (Figure 3) has the following form:

$$\ddot{\varphi}^{(central\ angle)} + \omega_0^2 \varphi^{(central\ angle)} = 0,$$

$$\omega_0^2 = g/l^{(pendulum)}$$

where  $\omega_0$  is the frequency having the dimension " $1/s$ ";  $l^{(pendulum)}$  is the length of the inextensible thread having the dimension " $m$ ";  $\varphi^{(central\ angle)}$  is the angle of displacement of the pendulum from the equilibrium position;  $g = 9,8 \frac{meter}{sec^2}$  is the gravitational acceleration, which has the dimension " $m/s^2$ ". This equation is a mathematical consequence of the following standard differential equation (i.e. the rotary motion dynamics equation):

$$m^{(M)} (l^{(pendulum)})^2 \ddot{\varphi}^{(central\ angle)} = -m^{(M)} g l^{(pendulum)} \sin \varphi^{(central\ angle)},$$

$$\sin \varphi^{(central\ angle)} \approx \varphi^{(central\ angle)}$$

where  $m^{(M)}$  is the mass of the material point  $M$ ;  $m^{(M)} g l^{(pendulum)} \sin \varphi^{(central\ angle)}$  is the rotational moment created by the force of gravity;  $m^{(M)} (l^{(pendulum)})^2$  is the moment of inertia of the pendulum;  $m^{(M)} (l^{(pendulum)})^2 \ddot{\varphi}^{(central\ angle)}$  is the momentum;

$m^{(M)} (l^{(pendulum)})^2 \ddot{\varphi}^{(central\ angle)}$  is the angular momentum (moment of momentum);  $\ddot{\varphi}^{(central\ angle)}$  is the angular acceleration.

The standard meaning of this dynamic equation is the following standard statement: the rate of change of angular momentum (i.e.,  $\dot{J}^{(M)} = m^{(M)} (l^{(pendulum)})^2 \ddot{\varphi}^{(central\ angle)}$ ) is equal to the moment of the acting force (i.e.,  $N^{(M)} = -m^{(M)} g l^{(pendulum)} \sin \varphi^{(central\ angle)}$ ). In other words, this equation expresses the condition of balance of the rate of change of the angular momentum (moment of momentum) and the moment of the acting force.

### OBJECTIONS TO THE STANDARD DIFFERENTIAL EQUATION OF HARMONIC OSCILLATIONS OF THE MATHEMATICAL PENDULUM

(a) The first gross error is that the dimensions of the quantities  $\sin \varphi^{(central\ angle)}$  and  $\varphi^{(central\ angle)}$  are different: the values of the quantity  $\sin \varphi^{(central\ angle)}$  are dimensionless numbers, and the values of the quantity  $\varphi^{(central\ angle)}$  have the dimension "degree".

(b) The second gross error is that the quantity  $l^{(pendulum)} \sin \varphi^{(central\ angle)}$  represents the imaginary side of the non-existent right-angled triangle.

(c) The third gross error is that the standard expression  $m^{(M)} (l^{(pendulum)})^2 \ddot{\varphi}^{(central\ angle)}$  contradicts to the following correct definition of the momentum of the material point  $M$ :

$$p^{(M)} \equiv m^{(M)} \dot{l}^{(M)}, \quad l^{(circular\ arc)} \equiv l^{(M)},$$

$$l^{(circular\ arc)} = \frac{l^{(circle)}}{360^\circ} \varphi^{(central\ angle)},$$

$$l^{(circular\ arc)} \neq l^{(pendulum)} \varphi^{(central\ angle)}.$$

(Explanation: The correct definition of the momentum of the material point  $M$  is based on the following fact: the trajectory (path) of the motion of the material point  $M$  is the arc of the

circle. The length of the arc of the circle is not a vector quantity. Change in the length of the arc of the circle determines the speed, acceleration and momentum of the material point  $M$ ).

(d) The fourth gross error is the assertion that the rate of change of momentum (i.e.,  $m^{(M)}(l^{(pendulum)})^2 \ddot{\varphi}^{(central\ angel)}$ ) is equal to the moment of the acting force (i.e.,  $-m^{(M)}gl^{(pendulum)}\sin\varphi^{(central\ angel)}$ ). But, from the point of view of formal logic and rational dialectics,

$$m^{(M)}(l^{(pendulum)})^2 \ddot{\varphi}^{(central\ angel)} \neq -m^{(M)}gl^{(pendulum)}\sin\varphi^{(central\ angel)}$$

because the left and right sides of the quantitative relationship have different qualitative determinacy (i.e., different measures, different meanings). In other words, the left and right sides of the standard relationship represent quantitative changes under non-identical qualitative determinacy. Really, the physical concepts "momentum of the material point  $M$ " and "moment of the force acting on the material point  $M$ " have different meanings, different qualitative determinacy, different measures. From the point of view of formal logic, the identification of the concepts "momentum" and "moment of force" is a violation of the law of the lack of contradiction:

"(momentum)  $\neq$  (moment of force)".

Consequently, the relationship

$$m^{(M)}(l^{(pendulum)})^2 \ddot{\varphi}^{(central\ angel)} = -m^{(M)}gl^{(pendulum)}\sin\varphi^{(central\ angel)}$$

represents a gross error.

(e) The fifth gross error is that the equation

$$\ddot{\varphi}^{(central\ angel)} + \omega_0^2 \varphi^{(central\ angel)} = 0$$

is not a logical consequence of the equation

$$m^{(M)}(l^{(pendulum)})^2 \ddot{\varphi}^{(central\ angel)} = -m^{(M)}gl^{(pendulum)}\sin\varphi^{(central\ angel)}$$

Really, from the point of view of formal logic, the concepts  $\ddot{\varphi}^{(central\ angel)}$  and  $\omega_0^2 \varphi^{(central\ angel)}$  are not identical to the concepts  $m^{(M)}(l^{(pendulum)})^2 \ddot{\varphi}^{(central\ angel)}$  and  $m^{(M)}gl^{(pendulum)}\sin\varphi^{(central\ angel)}$ , respectively.

(f) The sixth gross error manifests itself in the following. The quantities  $\ddot{\varphi}^{(central\ angel)}$  and  $\omega_0^2 \varphi^{(central\ angel)}$  in the standard equation  $\ddot{\varphi}^{(central\ angel)} + \omega_0^2 \varphi^{(central\ angel)} = 0$  represent the accelerations and have the dimension "degree/s<sup>2</sup>". If these quantities take numerical values and the sum of these values is equal to zero, then  $\ddot{\varphi}^{(central\ angel)} \equiv 0$  and  $\omega_0^2 \varphi^{(central\ angel)} \equiv 0$  in the region of neutral real numbers. Consequently, the standard equation is incorrect.

(g) The seventh gross error manifests itself in the next operation. The operation is based on the correct definition of acceleration. If one represents the standard differential equation in the form of the algebraic equation

$$\frac{\varphi^{(central\ angel)} - \varphi_0^{(central\ angel)}}{(t - t_0)^2} + \omega_0^2 (\varphi^{(central\ angel)} - \varphi_0^{(central\ angel)}) = 0$$

$$, \quad \varphi_0^{(central\ angel)} = 270^\circ$$

(where  $\varphi^{(central\ angel)} - \varphi_0^{(central\ angel)} \neq 0$  is the increment of the quantity of the angle;  $\frac{\varphi^{(central\ angel)} - \varphi_0^{(central\ angel)}}{(t - t_0)^2}$  is the definition of

acceleration), then the standard equation will take the following form:

$$1 \equiv -\omega_0^2 (t - t_0)^2, \quad \omega_0^2 = g/l^{(pendulum)}.$$

This algebraic expression is the proof that the standard differential equation represents nonsense.

Thus, the standard differential equation of harmonic oscillations of the mathematical pendulum is a gross error.

## DISCUSSION OF THE SOLUTION OF THE STANDARD DIFFERENTIAL EQUATION

Thus, the standard differential equation of harmonic oscillations is incorrect because the equation does not satisfy the laws of formal logic and rational dialectics. This fact means that the solution of the incorrect equation cannot be a scientific truth. Really:

1) As is known, the standard solution of the standard differential equation  $\ddot{x}^{(M)} + \omega_0^2 x^{(M)} = 0$  is found as follows. Substitution of the expression  $x^{(M)} = a \exp(\lambda t)$  into the standard differential equation leads to the characteristic equation  $\lambda^2 + \omega_0^2 = 0$  where the parameter  $\lambda$  and the constant  $a$  are not in a logical connection with the quantities  $x^{(M)}$  and  $\omega_0^2$ .

From the point of view of rational dialectics and formal logic, the standard mathematical (quantitative) expression  $x^{(M)} = a \exp(\lambda t)$  contradicts to the following assertions (statements, points) of the methodological basis: (a) the dialectical category of measure; (b) the formal-logical law of identity and the law of the lack of contradiction because the left and right sides of the mathematical (quantitative) expression  $x^{(M)} = a \exp(\lambda t)$  do not have identical qualitative determinacy (measures). In other words, the right side of the quantitative (mathematical) expression  $x^{(M)} = a \exp(\lambda t)$  is not a feature of the material point  $M$  (i.e.,  $a \exp(\lambda t)$  does not characterize the object  $M$ ). Therefore, the standard mathematical operation of change of the variable (i.e.,  $x^{(M)} = a \exp(\lambda t)$ ) is a gross methodological error.

From the point of view of formal logic, if  $\lambda^2 + \omega_0^2 = 0$ , then  $\lambda^2 \equiv 0$  and  $\omega_0^2 \equiv 0$  in the region of neutral real numbers because letters in mathematics designate (indicate,

denote, denominate, mean) numbers. Therefore, the expression  $\lambda^2 + \omega_0^2 = 0$  is a gross error.

From the standard point of view, the characteristic equation  $\lambda^2 + \omega_0^2 = 0$  has imaginary roots:  $\lambda_1 = +i\omega_0$ ,  $\lambda_2 = -i\omega_0$ . In this case, the standard general solution of the equation  $\ddot{x}^{(M)} + \omega_0^2 x^{(M)} = 0$  is:

$$x^{(M)} = (a/2)(\exp[i(\omega_0 t + \alpha)] + \exp[-i(\omega_0 t + \alpha)]) = a \cos(\omega_0 t + \alpha)$$

where  $a$  and  $\alpha$  are arbitrary constants that do not characterize the material point  $M$ . This solution leads to the following contradiction:

$$x^{(M)} = a \exp(\lambda t) = a \cos(\omega_0 t + \alpha).$$

From the mathematical point of view,  $\exp(\lambda t) \neq \cos(\omega_0 t + \alpha)$  because an exponential function is not a periodic function. Therefore, the general solution of the equation  $\ddot{x}^{(M)} + \omega_0^2 x^{(M)} = 0$  is a gross error.

The standard equation  $\ddot{x}^{(M)} + \omega_0^2 x^{(M)} = 0$  of harmonic oscillations is a special case of the standard equation of the general oscillatory motion. This means that the solution of the standard equation of the general oscillatory motion is a gross error.

2) In the case of the mathematical pendulum, the standard linear differential equation of harmonic oscillations of the mathematical pendulum has the following standard form:

$$\ddot{\varphi}^{(\text{central angel})} + \omega_0^2 \varphi^{(\text{central angel})} = 0, \\ \omega_0^2 = g/l^{(\text{pendulum})}$$

This equation is analogous to the equation  $\ddot{x}^{(M)} + \omega_0^2 x^{(M)} = 0$ . Therefore, the analysis of the solution of the equation  $\ddot{\varphi}^{(\text{central angel})} + \omega_0^2 \varphi^{(\text{central angel})} = 0$  leads to a similar conclusion: the solution of the equation  $\ddot{\varphi}^{(\text{central angel})} + \omega_0^2 \varphi^{(\text{central angel})} = 0$  is a gross error.

3) In my works [21-69], the following statements are proved:

- (a) the differential and integral calculus represents a gross error;
- (b) the numbers are neutral numbers; positive and negative numbers do not exist;
- (c) pure mathematics, standard trigonometry, complex number theory, and vector calculus represent gross errors.
- (d) Newton's second and third laws represent gross errors.

Consequently, the standard general theory of oscillations using standard trigonometry, complex number theory, vector calculus, Newton's laws, etc. is unfounded, groundless, unreasonable one because it represents gross error in science.

A particular (special) scientific theory cannot be substantiated (validated, well-founded, grounded) within the framework of particular (special) sciences because particular (special) sciences do not contain a criterion of truth. Moreover, the criterion of truth cannot be formulated within the framework of particular (special) sciences. The criterion of truth can only be formulated within the framework of the general sciences. The general sciences are formal logic and rational dialectics. Therefore, the unity of formal logic and rational dialectics represents the correct criterion of truth and the methodological basis for particular (special) sciences. Consequently, the substantiation (validation) of particular (special) scientific theories should be carried out within the framework of the correct methodological basis. The existence of methodological errors in particular (special) scientific theories means the collapse of these theories.

## CONCLUSION

Thus, the critical analysis of the foundations of the standard theory of harmonic oscillations within the framework of the unity of formal logic and rational dialectics leads to the conclusion that this theory represents gross error. The substantiation (validation) of this statement is the following main results.

I. In the case of the material point suspended on the elastic spring, the linear differential equation of harmonic oscillations is the equation (condition) of balance of Newton's force (Newton's second law) and "Hooke's force" ("Hooke's law" as pseudolaw). This equation contains the following gross methodological errors:

(a) the differential equation of motion of the material point does not satisfy the dialectical principle of the unity of the qualitative and quantitative determinacy of physical quantities (i.e., Newton's force and Hooke's force). In other words, the left and right sides of the differential equation (i.e., the equation of balance of the forces) have no identical qualitative determinacy: the left side of the the equation of balance of the forces represents Newton's force, and the right side of the the equation of balance of the forces represents the "Hooke's force" (as pseudolaw).

(b) the sum of Newton's force and the "Hooke force" (as pseudolaw) in the the equation of balance of the forces is equal to zero. This means that the sum of the numerical values of Newton's force and "Hooke's force" (as pseudolaw) is equal to zero. Consequently, the numerical values of Newton's force and "Hooke's force" (as pseudolaw) are equal to zero in the region of neutral real numbers. This means that the equation of balance of the forces is incorrect.

(c) "Hooke's force" (as pseudolaw) in the equation of balance of the forces represents the product of the spring constant (coefficient of stiffness of the spring) and the coordinate of the material point. In this case, "Hooke's force" (as pseudolaw) does not represent Hooke's law. "Hooke's force" (as pseudolaw) contradicts to Hooke's law because the coordinate of the material point does not determine the spring constant (coefficient of stiffness of the spring). "Hooke's force" (as pseudolaw) has the dimension of Newton's force. But, as the practice of measurement of Hooke's force with the help of a dynamometer shows, the dynamometer readings are real neutral numbers with the dimension "kilogram-force".

(d) the mathematical operation of division of the equation of balance of the forces by the mass of the material point leads to the linear equation of the balance of the accelerations of the material point. In this case, the mathematical operation gives rise to the term "frequency": (spring stiffness coefficient)-to-(mass) ratio is "squared frequency". But the spring stiffness coefficient is the constant that does not define the concept of frequency. Therefore, the quantity of the acceleration of the material point does not define the concept of the frequency of periodic motion.

(e) the solution of the linear differential equation of balance of the accelerations of the material point has imaginary roots. This leads to the following contradiction: the coordinate of the material point is both an exponential function and a trigonometric function.

II. In the case of oscillations of the mathematical pendulum, the linear differential equation of harmonic oscillations of the material point suspended on the inextensible thread represents a mathematical description of the angular displacement of the inextensible thread in the Cartesian coordinate system. This equation is a mathematical consequence of the standard differential equation of the rotational motion dynamics and contains the following gross methodological errors:

(a) the differential equation of motion of the material point suspended on the inextensible thread does not satisfy the dialectical principle of the unity of the qualitative and quantitative determinacy of physical quantities (i.e., the physical quantity of rate of change of the angular momentum (moment of momentum) and the physical quantity of moment of the acting force). This equation expresses the condition of balance of the rate of change of the angular momentum (moment of momentum) and the moment of the acting force. Gross error is that the left and right sides of the balance equation have no identical qualitative determinacy: the left side of the balance equation is the rate of change of the angular momentum (moment of momentum), and the right side of the balance equation is the moment of the acting force.

(b) the sum of the rate of change of the angular momentum and the moment of the acting force is equal to zero in the balance equation. This means that the sum of the numerical values of the rate of change of the angular momentum and the moment of the acting force is equal to zero. Consequently, the numerical values of the rate of change of the angular momentum and the moment of the acting force are equal to zero in the region of neutral real numbers. This means that the balance equation is incorrect.

(c) the mathematical operation of division of the equation of the balance of the rate of change of the angular momentum and the moment of the acting force by the mass of the material point and the square of the thread length results in the equation of balance of the angular accelerations. In this case, the mathematical operation results in the term "frequency" ("squared frequency"). But the quantity of the angular acceleration does not determine the frequency of the periodic motion;

(d) the linear differential equation of balance of the angular accelerations is analogous to the linear differential equation of balance of the accelerations of the material point suspended on the spring. Therefore, the solution of the linear differential equation of balance of the angular accelerations has imaginary roots and leads to the following contradiction: the angle of displacement of the pendulum from the equilibrium position is both an exponential function and a trigonometric function.

## REFERENCES

- [1] E. Madelung. (1957). Die Mathematischen Hilfsmittel Des Physikers. Berlin, Gottingen, Heidelberg: Springer-Verlag.
- [2] D.J. Struik. (1987). A Concise. History of Mathematics. New York: Dover Publications.
- [3] C.B. Boyer. (1991). A history of mathematics" (Second ed.). John Wiley & Sons, Inc. ISBN 0-471-54397-7.
- [4] M. Hazewinkel (2000). Encyclopedia of Mathematics. Kluwer Academic Publishers.
- [5] R. Nagel (2002). (ed.). Encyclopedia of Science. (2nd Ed.). The Gale Group.

- [6] W.B. Ewald. (2008). From Kant to Hilbert: A source book in the foundations of mathematics. Oxford University Press, US. ISBN 0-19-850535-3.
- [7] Robert M. Besançon (1990). The Encyclopedia of Physics. (eBook ISBN978-1-4615-6902-2. Springer, New York.
- [8] George L. Trigg (2004). Encyclopedia of Applied Physics. John Wiley. ISBN: 978-3-527-40478-0.
- [9] Rita G. Lerner, George L. Trigg. (2005). Encyclopedia of physics. (Edition: 3rd ed. completely rev.). Wiley-VCH, Weinheim.
- [10] Mary L. Boas. (2006). Mathematical Methods in the Physical Sciences. (3rd ed.), Hoboken, [NJ.]: John Wiley & Sons. ISBN 978-0-471-19826-0.
- [11] R.P. Feynman, R.B. Leighton, M. Sands, (1963). The Feynman Lectures on Physics. Vol. 1 – Mechanics. ISBN 978-0-201-02116-5.
- [12] C. Kittel, W.D. Knight, A. Ruderman. (1964). Berkeley Physics Course. V. 1 – Mechanics. McGraw-Hill Book Company.
- [13] L.D. Landau, E.M. Lifshitz. (1972). Course of Theoretical Physics. Vol. 1 – Mechanics. Franklin Book Company. ISBN 0-08-016739-X.
- [14] D. Kleppner, R.J. Kolenkow. (1973). An Introduction to Mechanics”. McGraw-Hill. ISBN 0-07-035048-5.
- [15] M. Alonso, J. Finn. (1992). Fundamental University Physics. Addison-Wesley.
- [16] H. Goldstein, P.P. Charles, L.S. John. (2002). Classical Mechanics. (3rd ed.). Addison Wesley. ISBN 0-201-65702-3.
- [17] T.Z. Kalanov. (2017). On the correct formulation of the starting point of classical mechanics. Advances in Physics Theories and Applications, 64, 27-46.
- [18] T.Z. Kalanov. (2017). On the correct formulation of the starting point of classical mechanics. International Journal of Advanced Research in Physical Science. 4(6), 1-22.
- [19] T.Z. Kalanov. (2017). On the correct formulation of the starting point of classical mechanics. International educational scientific research journal. 3(6), 56-73.
- [20] T.Z. Kalanov (2010). On a new analysis of the foundations of classical mechanics. I. Dynamics. Bulletin of the Amer. Phys. Soc. (April Meeting), 55(1).
- [21] T.Z. Kalanov (2011). Critical analysis of the foundations of differential and integral calculus. MCMS (Ada Lovelace Publications), 34-40.
- [22] T.Z. Kalanov (2011). Logical analysis of the foundations of differential and integral calculus. Indian Journal of Science and Technology, 4(12).
- [23] T.Z. Kalanov (2011). Logical analysis of the foundations of differential and integral calculus. Bulletin of Pure and Applied Sciences, 30E (2), 327-334.
- [24] T.Z. Kalanov (2012). Critical analysis of the foundations of differential and integral calculus. International Journal of Science and Technology, 1(2), 80-84.
- [25] T.Z. Kalanov (2012). On rationalization of the foundations of differential calculus. Bulletin of Pure and Applied Sciences, Vol. 31E (1), 1-7.
- [26] T.Z. Kalanov (2012). On logical error underlying classical mechanics. Bulletin of the Amer. Phys. Soc., (April Meeting), 57(3).
- [27] T.Z. Kalanov (2013). Critical analysis of the mathematical formalism of theoretical physics. I. Foundations of differential and integral calculus”. Bulletin of the Amer. Phys. Soc., (April Meeting), 58(4).
- [28] T.Z. Kalanov (2013). On the logical analysis of the foundations of vector calculus. International Journal of Scientific Knowledge. Computing and Information Technology, 3(2), 25-30.
- [29] T.Z. Kalanov (2013). On the logical analysis of the foundations of vector calculus. International Journal of Multidisciplinary Academic Research, 1(2).
- [30] T.Z. Kalanov (2013). On the logical analysis of the foundations of vector calculus. Journal of Computer and Mathematical Sciences, 4(4), 202-321.
- [31] T.Z. Kalanov (2013). On the logical analysis of the foundations of vector calculus. Journal of Research in Electrical and Electronics Engineering (ISTP-JREEE), (ISSN: 2321-2667), 2(5), 1-5.
- [32] T.Z. Kalanov (2013). On the logical analysis of the foundations of vector calculus. Research Desk, (ISSN: 2319-7315), 2(3), 249-259.
- [33] T.Z. Kalanov (2013). The foundations of vector calculus: The logical error in



- mathematics and theoretical physics. Unique Journal of Educational Research, 1(4), 054-059.
- [34] T.Z. Kalanov (2013). On the logical analysis of the foundations of vector calculus. Aryabhatta Journal of Mathematics & Informatics, (ISSN: 0975-7139), 5(2), 227-234.
- [35] T.Z. Kalanov (2013). Critical analysis of the mathematical formalism of theoretical physics. II. Foundations of vector calculus". Unique Journal of Engineering and Advanced Sciences (UJEAS, www.ujconline.net), 1(1).
- [36] T.Z. Kalanov (2013). Critical analysis of the mathematical formalism of theoretical physics. II. Foundations of vector calculus. Bulletin of Pure and Applied Sciences, 32E (2), 121-130.
- [37] T.Z. Kalanov (2014). Critical analysis of the mathematical formalism of theoretical physics. II. Foundations of vector calculus. Bulletin of the Amer. Phys. Soc., (April Meeting), 59(5).
- [38] T.Z. Kalanov (2014). On the system analysis of the foundations of trigonometry. Journal of Physics & Astronomy, (www.mehtapress.com), 3(1).
- [39] T.Z. Kalanov (2014). On the system analysis of the foundations of trigonometry. International Journal of Informative & Futuristic Research, (IJIFR, www.ijifr.com), 1(1), 6-27.
- [40] T.Z. Kalanov (2014). On the system analysis of the foundations of trigonometry. International Journal of Science Inventions Today, (IJSIT, www.ijst.com), 3(2), 119-147.
- [41] T.Z. Kalanov (2014). On the system analysis of the foundations of trigonometry. Pure and Applied Mathematics Journal, 3(2), 26-39.
- [42] T.Z. Kalanov (2014). On the system analysis of the foundations of trigonometry. Bulletin of Pure and Applied Sciences, 33E (1), 1-27.
- [43] T.Z. Kalanov (2014). Critical analysis of the foundations of the theory of negative number. International Journal of Informative & Futuristic Research (IJIFR, www.ijifr.com), 2(4), 1132-1143.
- [44] T.Z. Kalanov (2015). Critical analysis of the mathematical formalism of theoretical physics. IV. Foundations of trigonometry. Bulletin of the Amer. Phys. Soc., (April Meeting), 60(4).
- [45] T.Z. Kalanov (2015). Critical analysis of the mathematical formalism of theoretical physics. V. Foundations of the theory of negative numbers. Bulletin of the Amer. Phys. Soc., (April Meeting), 60(4).
- [46] T.Z. Kalanov (2015). Critical analysis of the foundations of the theory of negative numbers. International Journal of Current Research in Science and Technology, 1(2), 1-12.
- [47] T.Z. Kalanov (2015). Critical analysis of the foundations of the theory of negative numbers. Aryabhatta Journal of Mathematics & Informatics, 7(1), 3-12.
- [48] T.Z. Kalanov (2015). On the formal-logical analysis of the foundations of mathematics applied to problems in physics. Aryabhatta Journal of Mathematics & Informatics, 7(1), 1-2.
- [49] T.Z. Kalanov (2016). On the formal-logical analysis of the foundations of mathematics applied to problems in physics. Bulletin of the Amer. Phys. Soc., (April Meeting).
- [50] T.Z. Kalanov (2016). Critical analysis of the foundations of pure mathematics. Mathematics and Statistics (CRESCO, <http://crescopublications.org>), 2(1), 2-14.
- [51] T.Z. Kalanov (2016). Critical analysis of the foundations of pure mathematics. International Journal for Research in Mathematics and Mathematical Sciences, 2(2), 15-33.
- [52] T.Z. Kalanov (2016). Critical analysis of the foundations of pure mathematics. Aryabhatta Journal of Mathematics & Informatics, 8(1), 1-14 (Article Number: MSOA-2-005).
- [53] T.Z. Kalanov (2016). Critical Analysis of the Foundations of Pure Mathematics. Philosophy of Mathematics Education Journal, ISSN 1465-2978 (Online). Editor: Paul Ernest), No. 30 (October 2016).
- [54] T.Z. Kalanov (2017). On the formal-logical analysis of the foundations of mathematics applied to problems in physics. Asian Journal of Fuzzy and Applied Mathematics, 5(2), 48-49.
- [55] T.Z. Kalanov (2017). The formal-logical analysis of the foundation of set theory.

- Bulletin of Pure and Applied Sciences, 36E(2), 329 -343.
- [56] T.Z. Kalanov (2017). The critical analysis of the foundations of mathematics. Mathematics: The Art of Scientific Delusion. LAP LAMBERT Academic Publishing (2017-12-05). ISBN-10: 620208099X.
- [57] T.Z. Kalanov (2018). On the correct formulation of the starting point of classical mechanics. Physics & Astronomy (International Journal). 2(2), 79-92.
- [58] T.Z. Kalanov (2018). The formal-logical analysis of the foundation of set theory. Scientific Review, 4(6), 53-63.
- [59] T.Z. Kalanov (2019). Hubble Law, Doppler Effect and the Model of "Hot" Universe: Errors in Cosmology". Open Access Journal of Physics (USA), 3(2), 1-16.
- [60] T.Z. Kalanov (2019). Definition of Derivative Function: Logical Error in Mathematics. MathLAB Journal, 3, 128-135.
- [61] T.Z. Kalanov (2019). Definition of Derivative Function: Logical Error in Mathematics. Academic Journal of Applied Mathematical Sciences, 5(8), 124-129.
- [62] T.Z. Kalanov (2019). Definition of Derivative Function: Logical Error in Mathematics. Aryabhatta Journal of Mathematics & Informatics, 11(2), 173-180.
- [63] T.Z. Kalanov (2019). Vector Calculus and Maxwell's Equations: Logic Errors in Mathematics and Electrodynamics. Open Access Journal of Physics, 3(4), 9-26.
- [64] T.Z. Kalano (2019). Vector Calculus and Maxwell's Equations: Logic Errors in Mathematics and Electrodynamics. Sumerianz Journal of Scientific Research, 2(11), 133-149.
- [65] T.Z. Kalanov (2020). Definition of work: Unsolved Problem in Classical Mechanics. Bulletin of Pure and Applied Sciences (Section D -Physics), 39D (1), 137-148.
- [66] T.Z. Kalanov (2020). Definition of work: Unsolved Problem in Classical Mechanics. Open Access Journal of Physics, 4(1), 29-39.
- [67] T.Z. Kalanov (2021). Formal-logical analysis of the starting point of mathematical logic. Aryabhatta Journal of Mathematics & Informatics, 13(1), 01-14.
- [68] T.Z. Kalanov (2022). On fundamental errors in trigonometry. Bulletin of Pure and Applied Sciences (Section - E - Mathematics & Statistics), 41E(1), 16-33.
- [69] T.Z. Kalanov (2022). Theory of complex numbers: gross error in mathematics and physics". Bulletin of Pure and Applied Sciences (Section - E - Mathematics & Statistics), 41E(1), 61-68.

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