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Cosmological Model with Modified Chaplygin Gas and Dissipative Effects

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Abstract

A study of cosmological solutions in the presence of Modified Chaplygin gas (MCG) with dissipative term described by Eckart, truncated and full causal theories proposed by Israel and Stewart for a flat homogeneous isotropic Friedman-Robertson walker spacetime are given here. The interesting feature of the models is that it admits present acceleration phase as well as early inflationary phase with intermediate past deceleration phase of the evolution of the universe. The models may also help to predict future evolution of the universe. Statefinder diagnostic is also performed for the Eckart, truncated and full causal theories.

Keywords: dark energy, cosmology, viscosity, chaplygin gas.

1. INTRODUCTION

The data from different observational [1]-[2] strongly suggest that the present universe is filled with dark matter (~26.8%) and dark energy (~ 68.3%) in addition to the usual baryonic matter (~ 4.9%). Dark matters are weakly interacting massive particles (WIMPS) with zero effective pressure. Dark energy is responsible of the negative pressure in the universe. The cosmological constant (Λ) and its generalizations are a way to describe dark energy. However, the theoretical value of cosmological constant (Λ) is 60-120 orders greater than the observed value [3]. It is also considered self interacting scalar-field as a candidate for dark energy which is also known as quintessence field [4].

The Chaplygin gas (CG) is another candidate of Dark Energy (DE) models to explain the accelerated expansion of the universe. The CG plays a dual role at different epoch of the history of the universe: it can be as a dust like matter in the early time and as a cosmological constant at the late time [5]. The CG gas emerges as an effective fluid associated with D-branes [6] and can also be obtained from the Bron-Infeld action [7]. Recently, a modified form of Chaplygin gas (MCG) [8]-[9] has been considered to implement cosmological scenario. The MCG is more general and it is also consistent with observations [10].

Observational data [1]-[2] predict that the accelerating phase of present universe which might have emerged to this present state from an inflationary phase in the past. In the evolutions of the universe a number of processes might have occurred leading to a dissipative effect. Infact, some processes in cosmology and astrophysics can't be understood without a dissipative process. Under spatial homogeneity and isotropy spacetime, the bulk viscous pressure is the only admissible dissipative phenomenon [11]. Viscosity may be arises due to the decoupling of neutrinos from the radiation era, the decoupling of matter from radiation during the recombination era, creation of superstrings in the quantum era, particle collisions involving gravitons, cosmological quantum particle creation processes and formation of galaxies [12]. It has been predicted from observations that a non negligible dissipative bulk stress on cosmological scales at the late universe phase might be important [13]-[14]. To describe a relativistic theory of viscosity, Eckart [15] made the first attempt. Later Israel and Stewart [16] developed a fully relativistic formulation of the theory taking into account second order deviation terms in the theory, which is termed as "transient" or "extended" irreversible thermodynamics (in short, EIT). In the paper we consider the effects of possible existence of the bulk viscosity of the modified Chaplygin gas on the cosmological dynamics of the universe. The layout of the paper as: In sec. 2, we give the gravitational action and set up the relevant field equations, in sec. 3, cosmological solutions are presented. In sec. 4 state finder diagnostic are given. Lastly in sec. 5, we summarize the results obtained.

2. GRAVITATIONAL ACTION AND RELEVANT FIELD EQUATIONS

In standard general relativity hence Einstein's Field equation yields

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = -T_{\mu\nu} \;,$$

where $T_{\mu\nu}$ is the energy momentum tensor and we have choose a standard unit with $8\Pi G = c = 1$. The expression of energy momentum tensor with bulk viscous fluid is given by

$$T_{\mu\nu} = (\rho + p + \Pi)u_\mu u_\nu - (p + \Pi)g_{\mu\nu}$$

where ρ is the energy density and u_μ is the four velocity with normalization condition $u^\mu u_\nu - -1$. Here p is the equilibrium pressure [17] and Π is the bulk viscous pressure.

We consider the homogeneous and isotropic space-time given Friedmann Robertson-Walker (FRW) metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - k\, r^2} + r^2(d\theta^2 + sin^2\theta\, d\varphi^2)\right] \qquad (1)$$

where $a(t)$ is the scale factor of the universe. The constant k defined curvature of space $k = -1,\quad 0, 1$ represents open, flat and closed spaces respectively. The field and conservation equations are

$$H^2 = \frac{\rho}{3} - \frac{k}{a^2} \qquad (2)$$

$$2\dot{H} + 3H^2 = -p - \Pi - \frac{k}{a^2} \qquad (3)$$

$$\dot{\rho} + 3(\rho + p)H = -3\Pi H, \qquad (4)$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter, the over dot represents derivative with respect to cosmic time t. In *EIT,* the bulk viscous stress Π satisfies a transport equation given by

$$\Pi + \tau\dot{\Pi} = -3\zeta H - \frac{\epsilon}{2}\tau\Pi\left[3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\zeta}}{\zeta} - \frac{\dot{T}}{T}\right], \qquad (5)$$

where ζ is the coefficient of bulk viscosity, τ is the relaxation coefficient for transient bulk viscous effects and T is the absolute temperature of the universe. The parameter ϵ takes the value 0 or 1. Here $\epsilon = 0$ represents truncated Israel-Stewart theory, $\epsilon = 1$ represents full Israel-Stewart (FIS) causal theory and Eckart theory for $\tau = 0$. The cosmological fluid obeys a MCG equation of state [8]

$$p = B\rho - \frac{A}{\rho^\alpha}, \qquad (6)$$

where $B\ (0 \le B \le 1)$, $\alpha\ (0 \le \alpha \le 1)$ and $A\ (A > 0)$ are constant. The deceleration parameter (q) is related to H as

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1. \qquad (7)$$

The value of deceleration parameter is negative for accelerating and positive for decelerating phase of the universe. The observations of Cosmic Microwave Background (CMB) anisotropy indicate that the universe is flat and the total energy density is very closed to the critical $\Omega_{tot} \cong 1$. Hence in proceeding we use the concept of flat universe $(k = 0)$.
Using Eqs.(2), (3) and (6) we obtain

$$\Pi = -2\dot{H} - 3\left[1 + B - \frac{A}{3^{\alpha+1}} H^{2+2\alpha}\right]. \qquad (8)$$

The behavior of the temperature of the universe is obtained through Gibbs integrability condition [18], in the absence of particle creation the behavior of the same: (i) for a perfect fluid $(p - B\rho)$ is $T \sim \rho^{\frac{B}{1+B}}$ (ii) for Chaplygin is

$$T \sim [(1+B)\rho^{1+\alpha} - A]^{\frac{B+\alpha+\alpha B}{(1+\alpha)(1+B)}}\ \rho^{-\alpha}. \qquad (9)$$

3. COSMOLOGICAL SOLUTIONS

The system of eqs. (2)-(8) are employed to obtain cosmological solutions. The system of equations is not closed. The coefficients τ and ζ are in general functions of the time (or of the energy density). We assume the following widely accepted *adhoc* relations [19]

$$\zeta = \beta \rho^{\delta}, \qquad \tau = \beta \rho^{\delta-1}, \qquad (10)$$

where ζ (≥ 0), $\tau\,(\geq 0)$, β (≥ 0) and δ (≥ 0) are constant. In the next section we explore cosmologies with Eckart, TIS and FIS theory respectively.

3.1 Eckart Theory

In Eckart theory transport equation take the form

$$\Pi = -3\,\zeta\, H. \qquad (11)$$

Eq. (11) can be rewritten in terms of deceleration parameter as [20]

$$q = \frac{1}{2} + \frac{3B}{2} - \frac{1}{2}\frac{A}{3^{\alpha}} H^{2\alpha+2} - \frac{1}{2}\beta\, 3^{\delta+1} H^{2\delta-1} \qquad (12)$$

In the above equation A and β are dimensional constant, i.e., for a given value of α and δ it remains dimensionally correct. We plot q vs H for different values of other parameters. Assuming that cosmic comoving time (t) is determined by a monotonically decreasing function of Hubble parameter $\left(t \sim \frac{1}{H}\right)$. The earlier phase of evolution of the universe is described by higher value of H and the later phase of the evolution of the universe is described by smaller H. From the graphical plot of $q\ vs\ H$ the following points are noted:

(i) The variation of q with H for different values of Chaplygin parameter (A) is shown in fig. (1) for a given set of other parameters. It is evident that the universe has one deceleration phase and two accelerating phase. The early inflation phase and present accelerating phase are obtained due dissipation and chaplygin gas. The figure shows that for the smaller values of A the period of decelerating phase is larger. In the absence of CG (A=0) the evolution of the universe does not exhibits late acceleration phase.

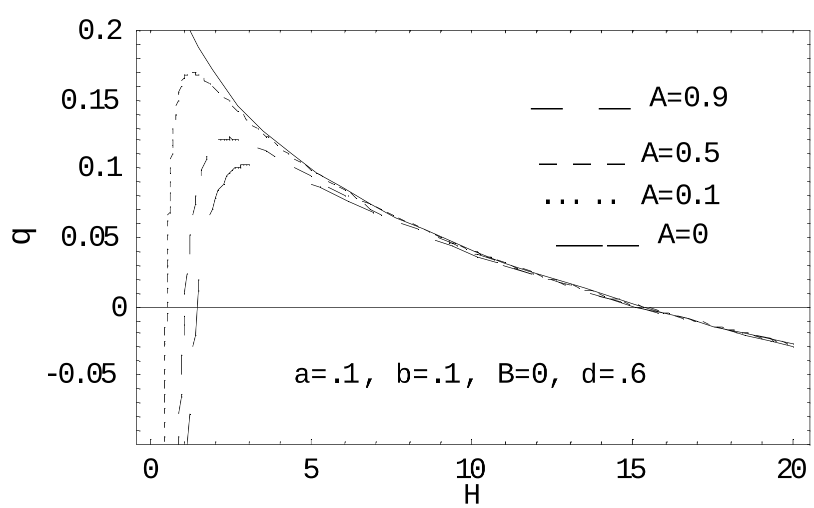

Fig. 1: Shows the variation $q\ vs\ H$ for different value of A with $\alpha = 0.1, \beta = 0.1, B = 0,$ and $\delta = 0.6$.

(ii) The variation of q with H for different values of Chaplygin gas parameter α is shown in fig. (2) for a given set of parameters A, B, β, δ . It is evident that the universe entered into the present accelerating phase followed by decelerating phase in addition to early inflation phase. The figure shows that for the smaller value of α the period of decelerating phase is shorter.

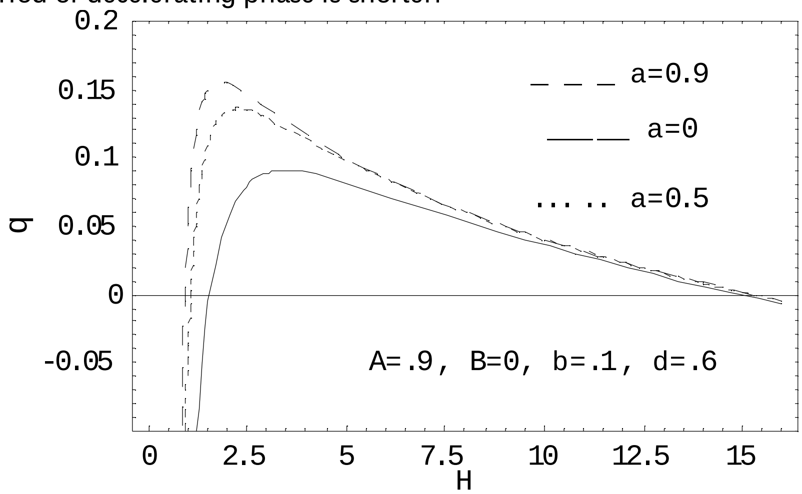

Fig. 2: Shows the variation $q \; vs \; H$ for different value of α with $A = 0.9, \beta = 0.1, B = 0$ and $\delta = 0.6$.

3. 2. Truncated Israel and Stewart (TIS) Theory

In TIS theory transport equation take the form

$\Pi + \tau \dot{\Pi} = -3\,\zeta\, H$. (13)

Using Eqs. (8) and (10), Eq. (13) yields

$$q' + \frac{2(q+1)}{H} - \frac{b_1}{H} + \frac{1}{(q+1)}\frac{b_2}{H} = 0, \qquad (14)$$

where

$$b_1 = 3(B+1) + \frac{A\,\alpha\,3^{-\alpha}}{H^{2\alpha+2}} + \frac{3^{1-\delta}}{\beta\,H^{2\delta-1}}, \qquad b_2 = \frac{(B+1)\,3^{2-\delta}}{2\,\beta\,H^{2\delta-1}} - \frac{9}{2} - \frac{A\,3^{1-\delta-\alpha}}{2\,\beta\,H^{2\delta+2\alpha+1}}$$

Where prime $(')$ represents the differentiation w. r. to H . To find a numerical solution we plot q vs H with initial condition. The variation of q with H are plotted for different configuration of the system. The model may be useful to predict satisfactory the future course of evolution of the universe. From the graphical plot of q *vs* H the following points are noted:

(i) The variation of q with H for different value of A is shown in fig. (3) for a given value of other parameters α , B , β , δ . It is evident that the universe entered into the accelerating phase in the recent past followed by a phase of deceleration and early inflation phase. The figure shows that for the smaller value of A the period of decelerating phase is longer. The universe enter into the present accelerating phase in the earlier time for the higher value of Chaplygin gas (A). One may note that the universe has no present accelerating phase in the absence of chaplygin gas $(A = 0)$.

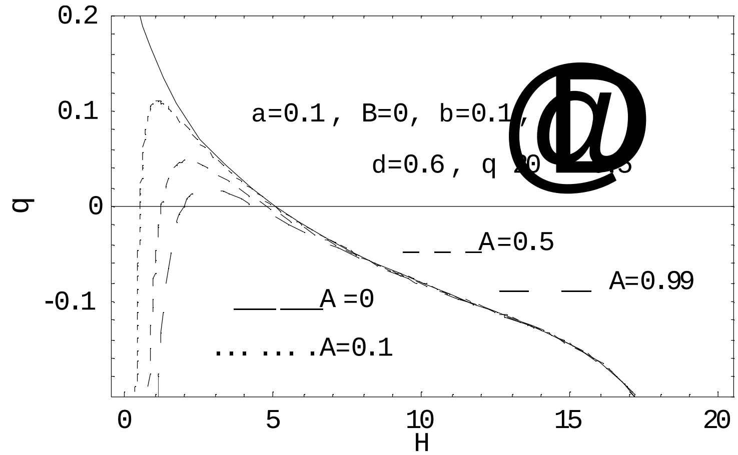

Fig. 3: Shows the variation q vs H for different value of A with $\alpha = 0.1, \beta = 0.1, B = 0, \delta = 0.6.$ and $q[20] = -0.5.$

(ii) The variation of q with H for different value of α is shown in fig. (4) for a given value of other parameters B , A , β , δ .

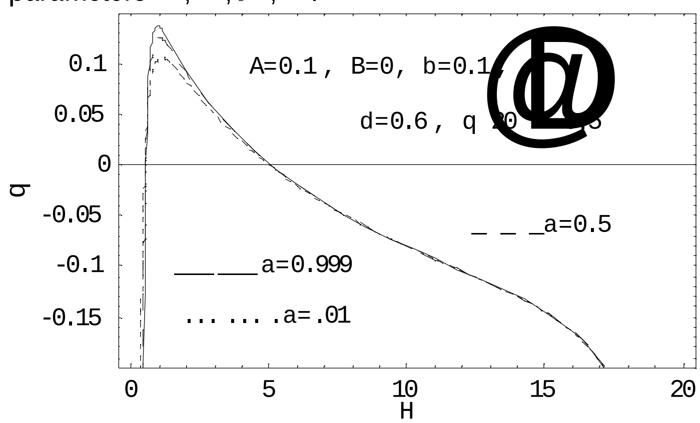

Fig. 4: Shows the variation q vs H for different value of α with $A = 0.1, \beta = 0.1, B = 0, \delta = 0.6.$ and $q[20] = -0.5.$

The above $(q - H)$ have been drawn using MATHEMATICA, the plots provides a sufficient data set. These numerical values may be used to determine a closed analytical mathematical form of q and H by assuming a polynomial function for q as $q = \sum_{0}^{n} a_i H^i$. The form to express $q(H)$ may be determined corresponding to the fig (3) with $A = 0.1$. In this case the corresponding analytical function may be expressed as

$$q = -0.18847 + 0.19307H - 0.04211H^2 + 0.00312H^3 - 7.67457 * 10^{-5}H^4$$

3. 3. Full Israel and Stewart (FIS) Theory

In FIS theory transport equation take the form

$$\Pi + \tau\dot{\Pi} = -3\,\zeta\,H - \frac{1}{2}\,\tau\,\Pi\left[3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\zeta}}{\zeta} - \frac{\dot{T}}{T}\right]. \qquad (15)$$

Using Eqs. (8), (9), (10) one can show that Eq. (15) reduces to

$$q' + [2 - a_2]\frac{q+1}{H} - \frac{a_3}{H} + \frac{a_4}{(q+1)H} = 0, \qquad (16)$$

where the co-efficient are

$$a_1 = 1 + B - \frac{A}{3^{1+\alpha}}\,H^{2\alpha+2},$$

$$a_2 = \frac{(1+2B)(3H^2)^{1+\alpha} + (\alpha-1)A}{(1+B)(3H^2)^{1+\alpha} - A}$$

$$a_3 = \left[\frac{9}{2} + 3B + \frac{A\,\alpha}{3^{\alpha}H^{2\alpha+2}} + \frac{3^{1-\delta}}{\beta H^{2\delta-1}} - \frac{3\,a_1 a_2}{2}\right] \;\&$$

$$a_4 = \left[\frac{a_1}{2\beta 3^{\delta-2}H^{2\delta-1}} - \frac{9}{2} + \frac{9\,a_1}{4}\right]$$

To solve the differential equation numerically, we follow the technique same as TIS theory. From the graphical plot of q vs H the following points are noted:

(i) The variation of q with H for different value of A is shown in fig. (5) for a given value of $\alpha, B, \beta,$ δ and $q[20]$. It is evident that the universe entered into the present accelerating phase in the recent past followed by early inflation and an intermediate deceleration phase. The figure shows that for the smaller value of A the period of deceleration phase is longer.

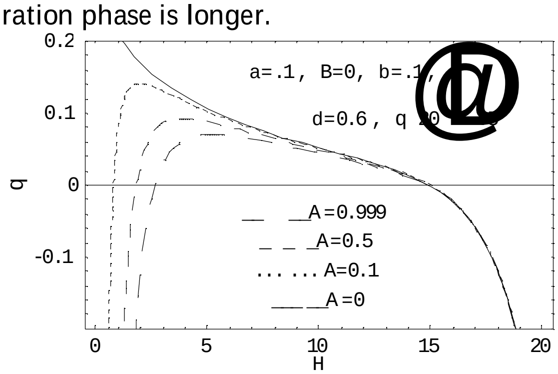

Fig. 5: Shows the variation q vs H for different value of A with $\alpha = 0.1, \beta = 0.1, B = 0, \delta = 0.6.$ and $q[20] = -0.5.$

(ii)The variation of q with H for different value of α shown in fig. (6) for a given value of other set of parameter.

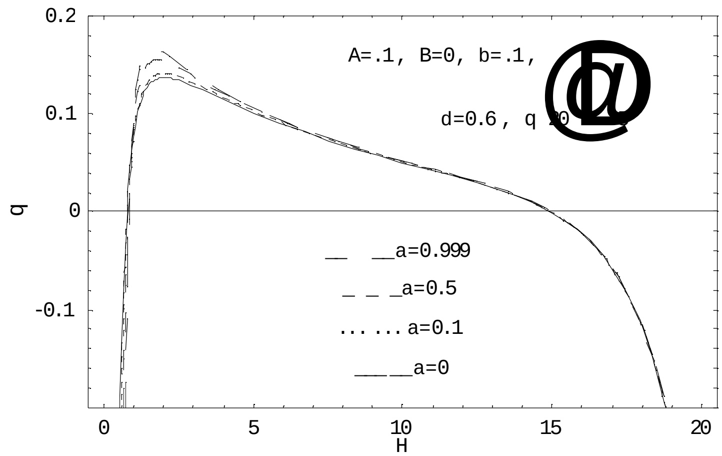

Fig. 6: Shows the variation $q \; vs \; H$ for different value of α with $A = 0.1, \beta = 0.1, B = 0, \delta = 0.6.$ and $q[20] = -0.5.$

The above $(q - H)$ plot may be used to determine a closed analytical mathematical form of q and H by assuming a polynomial function for q like TIS theory. The form to express $q(H)$ is determined corresponding to the fig (5) with $A = 0.1$. In this case the corresponding analytical function may be expressed as

$$q = -0.2991 + 0.28124\,H - 0.05488\,H^2 + 0.00396\,H^3 - 9.68649 * 10^{-5} H^4$$

4. STATE FINDER DIAGNOSTIC

Sahni *et. al* [21] proposed a new geometrical diagnostic pair, based on dimensionless parameter (r, s) which are function of scale factor and its higher order time derivatives are known as state finder diagnostic tools to study more general model of dark energy. The state finder pair (r, s) is defined as follows:

$$r = \frac{\dddot{a}(t)}{a} H^3; \quad s = \frac{r - 1}{3\left(q - \frac{1}{2}\right)} \quad . \qquad (17)$$

The plot of $r \; vs \; H$ in Eckart theory of different value of MCG is shown in the figure (7) for a given set of other parameter. The figure shows that the MCG (A) is more effective at the late phase of evolution of the universe.

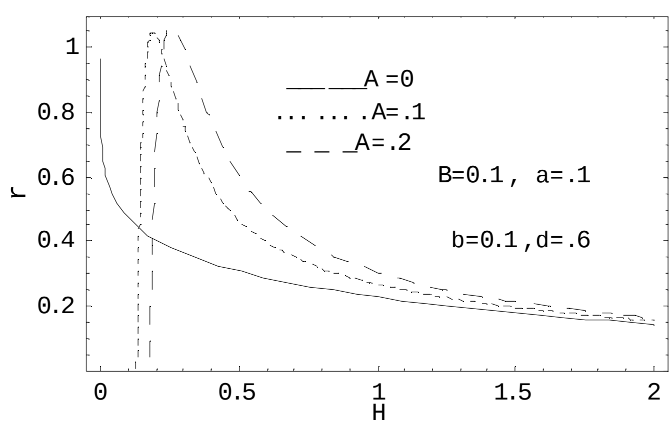

Fig. 7: Shows the variation $r \; vs \; H$ for different value of A with $\alpha = 0.1, \beta = 0.1, B = 0$ and $\delta = 0.6.$

The plot of $s\ vs\ H$ in Eckart theory of different value of Chaplygin gas is shown in the figure (8) for a given set of other parameter.

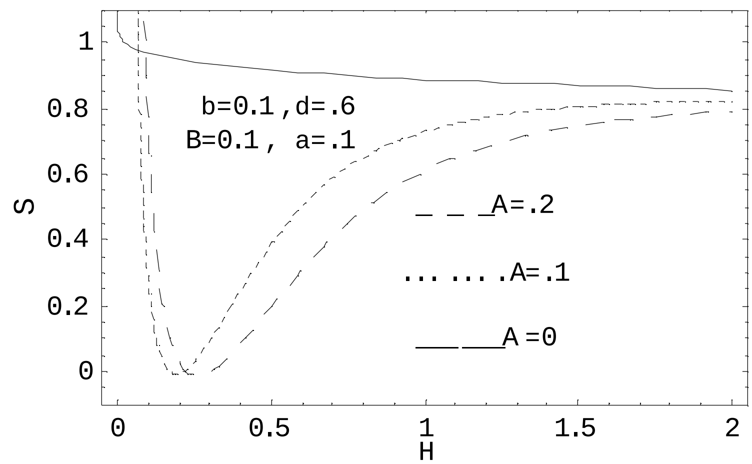

Fig. 8: Shows the variation $s\ vs\ H$ for different value of α with $A = 0.1, \beta = 0.1, B = 0$ **and** $\boldsymbol{\delta = 0.6}$.

To plot state finder parameters in the TIS & FIS theory, we used the analytical function of q given in respective theory. A comparison of the plot of $r\ vs\ H$ in three different models (i.e., Eckart, TIS and FIS theory) for a given set of other parameter is shown in figure (9). Like Eckart theory all the models show that late evolution of the universe is mostly influenced by Chaplygin gas.

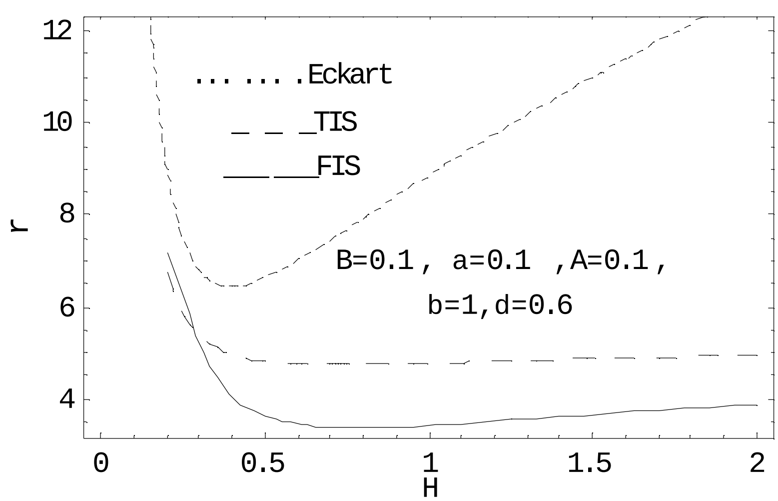

Fig. 9: Shows the variation $r\ vs\ H$ with $A = 0.1, \beta = 1, \alpha = 0.1, B = 0.1$ **and** $\boldsymbol{\delta = 0.6}$

A comparison of the plot of $s\ vs\ H$ in three different models (i.e., Eckart, TIS and FIS theory) for a given set of other parameter is shown in figure (10). The figure shows the late evolution of the universe is mostly influenced by Chaplygin gas.

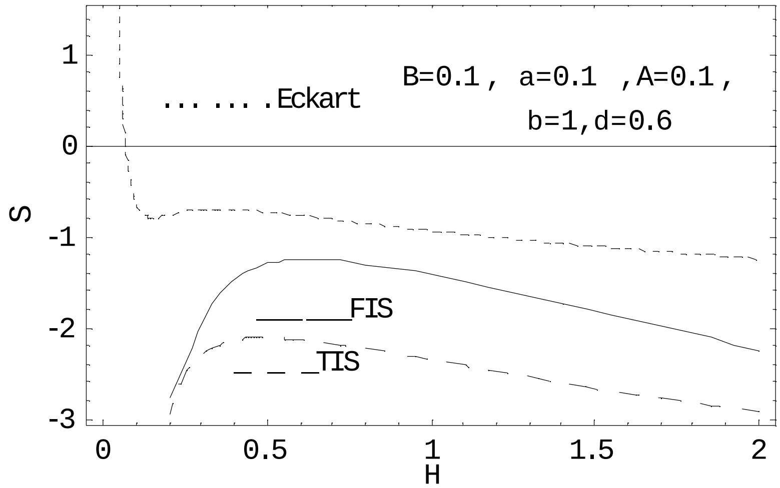

Fig. 10: Shows the variation s vs H
with $A = 0.1$, $\beta = 1, \alpha = 0.1, B = 0.1$ and $\delta = 0.6$.

5. CONCLUSIONS

Here, we study cosmological models of the universe in the presence of MCG with dissipative effects described by Eckart, truncated and full causal theories proposed by Israel and Stewart. In all the theories it is found the present accelerating phase of the universe is followed by the past decelerating phase and early inflation as shown in fig (1)-(6). The period of the deceleration phase is larger for smaller value of the MCG (namely, A). Figure (2) shows that for the larger value of α the period of intermediate deceleration phase is shorter. In the absence of MCG (namely, $A = 0$) there are no present acceleration phase followed by the early inflation phase of the evolution of the universe as shown in the fig(1), fig.(3) and fig.(5) for a given set of other parameter. The study of State finder parameters (r, s) are given in Eckart, TIS and FIS theory in fig. (7) - (10). The figures show MCG is more effective in the late phase of evolution of the universe. The studies of the state finder diagnostic suggest that among the three model TIS is more suitable one.

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