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Twistors and the Amplitudes in the Integrable Sector of Superstring Theory

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ABSTRACT Twistors can represent the spin in superstring theory and provide a theoretical explanation of the two-dimensional spherical model of elementary particles together with the surface distributions of the charges. The integrability of certain matter sectors of the strong interactions also may be traced to the action of the conformal group on collinear trajectories that occur in a twistor formulation. The amplitudes for these scattering processes in four dimensions have a representation with quantum states in a nonsupersymmetric theory defined by the low-energy limit of a solution to the N = 2 string equations with a Wick rotation of the metric with (2, 2) signature. The twistor form of an N = 2 scattering amplitude is given.

KEYWORDS Transport, Periodic, Doped, Metallic, Nanotube, Ballistic, Symmetry.

1. INTRODUCTION

The connection between twistor theory and string theory generalizes the spinor bilinear representation of the momentum in a ten-dimensional massless vector field theory. The twistor transform for the super-Yang-Mills and self-dual Yang-Mills equations has been developed in ten dimensions [1], and super-self-duality equations may be solved with supertwistors [2]. By dimensional reduction of the field equations, the integrability conditions on light like lines may be derived in N = 2 super-Yang-Mills theory in six dimensions [3] and N = 4 super-Yang-Mills theory in four dimensions. There is a correspondence between super-Yang-Mills actions, classical superstrings in 3, 4, 6 and 10 dimensions and the division algebras. It may be noted that, although the dimensions of other real division algebras must be 1, 2, 4 or 8 by Hurwitz's theorem, division algebras of other dimensions exist over finite fields [4][5][6] and the relation to discrete symmetries [7] in field theories may be considered.

The number of world sheet supersymmetries in D dimensions equals D - 2 because the super-Poincare algebra in D dimensions generates a D-2-superconformal algebra in two dimensions [8]. The heterotic string and Type IIA superstring theories have been written in twistor formalism and the N = 8 superconformal algebra may be parameterized by $\hat{S}^7$, the Kac-Moody extension of S^7 [9]. The occurrence of the string tension as an integration constant [10] in the ten-dimensional heterotic string theory has implications for its physical interpretation in relation to gradients of J vs. M^2 plots, minimal distances and the effective string coupling [11].

An S^7 transformation rule cannot be constructed for a pure Yang-Mills theory with the connection taking values only on the four-dimensional base space [12]. Since twistor variables that transform under Sp (4; $\mathbb{O}$) can be combined to transform parameterize S^7 [13], the problem of constructing a model with this invariance may be transferred to the action on twistors. It can be solved if the fundamental variables in the theory are interpreted in terms of the space S8 of lightlike lines of octonionic superparticles. This approach can be compared to an algebraic description of the supersymmetric Hopf fibration. When the base space is the super-sphere S_*^2, a supersymmetric version of the U (1) theory is found [14]. A generalization would exist for the base spaces S_*^4 and S_*^8.

The duality between points and lines in projective geometry, and the intepretation of points as lines in twistor space, would be expected to have implications for the superstrings in a formalism combining the two theories. This duality might be interpreted as a reflection of the two different descriptions of superstring dynamics through the path integral with a two-dimensional action for the weighting factor describing and the point-particle effective field theory in ten dimensions.

2. A SUMMARY OF THE TWISTOR FORMALISM

Dimensions for Twistor Transforms and Green-Schwarz Superstring

The number of fermion degrees of freedom in the division algebras is sufficient to be identified with one of the division algebras

$$\begin{aligned} SL(2,\mathbb{K}_\nu) &\simeq \overline{SO(1,\nu+1)} \qquad \nu = 1,2,4 \\ Sp(4;\mathbb{K}_\nu) &\simeq \overline{SO(2;\nu+2)}. \end{aligned} \tag{2.1}$$

Twistor transforms exist only in dimensions 3, 4, 6 and 10, when the identity

$$\lambda_1\gamma_\rho\lambda_2\gamma^\mu\lambda_3 + \lambda_3\gamma_\rho\lambda_1\gamma^\mu\lambda_2 + \lambda_2\gamma_\rho\lambda_3\gamma^\mu\lambda_1 = 0 \tag{2.2}$$

is required for supersymmetric invariance of the Green-Schwarz superstring [15]. The action of linear groups on division algebras can be found in the formulation of superstring theory [16]. The known particle multiplets also may be derived from the representations of the exceptional group, defined by division algebra modules, which also arise in the spinor space of the standard model.

The Hopf Fibration and Twistor Space

Let $[a^\dagger \upsilon b] = \upsilon_\mu\,(a\gamma^\mu b)$ and $[[x, y, z]] = 0$ Then $[PP] = 0$ for massless bosonic particles in dimensions $D = \nu + 2$, which is solved by $P = \psi\,\psi^\dagger$. If $\psi = [\xi\eta]$, then

$$P = \psi\psi^\dagger = \begin{bmatrix} \xi\xi^* & \xi\eta^* \\ \eta\xi^* & \eta\eta^* \end{bmatrix} \tag{2.3}$$

and

$$det\ P = (\xi\xi^*)(\eta\eta^*) - (\xi\eta^*)(\eta\xi^*) = 0,$$

reflecting the light-like nature of the spinor bilinear.

There is a symmetry algebra which leaves $\psi\psi^\dagger$ unchanged. It may verify that the symmetry algebra is $S^{\nu-1}$.
The Hopf fibrations are
$S^1 \to S^1/\mathbb{Z}_2 = \mathbb{RP}^1$, $S^3 \to S^2 = \mathbb{CP}^1$, $S^7 \to S^4 = \mathbb{HP}^1$ and $S^{15} \to S^8 = \mathbb{OP}^1 = \{\lambda\}/\sim$,

where $\lambda \sim \lambda'$ if $\lambda\lambda^\dagger = \lambda'\lambda'^\dagger$.

The equation relating twistor space and Minkowski space-time is
$\omega^\alpha = \psi_{\dot{\alpha}} X^{\alpha\dot{\alpha}}$.
The supertwistor is $Z^A = (\omega^\alpha, \psi_{\dot{\alpha},\xi})$ and the spin-shell constraint is

$\frac{1}{2}(\omega^\alpha \psi_\alpha + \psi_{\dot\alpha}\omega^{\dot\alpha}) + \frac{1}{2}\xi\bar\xi = 0.$

If $\psi\,\psi^\dagger = P$, $\omega\,\omega^\dagger = X[XP] - \frac{1}{2}P[XX]$ and $J^{MN} = \frac{1}{2} Z^{\dagger\Gamma}{}_{MN}Z$ generate the conformal group, while $\prod = -1/(\upsilon+1)\, J^{MN}\Gamma_{MN}Z$ provides conformally covariant generators for $S^{\nu-1}$. Given a supersymmetric extension of the symplectic metric g_{AB}, which is the invariant tensor of OSp $(N,4;K_\nu)$, the superconformal group in ν+2 dimensions, the spin-shell constraint is $\frac{1}{2} g_{AB}Z^A Z^B = 0$ [13].

The $S^{\nu-1}$ transformations, when $\nu \neq 8$, are $\psi \to \psi\,\Omega$, $\omega \to \omega$, with $[\Omega]=1$. The spin shell in the twistor space $\mathbf{K}\mathbb{P}^{\nu}{}_3$ is

$$\mathcal{N} = \{(\psi,\Omega) \in \mathbb{KP}^3_\nu | \tfrac{1}{2} g_{AB} Z^A Z^B = 0\},$$

where N has dimension 2ν+1. When $\nu = 8$, the condition may be chosen to be $T = (\psi\,\psi^\dagger)\,\omega - \psi\,[\psi^\dagger\,\omega] = 0$ with $\omega = \chi\psi$ [17].

The geometry of N may be deduced from fibrations of $S^{n\nu-1}$ to $\mathbf{K}\mathbb{P}^{n\nu-1}$. The fibres are S^7 when $n = 8$ and $\nu = 1$ or 2, and, if $n = 8$ and $\nu = 3$, S^7 transformations may be defined over a set of equivalence classes of coordinates on $\mathbb{OP}^2$ [18]. The coordinates of the supertwistor may be chosen such that one of the coordinates is aligned with the lightlike direction representing a point on the sphere at infinity S^{23} and $\nu - 2 = 6$ components are set equal to zero through the S^7 transformation, and the spin shell is diffeomorphic to S^{17}.

After the condition of equivalence under the symmetry algebra of the bilinear $\psi\,\psi^\dagger$ is imposed, the number of independent parameters is *9*, following from the invariance of the ten-dimensional vector $P = \psi\,\psi^\dagger$, and the topology of this space S would be S^9. It may be noted that the homology group of the twenty-six dimensional space E_6/F_4 is non-zero only for the indices *9* and *17*. From the dimensions, there would be a transformation from the space {S, *N*} to E_6/F_4. Then the E_6/F_4 bosonic string theory may be described in terms of octonionic twistor variables. The existence of a supersymmetry through the lattice representation of the E_6 supercharges has been given [17]. A closed superstring theory in the twistor formalism providing a theoretical basis for spinning particles then could be derived.

3. TWISTORS AND THE REPRESENTATION OF SPIN

Spin in the Supersymmetric Point Particle Action

The supersymmetric sigma model for the superstring is

$$I_{superstring} = -\frac{1}{4\pi\alpha'}\int d^2\xi \eta_{\mu\nu} e\Big[g^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu + i e^\alpha_a \bar\psi^\mu \rho^a \partial_\alpha \psi^\nu + 2e^\alpha_a e^\beta_b \bar\chi_\alpha \rho^b \rho^a \psi^\mu \Big[\partial_\beta X^\nu + \frac{1}{2}\psi^\nu \chi_\beta\Big]\Big], \tag{3.1}$$

where $\{\rho^\alpha\}$ are two-dimensional Dirac matrix includes the fermion fields with intrinsic spin [18]. However, there is no immediate description of fields of arbitrary spin in the Lagrangian until the fields are expanded in terms of vibrational modes. It is not a direct generalization of the spinning particle action with an extra term given by the Pauli spin matrices [19]

$$\begin{aligned}\mathcal{L}_{spinning\ point\ particle} &= \frac{1}{2}m\sum_i \dot s_i^2 + niTr(\sigma_3 s^{-1}\dot s) \\ &= \frac{1}{2}m\dot r^2 + \frac{1}{4}mr^2 Tr\dot{\hat X}^2 + niTr(\sigma_3 s^{-1}\dot s)\end{aligned} \tag{3.2}$$

such that $\dot X = \sigma^i \dot x_i = S\dot\sigma_3 S^{-1}$ parameterizes S^2 or the supersymmetric Lagrangian [20]

$$\begin{aligned}\mathcal{L}_{\substack{supersymmetric\ spinning\\ point\ particle}} &= \frac{im}{2}\frac{\partial}{\partial t}X_a d_\theta X_a - nTr(\sigma_3 s_*^\dagger d_\theta s_*)\\ &= \frac{1}{2}m\dot{x}_a^2 + \frac{i}{2}f_a\dot{f}_a + inTr(\sigma_3 s^\dagger \dot{s}) + 2ni\epsilon_{abc}\hat{x}_a\xi_b\xi_c,\end{aligned} \tag{3.3}$$

with f_a being an anticommuting Grassmann variable, while the spin vector is $S_a = -\frac{1}{2}\varepsilon_{abc} f_b f_c$ which can be coupled to the magnetic field.

Spin Condition from Spinning Twistor String Action

However, with a twistor formulation of superstring theory, the spin of the particle might be separately described. Consider the closed twistor string action [21]

$$I_{closed\ twistor\ string} = \int_S [e^{++}\wedge \overline{\Upsilon}_\Sigma \overline{\nabla}\Upsilon^\Sigma + d^2\xi L_G] \tag{3.4}$$

where $e^{\pm\pm}$ are the worldsheet zweibein one-forms, $\Upsilon^\Sigma = (\mu^{\dot\alpha}, \lambda_\alpha; \eta_i)$ is the supertwistor and L_G consists of free fermion terms related to the gauge group G. The constraint

$$\overline{\Upsilon}_\Sigma \Upsilon^\Sigma = \bar\lambda_{\dot\alpha}\mu^{\dot\alpha} - \bar\mu^\alpha\lambda_\alpha + 2i\bar\eta^i\eta_i = 0. \tag{3.5}$$

follows from the field equations. A solution [22,23] is

$$\mu^{\dot\alpha} = (x^{\alpha\dot\alpha} + i\theta_i^\alpha\bar\theta^{\dot\alpha i})\lambda_\alpha \qquad \eta_i = \theta_i^\alpha\lambda_\alpha \tag{3.6}$$

since

$$\begin{aligned} d\mu^{\dot\alpha} &= (dx^{\dot\alpha\alpha} + id\theta_i^\alpha\bar\theta^{\dot\alpha i} + i\theta_i^\alpha d\bar\theta^{\dot\alpha i})\lambda_\alpha\\ &\quad + (x^{\dot\alpha\alpha} + i\theta_i^\alpha\bar\theta^{\dot\alpha i})d\lambda_\alpha\\ d\eta_i &= d\theta_i^\alpha\lambda_\alpha + \theta_i^\alpha d\lambda_\alpha \end{aligned} \tag{3.7}$$

such that

$$\begin{aligned}\int_S e^{++}\wedge\bar\Upsilon_\Sigma d\Upsilon^\Sigma &= \int_S e^{++}\wedge(d\mu^{\dot\alpha}\bar\lambda_{\dot\alpha} - d\lambda_\alpha\bar\mu^\alpha - 2id\eta_i\bar\eta_i)\\ &= \int_S e^{++}\wedge[(dx^{\dot\alpha\alpha} + id\theta_i^\alpha\bar\theta^{\dot\alpha i} + i\theta_i^\alpha d\bar\theta^{\dot\alpha i})\bar\lambda_{\dot\alpha}\lambda_\alpha\\ &\qquad + (x^{\dot\alpha\alpha} + i\theta_i^\alpha\bar\theta^{\dot\alpha i})\bar\lambda_{\dot\alpha}d\lambda_\alpha - d\bar\lambda_{\dot\alpha}(x^{\dot\alpha\alpha} - \theta_i^\alpha\bar\theta^{\dot\alpha i})\lambda_\alpha\\ &\qquad - 2i(d\theta_i^\alpha\lambda_\alpha + \theta_i^\alpha d\lambda_\alpha)\bar\theta^{\dot\alpha i}\lambda_{\dot\alpha}]\\ &= \int_S e^{++}(dx^{\dot\alpha\alpha} - id\theta_i^\alpha\bar\theta^{\dot\alpha i} + i\theta_i^\alpha d\bar\theta^{\dot\alpha i})\bar\lambda_{\dot\alpha}\lambda_\alpha. \end{aligned} \tag{3.8}$$

and

$$\begin{aligned}\overline{\Upsilon}_\Sigma\Upsilon^\Sigma &= \bar{\lambda}_{\dot{\alpha}}(x^{\dot{\alpha}\alpha} + i\theta_i^\alpha\bar{\theta}^{\dot{\alpha}i})\lambda_\alpha - (x^{\dot{\alpha}\alpha} - i\theta_i^\alpha\bar{\theta}^{\dot{\alpha}i})\bar{\lambda}_{\dot{\alpha}}\lambda_\alpha + 2i\bar{\theta}^{\dot{\alpha}i}\bar{\lambda}_{\dot{\alpha}}\theta_i^\alpha\lambda_\alpha \\ &= 2i(\theta_i^\alpha\bar{\theta}^{\dot{\alpha}i} + \bar{\theta}^{\dot{\alpha}i}\theta_i^\alpha) \\ &= 0.\end{aligned} \tag{3.9}$$

given the anticommuting variables θ_i^α and $\bar{\theta}^{\dot{\alpha}i}$. The product

$$\overline{\Upsilon}_\Sigma\Upsilon^\Sigma = \tau \tag{3.10}$$

is a condition on a massless particle with spin $s = \frac{|\tau|}{2}$ [20], and the closed twistor string action does not describe such fields. The action of the spinning twistor string [24]

$$\begin{aligned}I_{spinning\ twistor\ string} &= \int_S \left[\nabla\overline{\Upsilon}_\Sigma \wedge {}_*\nabla\Upsilon^\Sigma + d^2\xi\Xi(\xi)(\overline{\Upsilon}_\Sigma\Upsilon^\Sigma - \tau)\right] \\ &= \int_S e^{++} \wedge e^{--}[\nabla_{++}\overline{\Upsilon}_\Sigma\nabla_{--}\Upsilon^\Sigma + \nabla_{--}\overline{\Upsilon}_\Sigma\nabla_{++}\Upsilon^\Sigma] \\ &\quad + \int_S d^2\xi\ \Xi(\xi)(\overline{\Upsilon}_\Sigma\Upsilon^\Sigma - \tau),\end{aligned} \tag{3.11}$$

yields the relation for non-zero spin with Ξ (ξ) being a Lagrange multiplier.

Projection of the Nonvanishing Spin of the Ten-Dimensional Twistor Superstring to Four Dimensions

The generalization of the twistor superstring to ten-dimensional superspace can be derived with spinor and vector Lorentz harmonics, $v^-_{\underline{\alpha}q}$, which span S^8, and $u^{--}_{\vec{a}}$, satisfying

$$\begin{aligned}&2v^-_{\underline{\alpha}q}v^-_{\underline{\beta}q} = u^{--}_{\underline{a}}\Sigma^{\underline{a}}_{\underline{\alpha\beta}} \\ &v^-_{\underline{\alpha}p}\Sigma^{\underline{\alpha\beta}}_{\underline{a}}v^-_{\underline{\beta}q} = \delta_{pq}u^{--}_{\underline{a}} \\ &\quad \underline{a},\ \underline{b} = 0,1,...,9,\ \underline{\alpha},\ \underline{\beta} = 1,...,16,\ p,q = 1,...,8,\end{aligned} \tag{3.12}$$

where $\{\Sigma^{\underline{a}}{}_{\underline{\alpha\beta}}\}$ is the set of Dirac matrices in ten dimensional Minkowski space-time [24]. The superspace action can be reduced to

$$I_{\substack{ten-dimensional \\ twistor\ superstring}} = \int_S e^{++} \wedge (d\mu_q^{-\underline{\alpha}}v^-_{\underline{\alpha}q} - \mu_q^{-\underline{\alpha}}dv^-_{\underline{\alpha}q} - id\chi_q^-\chi_q^-) \tag{3.13}$$

Where

$$\mu_q^{-\underline{\alpha}} = X^{\underline{a}}\Sigma^{\underline{\alpha\beta}}_{\underline{a}}v^-_{\underline{\beta}q} - \frac{i}{2}\Theta^{\underline{\alpha}}\Theta^{\underline{\beta}}v^-_{\underline{\beta}q} \qquad \chi_q^- = \Theta^{\underline{\alpha}}v^-_{\underline{\alpha}q}, \tag{3.14}$$

and the inner product must be

$$\mu_q^{-\underline{\alpha}} v_{\underline{\alpha}q}^{-} + i\chi_q^{-}\chi_q^{-}. \tag{3.15}$$

By the relation [22]

$$\mu_q^{-\underline{\alpha}} v_{\underline{\alpha}p}^{-} = X^{\underline{a}} u_{\underline{a}}^{--}\delta_{pq} + \frac{i}{2}\chi_q^{-}\chi_p^{-}, \tag{3.16}$$

it follows that

$$\mu_q^{-\underline{\alpha}} v_{\underline{\alpha}q}^{-} = 8X^{\underline{a}} u_{\underline{a}}^{--} + \frac{i}{2}\chi_q^{-}\chi_q^{-}. \tag{3.17}$$

Then

$$\mu_q^{-\underline{\alpha}} v_{\underline{\alpha}q}^{-} + i\chi_q^{-}\chi_q^{-} = 8X^{\underline{a}} u_{\underline{a}}^{--} + \frac{3i}{2}\chi_q^{-}\chi_q^{-} = 8X^{\underline{a}} u_{\underline{a}}^{--} \tag{3.18}$$

Because χ_q^{-} is an odd Grassmann variable and $\chi_q^{-}\chi_q^{-} = 0.$ The ten-dimensional equivalent of $\bar{\Upsilon}_\Sigma\Upsilon^\Sigma$ is nonvanishing and represents non-zero spin. The expansion into spinorial Lorentz harmonics is sufficient to represent any rotating configuration corresponding to a particle with spin. A reduction of this model to four dimensions would yield an action that includes

$$I_{four-dimensional\ twistor\ superstring} = \int e^{++} \wedge \Pi^{\dot{\alpha}\alpha}\bar{v}_{\dot{\alpha}}^{-}v_{\alpha}^{-}$$
$$\Pi^{\dot{\alpha}\alpha} = dx^{\dot{\alpha}\alpha} - id\theta_i^{\alpha}\bar{\theta}^{\dot{\alpha}i} + i\theta_i^{\alpha}d\bar{\theta}^{\dot{\alpha}i} \qquad v_{\alpha}^{-} = \lambda_{\alpha} \tag{3.19}$$

together with the supertwistor relation $\bar{\Upsilon}_\Sigma\Upsilon^\Sigma = \tau$ for non-zero spin, and the sphere of spinorial harmonics would be projected to S^2. The twistor formalism provides an intrinsic rotational motion of the string which would represent the spin. Consequently, the two-dimensional spherical model of elementary particles with charge and spin is verified.

The N = 4 super-Yang-Mills theory may be found from a boundary theory of the reduction of ten-dimensional superstring through the compactified solution AdS5 x S^5. The isometry group of then ten-dimensional space is *SO* (4, 2) x *SO* (6) that is isomorphic within a $\mathbb{Z}_2$ factor to the bosonic part of the symmetry group of N = 4 super-Yang-Mills theory, which has an invariance under conformal and internal *SU* (4) transformations. There is a twistor formulation of the super-Yang-Mills theory, which will be constrained to be consistent with boundary conditions for spin-s fields on AdS_5. Peeling theorems have been adapted to asymptotically anti-de Sitter space-times to determine the fall-offs of conformally related massless spin-s fields on conformally compactified anti-de Sitter space-times [25]. The dependence may be given for five-dimensional anti-de Sitter space by replacing the dimension d = 4 by d + 1 = 5. The gauge symmetry results from the isometry group of *SO* (6), and therefore, boundary conditions for the entire bundle must be given. The group *SU* (3) is an S^3 bundle over S^5 [26], and the boundary conditions for the spin-s *SU* (2) fields in the superstring theory on AdS_5 x S^5 then must be defined to be consistent with the transition functions of this bundle to describe *SU* (3) fields on *I* and Minkowski space-time.

4. INTEGRABILITY OF A SECTOR OF THE THEORY OF THE STRONG INTERACTIONS

Collinear Trajectories and Solutions to the Super-Yang-Mills Field Equations by Twistor Transform

The existence of solutions to the super-Yang-Mills field equations in four [27], six [3] and ten dimensions [1] through a twistor transform requires vanishing curvature along the super null lines in super-Minkowski space-time. The derivation of the *N*= 4 super-Yang-Mills theory in four-dimensional Minkowski space-time by dimensional reduction over T^6, with the isometry group U $(1)^6$ [28], follows from the orthogonal splitting of Minkowski space-time that yields a sequence of integrability conditions for the field equations between four and ten dimensions.

The spaces of light like lines in four, six and ten-dimensional Minkowski space-time are S^2, S^4 and S^8, which represent the base spaces of three of the Hopf fibrations. The action of S^1, S^3 and S^7 on these Hopf bundles [13] generate differential systems describing the integrable parallelisms of tangent vector fields on the fibres. The integrability conditions for field equations of the super-Yang-Mills theories may be solved similarly if the superconnection has a vanishing supercurvature.

Yang-Mills Theory constructed from a Hopf Fibration

The Spin (9)-invariant theory representing the Hopf fibration $S^{15} \overset{S^7}{\rightarrow} S^8$ is

$$\begin{aligned}
&\int_{S^8} d\Omega\ Tr(\hat{F}^{[abc]}\hat{F}_{[abc]}) \\
&\hat{F}_{abc} = \left(\hat{x}_a \frac{\partial}{\partial \hat{x}_b} - \hat{x}_b \frac{\partial}{\partial \hat{x}_a}\right)\hat{A}_c + \hat{x}_a[\hat{A}_b, \hat{A}_c] \\
&\hat{A}_a = i\alpha\hat{\Sigma}_{ab}\hat{x}_b
\end{aligned} \tag{4.1}$$

where $\hat{x}_a$, α = 1, ..., 9 are the coordinates of $\mathbb{R}^9$, the sphere is represented by $\hat{x}_a\hat{x}_a$ = 1 and $\hat{A}_a\hat{x}_a = 0$ [29]. The solutions to the Euclidean equations will represent finite-action instantons of the theory on $\mathbb{R}^8$ derived by stereographic projection.

By using the transformations,

$$\begin{aligned}
\hat{x}_\mu &= \frac{2x_\mu}{1+x^2} \qquad \hat{x}_9 = \frac{1-x^2}{1+x^2} \\
\frac{\partial}{\partial x_\mu} &= \frac{\partial \hat{x}_\nu}{\partial x_\mu}\frac{\partial}{\partial \hat{x}_\nu} = \frac{2(1+x^2)\delta_\nu{}^\mu - 4x^\mu x_\nu}{(1+x^2)^2}\frac{\partial}{\partial \hat{x}_\nu} \\
A_\mu &= \frac{2}{1+x^2}(\hat{A}_\mu - x_\mu\hat{A}_9),
\end{aligned} \tag{4.2}$$

it follows that

$$\begin{aligned}
F^{\mu\nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu + [A^\mu, A^\nu] \\
&= \left[\frac{2}{1+x^2}\frac{\partial}{\partial \hat{x}_\mu} - 4\frac{x^\mu x_\rho}{(1+x^2)^2}\frac{\partial}{\partial \hat{x}_\rho}\right]\frac{2(\hat{A}^\nu - x^\nu\hat{A}^9)}{(1+x^2)^2} \\
&\quad - \left[\frac{2}{1+x^2}\frac{\partial}{\partial \hat{x}_\nu} - 4\frac{x^\nu x_\rho}{(1+x^2)^2}\frac{\partial}{\partial \hat{x}_\rho}\right]\frac{2(\hat{A}^\mu - x^\mu\hat{A}^9)}{(1+x^2)^2} \\
&\quad + [\hat{A}^\mu, \hat{A}^\nu] - x^\nu[\hat{A}^\mu, \hat{A}^9] + x^\mu[\hat{A}^\nu, \hat{A}^9],
\end{aligned} \tag{4.3}$$

$$\hat{F}_{\mu\nu\rho} = 2\left(\frac{x_\mu}{1+x^2}\frac{\partial}{\partial\hat{x}_\nu} - \frac{x_\nu}{1+x^2}\frac{\partial}{\frac{\partial}{\partial\hat{x}_\mu}}\right)\hat{A}_\rho + \frac{2x_\mu}{1+x^2}[\hat{A}_\nu, \hat{A}_\rho]$$

$$\hat{F}_{\mu\nu 9} = 2\left(\frac{x_\mu}{1+x^2}\frac{\partial}{\partial\hat{x}_\nu} - \frac{x_\nu}{1+x^2}\frac{\partial}{\frac{\partial}{\partial\hat{x}_\mu}}\right)\hat{A}_9 + \frac{2x_\mu}{1+x^2}[\hat{A}_\nu, \hat{A}_9], \qquad (4.4)$$

and the action (4.1) is equivalent to

$$\int_{\mathbb{R}^8} d^8x\ Tr(F^{\mu\nu}F_{\mu\nu}) \qquad (4.5)$$

when a single point is removed from S^8. This integral is invariant under *SO* (8) x E^8, which has the same dimension as *SO* (9). The translation symmetry is considered to be separate from the compact gauge group that has been reduced from *SO* (9) to *SO* (8).

This integral may be cast in the form

$$\frac{1}{4}\int d^8x Tr(\xi_m\xi_m) + \frac{1}{2}C_{\mu\nu\rho\sigma}Tr(F^{\mu\nu}F^{\rho\sigma})$$
$$\xi_m = F_{m8} - \frac{1}{2}c_{mnp}F_{np} \qquad (4.6)$$
$$C^{mnp8} = c^{mnp} \qquad C^{mnpq} = \frac{1}{2}\epsilon^{mnpqrst}c_{rst}$$

where c_{mnp} are the octonion structure constants, and a method for finding instanton solutions with the potential and field strength

$$A_M(x) = -\frac{2}{3}\frac{1}{1+x^2}G_{MN}x_N$$
$$F_{MN}(x) = \frac{2}{3}\frac{2+x^2}{(1+x^2)^2}G_{MN} + \frac{4}{3}\frac{1}{(1+x^2)^2}x_{[M}G_{N]P}x_P \qquad (4.7)$$
$$-\frac{2}{9}\frac{1}{(1+x^2)^2}x^P x^Q C_{MNPR}{G_Q}^R$$

and finite topological charge defined over a four-dimensional hypersurface, begins with setting $\xi_m = 0$ [30], where G_{MN} are generators of *SO* (7). Since

$$
\begin{aligned}
F_{MN}F^{MN} = & \frac{4}{9}\frac{(2+x^2)^2}{(1+x^2)^4}G_{MN}G^{MN} + \frac{16}{9}\frac{2+x^2}{(1+x^2)^4}G^{MN}x_{[M}G_{N]P}x^P \\
& - \frac{8}{27}\frac{2+x^2}{(1+x^2)^4}G^{MN}x^P x^Q C_{MNPR}G_Q{}^R \\
& + \frac{16}{9}\frac{1}{(1+x^2)^4}x^M G^{N]Q}x^Q x_{[M}G_{N]P}x^P \\
& - \frac{16}{27}\frac{1}{(1+x^2)^4}x^{[M}G^{N]P}x_P x^R x^S C_{MNRT}G_S{}^T \\
& + \frac{4}{81}\frac{1}{(1+x^2)^2}x^P x^Q C_{MNPR}G_Q{}^R x^S x^T C^{MN}{}_{SU}G_T{}^U,
\end{aligned} \tag{4.8}
$$

$$
Tr(F_{MN}F^{MN}) = \frac{1}{(1+x^2)^4}\left[\frac{13156}{81}x^4 + \frac{2156}{9}x^2 + \frac{784}{3}\right] \tag{4.9}
$$

and the integral over the Euclidean space diverges. The remaining term, however, is a total derivative, and the boundary integral vanishes if

$A_M \sim \frac{1}{(1+x^2)^\alpha}G_{MN}x^N$

and $\alpha > 2$. Then the action also equals

$$
\frac{1}{2}\int d^8x\ Tr(\xi_m\xi_m) \tag{4.10}
$$

which is invariant under *SO* (7). When the energy of the Yang-Mills field is conserved, the symmetry group may be enlarged to *SO* (8) [31]. Therefore, the action is invariant under *SO* (8) / *SO* (7) transformations or octonion multipication on S^7. This method may be contrasted with the use of auxiliary fields which belong to a division algebra in supersymmetric Yang-Mills theories for closure of the superalgebra and do not occur in the on-shell action [32].

The sphere in nine dimensions is diffeomorphic to a compactified, complexified four-dimensional Euclidean space. Therefore, the model may be projected to a theory described geometrically by an S^7 fibre on $\mathbb{R}^4$ after imposing additionally conditions on the dependence of the potential on the complex coordinates. This action would describe a viable model of the vector bosons of the strong interactions, since the energy dependence of the coupling is unaffected by the replacement of the structure constants of *SU*(3) by that of the octonions [33].

Twistors arise both in the classical solutions [34] and the quantum amplitudes [35] of a self-dual Yang-Mills theory. It has been found that two-dimensional symmetric coset space sigma models and the supersymmetric generalizations with a $\mathbb{Z}_4$ automorphism can be derived from self-dual Yang-Mills actions in four and eight dimensions respectively [36]. Rotations in the flat space including the octonion algebra leave invariant a quadratic form of the latter integral [30]. This S^7 symmetry also arises as the set of transformations of the spinors that fix the vector fields in the twistor form of the heterotic string [37]. It is known also that the heterotic string effective action to quartic order in the curvature tensor can be cast in the form of a ten-dimensional higher-derivative super-Yang-Mills theory [38]. Reduction of the self-dual sector to eight dimensions again yields the eight-dimensional quadratic integral.

There exist sectors of quantum chromodynamics which can be characterized by integrability. It is found in the equations describing the action of the collinear conformal group on operators representing products of certain matter fields. A spectral curve of constant energy of states in these sectors is known to form a Riemann surface with the genus equal to the lengths of an integral Heisenberg chain and demonstrates the integrability. The BMN states arising from the quantization of the parallel plane wave limit of the $AdS_5 \times S^5$ superstring require surfaces of infinite genus [39]. The sum over the genus, which occurs in the superstring perturbation expansion, reflects the equivalence with the superstring sigma model with a coset space that includes $SU(3)$ after a restriction to the nonsupersymmetric sector. The integrability of that sector of quantum chromodynamics also follows from the finiteness of the superstring perturbation series. This algebra of the collinear group and the dilations can be embedded in a higher-dimensional real plane, which is consistent with integrability of the self-dual Yang-Mills theory in this space. There exists a gauge such that the twistor action for supersymmetric gauge theories consists only of a nonlocal term representing maximum helicity violating vertices and the propagator for the anti-holomorphic derivative on twistor space yields a delta function that has support on sets of collinear points [40]

Twistor Form of $N = 2$ String Amplitudes

The $N = 2$ string theory is formulated in a space-time of signature (2,2) and describes a self-dual $N = 4$ super-Yang-Mills theory [41]. A higher-genus surface can be decomposed into a topological sum of thrice-punctured spheres and connecting cylinders allowing a twistor description. In the Euclidean metric, the open string field theory formulated on CP^3 is

$$S_{N=2\ open\ string} = \int d\zeta \int d\theta^1 d\theta^2 d\theta^3 \theta^4 \langle tr\left(\mathcal{A}Q\mathcal{A} + \frac{2}{3}\mathcal{A}^3\right)\rangle \qquad (4.11)$$

where the expectation value refers to the integral over four-dimensional spatial and Grassmann coordinates, A is the string field defined by

$$\mathcal{A} = \mathcal{A}_{\bar{D}} + \mathcal{A}_{\bar{\zeta}} d\bar{\zeta},\ \mathcal{A}_{\bar{D}} = \zeta^\alpha A_\alpha + \theta^i \bar{\chi}^i +$$
$$\frac{\nu}{2!}\theta^{ij}\hat{\zeta}^\alpha \phi_{\alpha ij} + \frac{\nu^2}{3!}\theta^{ijk}\hat{\zeta}^\alpha\hat{\zeta}^\beta \chi_{\alpha\beta ijk} + \frac{\nu^3}{4!}\theta^{ijk\ell}\hat{\zeta}^\alpha\hat{\zeta}^\beta \chi_{\alpha\beta ijk} + \frac{\nu^3}{4!}\theta^{ijk\ell}\hat{\zeta}^\alpha\hat{\zeta}^\beta\hat{\zeta}^\gamma G_{\alpha\beta\gamma ijk}$$
$$\mathcal{A}_{\bar{\zeta}} = \frac{\nu^2}{2!}\theta^{ij}\phi_{ij} + \frac{\nu^3}{3!}\theta^{ijk}\hat{\zeta}^\alpha \chi_{\alpha ijk} + \frac{\nu^4}{4!}\theta^{ijk\ell}\hat{\zeta}^\alpha\hat{\zeta}^\beta G_{\alpha\beta ijk\ell},$$

with ζ being a coordinate on $\mathbb{CP}^1$,

$$\hat{\zeta}^\alpha = \begin{pmatrix} -1 \\ -\bar{\zeta} \end{pmatrix},\ \nu = (1+\zeta\bar{\zeta})^{-1},\ G_{\alpha\beta} = \frac{1}{4!}\epsilon^{ijk\ell} G_{\alpha\beta ijk\ell} \text{ and } \chi^i_\alpha = \frac{1}{3!}\epsilon^{ijk\ell}\chi_{\alpha jk\ell}$$

[42], yielding the component action

$$\left\langle tr\left(G^{\alpha\beta}F_{\alpha\beta} + 2\,\chi^{\alpha i}\nabla_\alpha\tilde{\chi}_i + \frac{1}{4}\phi^{ij}\nabla_\alpha\nabla^\alpha\phi_{ij} + \ \phi^{ij}\tilde{\chi}_i\tilde{\chi}_j\right)\right\rangle$$

[43]. It has been demonstrated that string amplitudes may be derived from a holomorphic Chern-Simons theory on twistor space [44]. The N = 2 open string amplitudes [45] then can be expressed in the twistor formalism. It follows then that a one-loop n-point amplitude in twistor space would be

$$\prod_{i=1}^{n}\int_{-\infty}^{\infty} d\sigma_i d^2\mu_i{}^{\dot a} d^4\psi_{1i}^A d^4\psi_{2i}^A V_i(\sigma_i,\mu_i^{\dot a},\psi_{ri}^A)\int \frac{D^{4|8}Z_A\wedge D^{4|8}Z_B}{Vol(GL(2;\mathbb{C}))}$$
$$\int d^{3|8}Z_1 d^{3|8}Z_2 d^{3|8}Z_3 d^{3|8}Z_4 \prod_{i=1}^{n}\delta^3(\mu_{i\dot a}+x_{a\dot a}\lambda_i^a)\prod_{r=1}^{2}\delta^4(\psi_{ri}^A+\theta_{ra}^A\lambda_i^a)$$
$$\prod_{i=1}^{n}\frac{1}{\sigma_{i+1}-\sigma_i}\Delta_{12}\Delta_{34} \qquad (4.12)$$

where L_{AB} is the line between Z_A and Z_B corresponding to (x, θ) in the supertwistor space, the lines $\mu_{i\dot a}+x_{a\dot a}\lambda_i^a=0$ and $\psi_i^A+\theta_{ra}^A\lambda_i^a$ pass through { $\sigma\ i$ }, the propagator in twistor space is

$$\Delta_{ij}=\bar\delta^{2|4}(Z_i,*,Z_j)=\int_{\mathbb{C}\times\mathbb{C}}\frac{ds\ dt}{st}\delta^{4|4}(Z_i+sZ_*+tZ_j)$$

and * refers the the twistor at infinity $Z\ *$ and $V_i(\sigma_i,\mu_{ri}^{\dot a},\psi_i^A)$ represents the i^{th} vertex operator defined through the expansion of the N = 2 string fields in the action [46].

The action representing a nonlinear sigma model on the supersphere S (2,2) arising from the supersymmetric Hopf fibration, U(1) → $S^{(3,2)}$ → $S^{(2,2)}$ has been constructed [47]. The integrability of the supersymmetric generalization of the theory with the fibre S^7 would verify its finiteness in contrast with the problems related to Borel summability of the perturbation series in quantum chromodynamics. The integrability of the supersymmetric generalization of the theory described by action in Eq. (32) may be contrasted with the problems related to Borel summability of the perturbation series in quantum chromodynamics [48] [49] in the bosonic sector. Even though the spectrum of the closed symmetric N = 2 string consists of a single massless scalar, it is increased in the N = 2 heterotic string theory to a set of particles corresponding to the sum of the number of elements of a self-dual even lattice in 24 dimensions and the 24 internal oscillations, and the existence of an invariant null vector causes the reduction of the effective field theory to two or three dimensions still allows the dynamics to occur in an embedding four-dimensional space-time through a local variation of the null vector [50]. Since the signatures of the submanifolds are (1, 1) and (1, 2) respectively, and embedding in a manifold with Lorentzian signature is feasible, which would be equivalent to a Wick rotation of one of the time coordinates in the geometry of (2,2) signature. The calculations of the amplitudes may be extended to a larger number of vertex operators by not utilizing methods of cancellation of the amplitudes of massless scalars describing self-dual Yang-Mills fields. The twistor formulation then may be derived from the AdS_5 x S^5 superstring [51] [52], the restriction of N=2 string amplitudes of the form given in Eq.(33) to a nonsupersymmetric sector and gauge/string duality [53].

CONCLUSION

The twistor formulation of superstring theory can be defined such that the spin of the physical state is given. It arises as a constraint in the spinning twistor action. The description of elementary particles with spin is supported by a physical two-dimensional spherical model. The fields in this ten-dimensional action may be expanded in spherical harmonics on the space of lightlike lines. When reduced to four dimensions, the spherical harmonics representing particles with spin would be defined on the sphere S^2. Therefore, the identification of the spherical model of the particle follows. It remains to establish the relation with the action of rotating superstring state for a charged particle. The charge density would appear to be uniformly distributed over a sphere for a sufficiently rapid rotation.. The formula for the mass in terms of the charge on the sphere [54] in the Lorentz model of an elementary particle such as the electron could be compared with the parameters of the twistor string. The equation for the tension of the heterotic string and the gauge coupling must be extended to energies for which the strength of the electromagnetic force achieves its value in elementary particle phenomenology.

Since the boundary values must be equated to super-Yang-Mills fields on Minkowski space-time, the twistors on I in the conformally compactified five-dimensional antide Sitter space must be conformally tranformed to twistors on Minkowski space-time to determine the components. The massless fields of the super-Yang-Mills theory then may be derived after the mapping supersymmetry transformation rules on I to a flat Lorentzian manifold.

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