Bulletin of Pure and Applied Science
Physics, Vol.44D No.1,
January-June 2025 P.33-37
Original Research Article

Print version ISSN 0970 6569
Online version ISSN 2320 3218
Available online at www.bpasjournals.com

Study of Mathematical Model for the Change in Entropy of AGN BHs of Spin Parameter a*=+5/2 & -5/2

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ABSTRACT	In the present work, a mathematical model for the change in entropy of fermionic fields of black holes with respect to the mass as given by $\delta S/\delta M = 8\pi M(1-2\Omega Ma^* + a^{*2}/2 - M\Omega a^{*3})$ is used to study the behavior of BHs in the AGN for spin a*=+5/2 & -5/2 and also the calculation of their values for different masses. The outcome of this work concludes that there is decrease and increase in entropy due to co - rotation and counter rotation of Fermionic field of BHs respectively.

KEYWORDS	Fermionic fields, Spin Parameters and AGN

Received on 20.12.2024, Revised on 31.03.2025, Accepted on 12.04.2025

How to cite this article: Kumari V., Amar A., Yadav A., Singh BP. and Mahto D. (2025). Study of Mathematical Model for the Change in Entropy of AGN BHs of Spin Parameter a*=+5/2 & -5/2. *Bulletin of Pure and Applied Sciences- Physics,* 44D (1), 33-37

INTRODUCTION

The classical theory fails to explain the emission of radiation from black holes, while the quantum theory is able to explain the emission of radiation from a black body(1975).Many discussions have been done regarding the entropy of BHs by Bardeen et al.(1973), Strominger & Vafa (1996), Transchen (2000), Narayan (2005), Dabholkar (2005), Bekenstein(2008), Carlips,(2009), Mahto et al., (2012), Mahto et al. (2013).Mahto and Kumari (2018).Kumari et al. (2023), Mahto et al. (2020) and Paswan et al.(2024).

The paper discusses about a mathematical model for the change in entropy of fermionic fields of black holes w. r. t. the mass of spin(a*) =+5/2 & -5/2 and calculates their values in AGN.

MODEL OF THE WORK

For the purpose of this work, the mathematical model for the change in entropy is given as **(**Mahto and Kumari 2018).

$$\delta S / \delta M = 8\pi M(1 - 2\Omega M a* + a*^2 / 2 - M\Omega a*^3) \quad (1)$$

Where M, Ω and a* denoting the mass, angular velocity and spin parameter respectively.

Now the spinning parameters a*= +5/2 and -5/2 are used in the above equation**(Tayal 2009, Yash 2022)**, then we have

$$\left(\frac{\delta S}{\delta M}\right)_{+5/2} = -\pi M(165M - 33) \quad (2)$$

$$\left(\frac{\delta S}{\delta M}\right)_{-5/2} = \pi M(165M + 33) \quad (3)$$

Applying the condition of maxima and minima to obtain the values of mass, we have

$$M(165M - 33) = 0 \quad (4)$$

$$M(165M + 33) = 0 \quad (5)$$

The equations(4) and (5) are the quadratic in nature and hence their solutions are given as follows:

M=0 or M=33/165 (6)

M=0 or M=-33/165 (7)

The solution of equations(6) and (7) gives 0 & 33/165 and 0 and -33/165respectively.The positive (+), negative (-) & zero (0)mass explains the gravity, naked singularity &dark matter and quantum theory. Using the proper data in the models represented by equations (2) and (3), the change in entropy w.r.t. the mass for AGN is listed in the table 1.

Table1:

Change in Entropy of the BHs of Spin(a*) = +5/2 and -5/2				
S. No.	Mass of BH_s (M) in $M_\odot$	Mass of BHs in terms of $10^7 M_\odot$	$\left(\frac{\delta S}{\delta M}\right)_{+5/2} = -\pi M(165M - 33)$ [Joule/Kelvin/kg]× 10^{78}	$\left(\frac{\delta S}{\delta M}\right)_{-5/2} = \pi M(165M + 33)$ [Joule/Kelvin/kg]× 10^{78}
1	1 x $10^6 M_\odot$	0 . 1	-0 . 00207	0 . 00207
2	2 x $10^6 M_\odot$	0 . 2	-0 , 00828	0 , 00828
3	3 x $10^6 M_\odot$	0 . 3	-0 . 01865	0 . 01865
4	4 x $10^6 M_\odot$	0 . 4	-0 . 03315	0 . 03315
5	5 x $10^6 M_\odot$	0 . 5	‘0 . 08181	0 . 08181
6	6 x $10^6 M_\odot$	0 . 6	-0 . 074606	0 . 074606
7	7 x $10^6 M_\odot$	0 . 7	-0 . 10155	0 . 10155
8	8 x $10^6 M_\odot$	0 . 8	-0 . 13263	0 . 13263
9	9 x $10^6 M_\odot$	0 . 9	-0 . 16786	0 . 16786

10	1 x $10^7 M_\odot$	1. 0	-0 . 20724	0 . 20724
11	2 x $10^7 M_\odot$	2. 0	-0 . 82896	0 . 82896
12	3 x $10^7 M_\odot$	3. 0	-1 . 8652	1 . 8652
13	4 x $10^7 M_\odot$	4. 0	-3 . 3158	3 . 3158
14	5 x $10^7 M_\odot$	5. 0	-5 . 1810	5 . 1810
15	6 x $10^7 M_\odot$	6. 0	-7 . 4606	7 . 4606
16	7 x $10^7 M_\odot$	7. 0	-10 . 1550	10 . 1550
17	8x $10^7 M_\odot$	8. 0	-13 . 2630	13 . 2630
18	9 x $10^7 M_\odot$	9. 0	-16 . 7860	16 . 7860
19	1 x $10^8 M_\odot$	10. 0	-20 . 7240	20 . 7240
20	2 x $10^8 M_\odot$	20. 0	-82 . 8960	82 . 8960
21	3 x $10^8 M_\odot$	30. 0	-186 . 552	186 . 552
22	4 x $10^8 M_\odot$	40. 0	-331 . 580	331 . 580
23	5 x $10^8 M_\odot$	50. 0	-518 . 100	518 . 100
24	6 x $10^8 M_\odot$	60. 0	-746 . 060	746 . 060
25	7 x $10^8 M_\odot$	70. 0	-1015 . 000	1015 . 000
26	8 x $10^8 M_\odot$	80.0	-1326 . 300	1326 . 300
27	9 x $10^8 M_\odot$	90.0	-1678 . 600	1678 . 600
28	1 x $10^9 M_\odot$	100. 0	-2072 . 400	2072 . 400

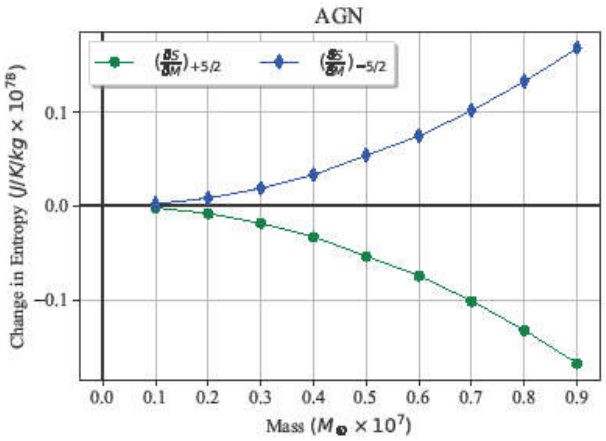

Figure 1:Change in entropy w.r.t. change in mass of range 0.1 to 0.9 times $10^7 M_\odot$ of spin +5/2 and -5/2.

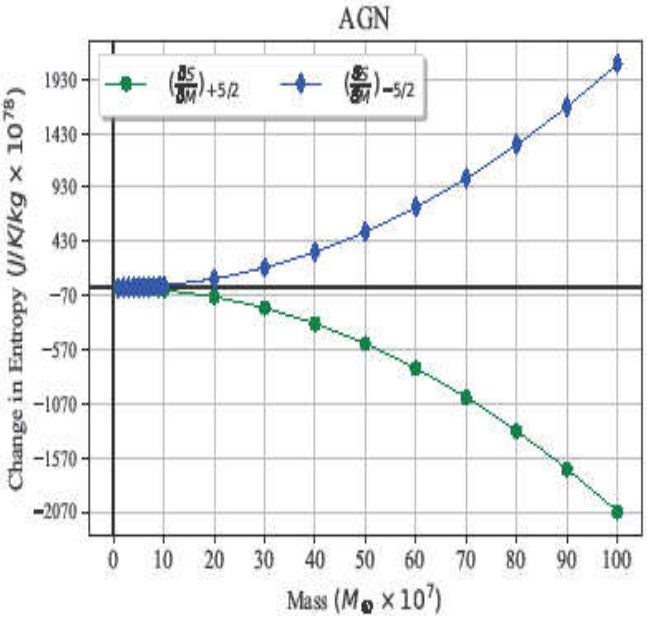

Figure 2: Change in entropy w.r.t. the change in mass of range 10 to 100 times $10^7 M_\odot$ BHs of spin +5/2 and -5/2.

RESULT AND DISCUSSION

The mathematical model for discussion for the change in entropy with respect to mass is given by

$$\delta S / \delta M = 8\pi M(1 - 2\Omega M a^* + a^{*2}/2 - M\Omega a^{*3})$$

The observation of above equation indicates that the change in entropy depends on (1) angular velocity, (2) spin parameters and (3) mass of black holes.

When the equation (1) is used for the spin like a*= +5/2 & -5/2, it takes its form represented by the equations(2) and (3). Now these two models are taken into consideration for discussions. From discussion of our results, we see that the change in entropy with respect to the change in mass decreases for a*= +5/2, while increases for a*= -5/2, The situation is shown in the table 1.

The graph is plotted between the mass of black holes and corresponding change in entropy with change in mass using the equations (2) & (3). This is shown in the Figs.1(a) and 1(b). We

observe that the variation is symmetrical for either types of spin parameters for co-rotation and counter rotations, but differs in their gradients.

CONCLUSION

The change in entropy w.r.t mass for a*= +3/2, to show the decrease in entropy change, while increases the entropy change for a*= +3/2.The present model also explains the naked singularity and dark matter. Hence we see that the conclusion drawn from this work is the same to that of the character of BHs as explained for XRBs.

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