Bulletin of Pure and Applied Science *Physics, Vol.42D No.2S (Special Issue)* *NCAEST-2023 P.40-43* *Original Research Article*

Print version ISSN 0970 6569 *Online version ISSN 2320 3218* *DOI: 10.62427/bpas.2023.42D.2S.5* *Available online at www.bpasjournals.com*

Relativistic Surface Gravity of Black Holes for Maximum Spin Parameter in XRBs

[1]Niharika Kumari, [2]Ramanand Raman and [3]Dipo Mahto*

Author's Affiliations:

[1]Research Scholar, University Department of Physics, T.M.B.U. Bhagalpur, Bihar 812007, India
[2]Guest Faculty, Department of Mathematics, Marwari College Bhagalpur, TMBU Bhagalpur, Bihar 812007, India
[3]Professor & Head, Department of Physics, Bhagalpur College of Engineering Under Department of Science and Technology, Govt. of Bihar, Bihar 812001, India & Former Head, Department of Physics, Marwari College Bhagalpur, TMBU Bhagalpur, Bihar 812007, India

***Corresponding author:**

Dipo Mahto
Professor & Head, Department of Physics, Bhagalpur College of Engineering Under Department of Science and Technology, Govt. of Bihar, Bihar 812001, India
E-mail: dipomahto@hotmail.com

ABSTRACT The surface gravity of a black hole depends on its mass, angular momentum and charge, but mainly dependent on the mass and angular momentum. The present paper provides a model for the surface gravity of spinning black holes due to relativistic correction for maximum spin parameter and concludes that the relativistic surface gravity attains the maximum values for the lower masses and vice-versa.

KEYWORDS Relativistic Surface gravity, Maximum Spin and XRBs

How to cite this article: Kumari N., Raman R. and Mahto D. (2023). Relativistic Surface Gravity of Black Holes for Maximum Spin Parameter in XRBs. *Bulletin of Pure and Applied Sciences- Physics,* 42D (2 Special Issue), 40-43.

1. INTRODUCTION

The surface gravity is a force at spatial infinity pulling a massive point particle away from the black hole (Poisson, 2004) and directly related to the black hole temperature given as $\kappa = T/2\pi$ (Pfeifer and Schuster, 2021). The surface gravity of non-spinning black hole is inversely proportional to its own mass (Narayan, 2006). The Kerr-Newman solution of surface gravity of black hole depends on its mass, angular momentum and charge (Newman & Janis 1965, Bardeen, Carter and Hawking 1973). Mahto and Kumari, gave a model for the surface gravity of black holes in terms of mass and its spin parameter (Kumari & Mahto, 2019). The relativistic surface gravity of black holes of quarter and three- quarter spin parameters in XRBs has been studied to discuss that the maximum values of the surface gravity is obtained for the lower mass and vice - versa (Chandan, 2023).

The present research paper provides a model the surface gravity of spinning black holes due to relativistic correction for maximum spin parameter and discusses some test black holes of maximum spin parameters in XRBs.

2. THEORETICAL DISCUSSION

The surface gravity of spinning black holes in terms of spin parameter (a*) is given by the following equation (Kumari & Mahto, 2019).

$$\kappa = \frac{1}{4M}\left[1-\frac{a^{*2}}{2}\right] \quad (1)$$

The above equation can be written as:

$$\kappa = \frac{1}{4}\left[1-\frac{a^{*2}}{2}\right](M)^{-1} \quad (2)$$

The spinning speed of black holes is ranging from 50% to about 99.99% of the speed of light (Reich, 2013). This limit of velocity is comparable to the velocity of light and hence the relativistic correction is possible. Apply the variation of mass of special theory of relativity (Bergmann, 1969).

$$M = \frac{M_0}{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}} \quad (3)$$

or

$$M = M_0\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

or

$$(M)^{-1} = (M_0)^{-1}\left[\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}\right]^{-1} \quad (4)$$

Using binomial theorem to solve the above equation and neglecting higher power of terms, we have

$$(M)^{-1} = \frac{1}{M_0}\left(1-\frac{v^2}{2c^2}\right) \quad (5)$$

Putting above value in the equation (2), we have

$$\kappa_{rel} = \frac{1}{4M_0}\left(1-\frac{a^{*2}}{2}\right)\left(1-\frac{v^2}{2c^2}\right). \quad (6)$$

The equation (6) gives a relativistic model for the surface gravity of spinning black holes. The spin parameters a* = +1 and -1 represent maximum spin. The sign (+) and (-) are taken into consideration for co-rotation and counter rotation of spinning black holes. For either rotation of the black holes, the net surface gravity of black holes remains the same for the particular test black holes due to the square of the spin parameter (a*). The surface gravity of some test black holes are calculated and listed in the following tables.

Table 1: Calculation of term $\left(1-\frac{v^2}{2c^2}\right)$ **Velocity of light (c) =** 3.00x10^{8}m/s

S. No.	Velocity of black holes (x10^{8}m/s) (v)	$\frac{v}{c}$	$\frac{v^2}{c^2}$	$\frac{v^2}{2c^2}$	$\left(1-\frac{v^2}{2c^2}\right)$
1.	1.50	0.50	0.2500	0.12500	0.87500
2.	1.65	0.55	0.3025	0.15125	0.84875
3.	1.80	0.60	0.3600	0.18000	0.82000
4.	1,95	0.65	0.4225	0.21125	0.78875
5.	2.10	0.70	0.4900	0.24500	0.75500
6.	2.25	0.75	0.5625	0.28125	0.71875
7.	2.40	0.80	0.6400	0.32000	0.68000
8.	2.55	0.85	0.7225	0.36125	0.63875
9.	2.70	0.90	0.8100	0.40500	0.59500
10.	2.85	0.95	0.9025	0.45125	0.54875
11.	2.97	0.99	0.9801	0.49005	0.50995

Table 2: Relativistic surface gravity of spinning black holes of spin parameter ($a^* = \pm 1$) in XRBs

S. N.	(%) Velocity	Velocity (10^8m/s)	$\kappa_{rel} = \frac{1}{8M_0}\left(1-\frac{v^2}{2c^2}\right)\times 10^{-34}$			
			M$_0$=5 M$_\Theta$ $\frac{1}{8M_0} = 0.125\times 10^{-31}$	M$_0$=10 M$_\Theta$ $\frac{1}{8M_0} = 0.0625\times 10^{-31}$	M$_0$=15 M$_\Theta$ $\frac{1}{8M_0} = 0.0416\times 10^{-31}$	M$_0$=20M$_\Theta$ $\frac{1}{8M_0} = 0.0312\times 10^{-31}$
1.	50	1.50	109.375	54.687	36.400	27.300
2.	55	1.65	106.093	53.046	35.308	26.481
3.	60	1.80	102.500	51.250	34.112	25.584
4.	65	1.95	098.593	49.296	32.812	24.609
5.	70	2.10	094.375	47.187	31.408	23.556
6.	75	2.25	089.843	44.921	29.900	22.425
7.	80	2.40	085.000	42.500	28.288	21.216
8.	85	2.55	079.843	39.921	26.572	19.929
9.	90	2.70	074.375	37.187	24.752	18.564
10.	95	2.85	068.593	34.296	22.828	17.121
11.	99	2.97	063.743	31.871	21.213	15.910

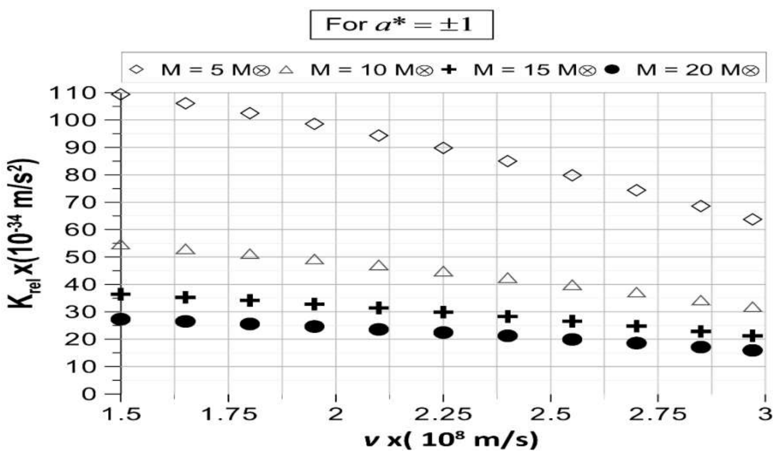

Figure 1: Relativistic surface gravity of black holes of spin parameter ($a^* = \pm 1$) in XRBs

3. RESULT AND DISCUSSION

The present work discusses the relativistic surface gravity of black holes in terms of mass, spin parameter and velocity of light given by $\kappa_{rel} = \frac{1}{4M_0}\left(1-\frac{a^{*2}}{2}\right)\left(1-\frac{v^2}{2c^2}\right)$.

This model is obtained by using the equation (1) for the surface gravity of the black holes and the variation of mass with velocity.

In the present work, we have calculated the relativistic surface gravity for different spin parameters (a* = +1, -1) of different test black holes of 5, 10, 15, 20 of solar mass in XRBs. For the heavier masses, the relativistic surface gravity decreases with increase of spinning velocity of black holes. The same change also occurs for other test black holes having lower masses, but the difference in amount of change in the relativistic surface gravity are greater for the lower masses and smaller for heavier masses. The tables 2 as well as figures1 show clearly that with increasing the velocity of black holes, the surface gravity decreases approximately linearly for the same mass of the black holes. The observations from our work mentioned in the table 2, it is clear that the relativistic surface gravity attains the maximum values for the lower masses and minimum value for higher masses. Hence we conclude that the velocity of black holes is mainly responsible for either increasing or decreasing the surface gravity of black holes.

4. CONCLUSION

1. For the lower masses of black holes, the relativistic gravity attains the maximum values and vice-versa.
2. The spinning velocity of black holes is mainly responsible to characterize the black holes in addition to their mass, spin and charge.
3. Due to the effect of co-rotation and counter rotation, there is no any change in the relativistic surface gravity of black holes.

****

REFERENCES

1. Bergmann P.G (1969). Introduction to the theory of relativity, Prentice Hall of India Pvt. Limited, New Delhi.
2. Bardeen J. M, Carter B and Hawking S.W. (1973). The Four Laws of Black Hole Mechanics, Communications in Mathematical Physics, 31(2), 161–170.
3. Chandan M. et al. (2023). Relativistic Surface Gravity of Black Holes of Quarter and Three- Quarter Spin parameters in XRBs, Journal of Chemical, Biological and Physical Sciences, 13(3), 270-274.
4. Mahto D and Kumari A. (2019). Study of Some Thermodynamic Parameters of Spinning Black Holes, A thesis submitted to TMBU Bhagalpur.
5. Newman .T. and Janis A.I. (1965), Note on the Kerr Spinning Particle Metric, Journal of Mathematical Physics, 6, 915.
6. Narayan R (2005), Black holes in Astrophysics, New Journal of Physics 7(19), 1-31.
7. Poisson E. (2004), A Relativist's Toolkit. Cambridge University Press.
8. Pfeifer C. and Schuster S (2021): Static spherically symmetric black holes in weak f (T)-gravity, arXiv: 2104.00116v2 [gr-qc].
9. Reich E.S. (2013), Spin Rate of Black Holes Pinned Down, Nature, Macmillan Publishing limited, 500, 135.