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Traveling Signals of Electromagnetic Waves through the Isolated Infinitely Long Carbon Nanotubes

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ABSTRACT

The traveling signals of electromagnetic waves through carbon nanotubes are manifested with Helmholtz equation that gives us group velocity with inductance and capacitance and it is found with sinusoidal magnetic and electric field given by Maxwell's equation and Stoke's theorem. The group and phase velocities for electromagnetic waves in transmission line affected by kinetic inductance and intrinsic capacitance that gives as results, the propagation velocity of electromagnetic signal is equal to the Fermi velocity of electrons and the signal velocity is also affected by classical capacitance and magnetic inductance. The results are good agreement with previously found results by others.

KEYWORDS

Electromagnetic Waves, Carbon Nanotubes, Helmholtz Equation, Group Velocity, Inductance, Capacitance.

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INTRODUCTION

The Helmholtz wave equation is solved by Kumar [1,3,19-21] with different approaches and shown the nature of waves as transverse, plane and monochromatic waves in the closed bundle of single wall carbon nanotubes [2] and gold core multi-wall carbon nanotubes as antennas [4]. The propagation frequency is discussed with wave-number and parameters [5]. The surface Plasmon wave in metallic carbon nanotube is manifested by numerically [6] and the stimulation of Plasmon with low frequency is given in surface wave radiation [7-9]. The property of the slow-wave propagation at high impedance with kinetic-inductive effect is shown in the range of low frequency [10]. The propagation of electromagnetic wave is in the waveguide of photonic-crystal [11]. The surface-wave- Plasmon, lower frequency, surface-intraband conductivity and gold core multiwall carbon nanotube in infrared as well as visible-regimes are examined [12,15]. The potential of nanotubes [16] interactive for electromagnetic wave and the Gaussian laser beams propagation is described [13-14]. The transmission characteristics are shown on wave-guide with approach of surface-scattering and found scattering-amplitude [17]. The impedance for device of waveguide is described with regime of optic plasmonic frequency [18]. The function of normalized with frequency parameter was found that described the mode of linearly polarization as transverse electric (TE) mode and transverse magnetic (TM) mode and this is manifested by root of wave equation with Bessel's functions [20-21]. The propagation of guided wave in finite length of carbon nanotube bundle is manifested with integral equation method and guided wave propagation dispersion equation [22].

METHODS

The Helmholtz equation is the partial differential wave equation. This is written over Eigen function, $\prod_m(r)$, as

$$\nabla^2 \prod_m(r) = -\omega^2 \prod_m(r)\,\mu_0\varepsilon_0 \quad (1)$$

Where, ω is angular frequency. The electric Hertz vector, $\prod_m(r) \equiv \prod_m(r)\,\hat{a}_z$, vary in z-direction that created by axial-current-density on surface of carbon nanotube and $\hat{a}_z$ is unit vector along the axis of the carbon nanotube and be the function of z and t only, so, may be written as

$$\frac{\partial^2 \prod_m(r)}{\partial t^2} = -\omega^2 \prod_m(r) \quad (2)$$

From (1) and (2) we have

$$\nabla^2 \prod_m(r) = \mu_0\varepsilon_0 \frac{\partial^2 \prod_m(r)}{\partial t^2} \quad (3)$$

This equation (3) is as the general form of wave equation, $\nabla^2 y = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$, and on comparing with it and taking $v = group\ velocity(v_g)$, we have

$$v_g = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{1}{\sqrt{\frac{\mu_0 a}{b}\frac{\varepsilon_0 b}{a}}} = \frac{1}{\sqrt{LC}} \quad (4)$$

Where L is the total inductance in henry per unit length and C is the total capacitance in farad per unit length and they are expressed as $L = \frac{\mu_0 a}{b}\ and\ C = \frac{\varepsilon_0 b}{a}$ and obtained by the Maxwell's equations and Stokes Theorem (see Appendix A). The equation (4) is expressed with phase velocity, v_p , because of group velocity is equal to phase velocityfor the dissipation less line, $R = G = 0$, i.e., $v_g = v_p,\ v_g v_p = c^2$, so, for maximum frequency the group and phase velocities should be maximum and represents the velocity of electromagnetic wave in transmission line consisting of carbon nanotube that independent of geometrical line. The signal of electromagnetic wave with maximum frequency travels in the carbon nanotube transmission line and affected by kinetic inductance. The velocity of wave for transmission of signal in carbon nanotube transmission line given as

$$v = \sqrt{\frac{C_t}{L_k + L_m}} \quad (5)$$

Where, $C_t = \frac{C_c C_i}{C_c + C_i}$, C_c is the classical or electrostatic capacitance, C_i is the intrinsic capacitance, L_m is the magnetic inductance and L_k is the intrinsic or kinetic inductance. For nanotube, the classical capacitance is defined over per unit length as $C_c = \frac{2\pi\varepsilon}{\ln\left(\frac{4h}{d}\right)}$, here d is diameter and h is distance to the conducting plate, obtained by fixed charge on carbon nanotube and equivalent to fixed position of Fermi level for infinite density of state. For change in potential, the charge per unit length of carbon nanotube is changed and we have obtained the intrinsic capacitance as $C_i = e^2 D(E_f) = \frac{4e^2}{\pi \hbar v_f}$. C_i is larger than C_c and both capacitance add as capacitors in the series that expressed by electrostatic capacitance if $D(E_f)$ is infinite.

Using *Ampere's law and Images method*, we may be calculated the magnetic inductance of carbon nanotube per unit length carrying the steady electric current over plane as $L_m = \frac{\mu}{2m} ln \frac{4h}{d}$. The intrinsic inductance, L_k , of metallic carbon nanotube is obtained by energy of flow of ballistic electrons in shaded band of $source\ Fermi\ level$ and $drian\ Fermi\ level$ and expressed as $L_k = \frac{\pi^2\hbar^2 D(E_f)}{16e^2} = \frac{\pi\hbar}{4e^2 v_F}$. Here, $v_F$is the Fermi velocity and related to Fermi distribution expressed as $f(E) = 1/\{e^{(E-E_f)/K_BT} + 1\}$. It is larger than L_m . If the biases are larger than 160 mV, the scattering decreases conductance due to optic phonon. If L_k dominates L_m and C_i is larger than C_c, the propagation velocity of electromagnetic-signal is given as

$$v \approx \sqrt{\frac{1}{L_k C_i}} = v_F \quad (6)$$

So, signal velocity is same as the Fermi velocity of electrons. If the nanotubes are parallel to each other and C_c influences and $C_i\ \&\ L_k$ are neglected, so, the propagation velocity of signal reduced to

$$v = \sqrt{\frac{1}{L_m C_c}} \quad (7)$$

RESULTS AND DISCUSSIONS

The velocity of wave calculated by Helmholtz equation is approximately equal to the speed of light that travels through carbon nanotubes. The group as well as phase velocity is equal to square root of product of inductance and capacitance in transmission line of carbon nanotubes that is found by Maxwell's equation by applying Stokes theorem (see Appendix A). The electric field depends on diameter of nanotube near the contact. At centre, the electric field in carbon nanotube increases with increasing diameter due to the density of state decreases per atom with increasing diameter shown in Figure (1). In this figure, the applied bias drops across two ends of carbon nanotube. The diameters 0.94 nm and 18.80 nm are the (12, O) zig-zag carbon nanotube and (240, O) zig-zag carbon nanotube respectively and applied bias is 100 mV for 213 nm of length of nanotube. The screaming is poorer for larger diameter. When increasing applied bias, drops the electrostatic potential over the tube length that is the mean free path and allows optical zone boundary phonons see Figure (2). The function of position as potential for 42.6 nm and 213 nm of length of zig-zag (12, O) shown by solid line for scattering and the dashed line shows the ballistic limit. The inductance and capacitance per unit length is found {that shown by equations (19A and 20A) in Appendix A}, when loop resistance and shunt conductance are zero. With this, the phase and group velocities are equal and represent the electromagnetic wave velocity in transmission line affected by kinetic inductance for signal in carbon nanotubes. If kinetic inductance influences magnetic inductance and intrinsic capacitance is greater than classical, so, the propagation velocity is equal to the Fermi velocity. For parallel carbon nanotubes with neglecting the intrinsic capacitance and kinetic inductance, we have the signal velocity as in equation (7). Actually, the signal velocity is described by the group velocity that is given by expression (4), so, signal velocity with inductance and capacitance is approximately equal to speed of light and in case of inductance, it is approximately equal to Fermi-velocity. The nature of propagating signal through carbon nanotubes is shown with Mathway software in Figure (3). These results are very similar to the previous work.

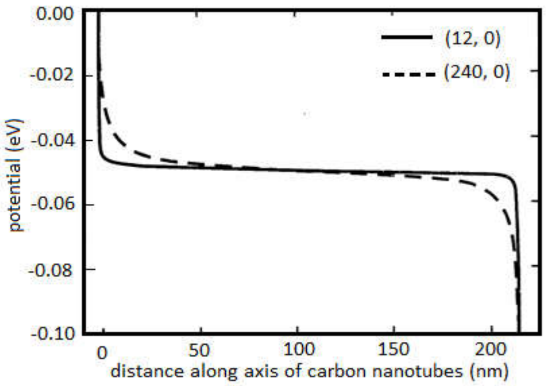

Figure1: Electrostatic potential along axis of carbon nanotube (12, 0) and (240, 0) that have radius 0.47nm and 9.4nm and screening is poorer.

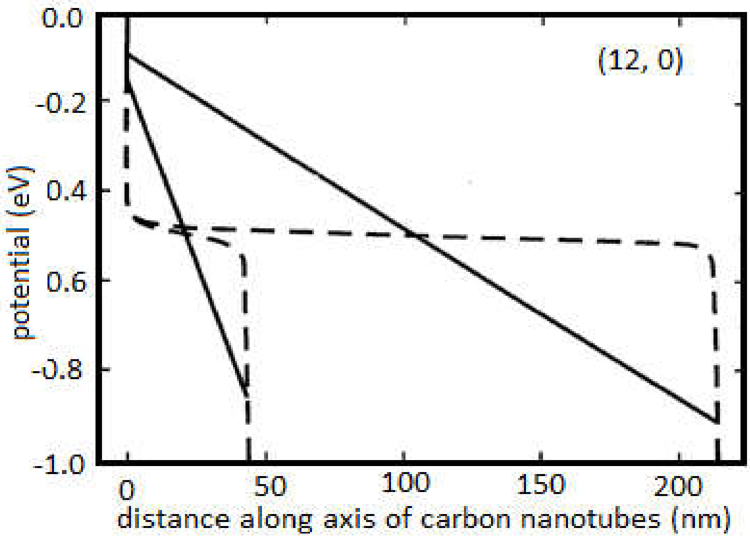

Figure2: Electromagnetic potential as position-function for (12, O) carbon nanotubes in the scattering represented by solid line and in ballistic-limit represented by the dashed line.

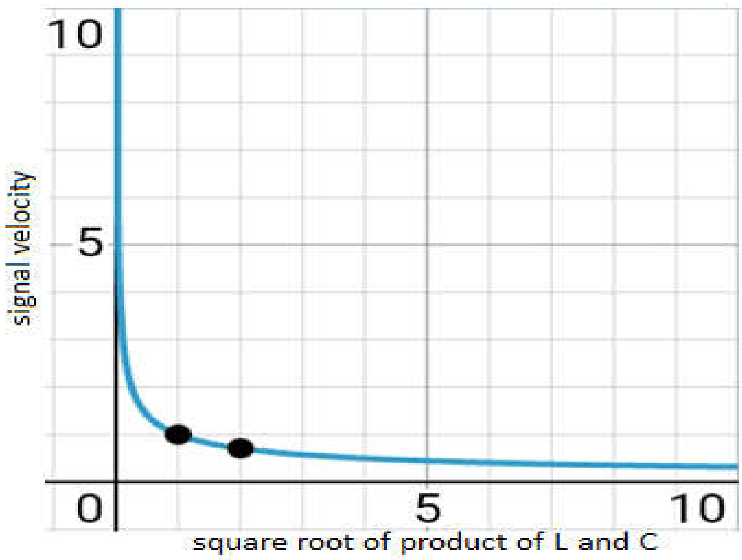

Figure 3: Nature of propagating signal through carbon nanotubes.

Appendix A: Derivation for Inductance (L) and Capacitance (C)

Suppose a transmission line on guided waves with Maxwell's field equations along z-direction. The transmission line consists of two conducting plates at separation $'a'$in x-direction and $'b'$in y-direction as shown in Figure (A(a)).

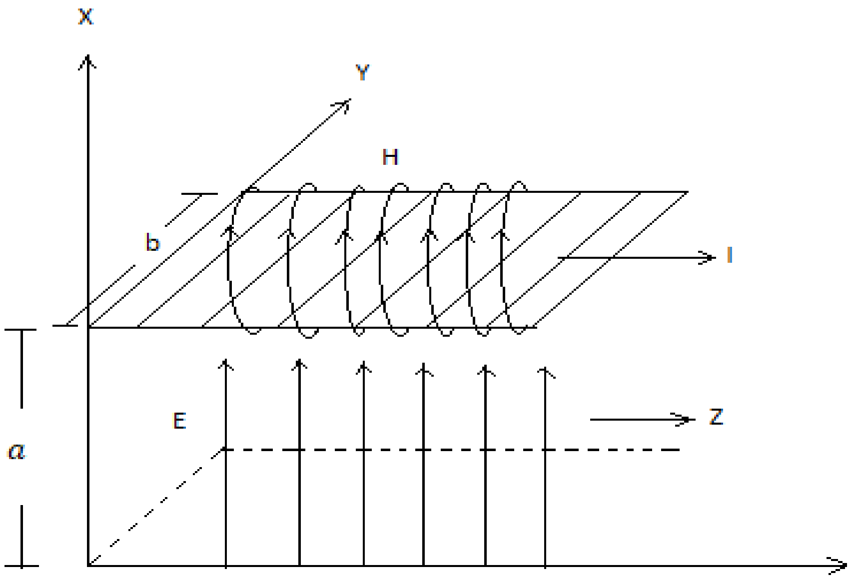

Figure 4(a): Electric and Magnetic fields about the conductors of parallel plates for transmission line on guided waves along z-direction.

We have Maxwell's equations given as

$\nabla \times \boldsymbol{E} = \frac{\partial \boldsymbol{B}}{\partial t}$ (A1)

$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{J}$ (A2)

Applying Stoke's theorem, we have

$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\int \frac{\partial \boldsymbol{B}}{\partial t} \cdot d\boldsymbol{A}$ (A3)

$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = \int \left(\frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{J}\right) \cdot d\boldsymbol{A}$ (A4)

For sinusoidal magnetic and electric field,

$\boldsymbol{B} = \boldsymbol{B}_0 e^{j\omega t}$ (A5)

And$\boldsymbol{D} = \boldsymbol{D}_0 e^{i\omega t}$ (A6)

So that$\frac{\partial \boldsymbol{B}}{\partial t} = j\omega \boldsymbol{B}$ (A7)

And $\frac{\partial \boldsymbol{D}}{\partial t} = j\omega \boldsymbol{D} = j\omega\varepsilon \boldsymbol{E}$ (A8)

Where, ε is permittivity of space between plates. Consider XZ-plane and YZ-plane of FigureA(a) for field, voltage and current shown in Figure A(b) and Figure A(c). For loop ABCD and FGNK, $dA = AB \cdot BC = adz,$ $V_{AD} = V_{BC} = 0$, $V_{CD} - V_{AB} = dV$, $\boldsymbol{H} = \hat{y}H_y$ and $dA = bdz$, $J \cdot dA = 0$, $H_{GN} \cdot dl = H_{FK} \cdot dl = 0$, $H_{FG} - H_{NK} = -dH_y$, $\boldsymbol{E} = \hat{x}E_x$respectively and we may write equations (A3) and (A4) as

$\frac{dV}{dz} = -j\omega B_y a$ (A9)

And $\frac{d(bH_y)}{dz} = \frac{dI}{dz} = -j\omega\varepsilon E_x b$ (A10)

The field exists along y-direction by equation (A9) so, $B_y(= \mu H_y)$ components of magnetic flux density will exist and where, $\mu = \mu_r \mu_0$and $bH_y = bJ_{sz} = I$, $J_{sz}$is linear surface density and equal to $H_y = \frac{I}{b}$ and putting in (A9), we have

$\frac{dV}{dz} = -j\omega\mu \frac{Ia}{b}$ (A11)

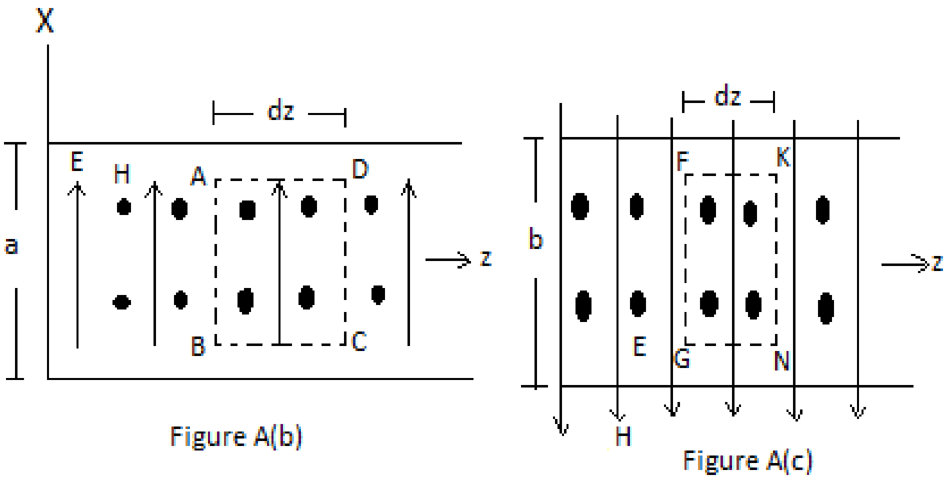

Figure 4(b): XZ- plane for parallel plate conductor. Figure 4(c): YZ- plane for parallel plate conductor.

Fall in voltage and current with wave propagation for loop resistance, loop inductance, conductance and capacitance are R, L, G and C respectively per unit length of transmission-line. If R = 0 and G = 0, then for perfect direction

$$\frac{dV}{dz} = -j\omega LI \tag{A12}$$

And $$\frac{dI}{dz} = -j\omega CV \tag{A13}$$

Since, $V = E_x a$, equation (A13) may written as

$$\frac{dI}{dz} = -j\omega CE_x a \tag{A14}$$

From equations (A11) and (A12), we have the inductance per unit length of transmission-line as

$$L = \frac{\mu a}{b} \tag{A15}$$

From equations (A10) and (A14), we have the capacitance per unit length of transmission-line as

$$C = \frac{\varepsilon b}{a} \tag{A16}$$

CONCLUSIONS

Helmholtz equation has been given the expression for signal wave velocity. The group as well as phase velocity is equal to inverse of square root of product of inductance and capacitance in transmission line of carbon nanotubes that approximately equal to the speed of light. The electric field around the carbon nanotube depends on diameter of nanotube. At centre, the electric field in carbon nanotube increases with increasing diameter due to the density of state decreases per atom with increasing diameter. The electrostatic potential drops over the tube length due to increasing applied bias. The inductance and capacitance per unit length is found, when loop resistance and shunt conductance are zero. And the phase and group velocities are equal and represent the electromagnetic wave velocity in transmission line affected by kinetic inductance for signal in carbon nanotubes. So, we have found that the propagation velocity is equal to the Fermi velocity. For parallel carbon nanotubes, the signal velocity is described by the group velocity that is approximately equal to speed of light. This work is very similar to the previous work.

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