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Influence of Electron-Phonon Interaction on Transport Characteristics of Bound States of Nanowires

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ABSTRACT We have studied the influence of electron-phonon interaction on transport characteristics in the presence of Majorana bound states. In the absence of electron-phonon interaction the quantum dot spectral function showed a line shape characterizing the presence of Majorana bound states. The location of antiresonance points which occurred in the spectrum was determined by the overlap energy between Majorana bound states. It was found that when the Majorana bound states did not overlap the linear conductance was in equal manner attributed to electron tunneling and local Andreev reflection processes at zero temperature. The linear conductance and its components were 2π periodic functions of magnetic flux phase and did not depend on dot energy or finite values of dot Majorana bound state coupling at zero temperature. It was established that the electron tunneling and local Andreev reflection components of linear conductance responded to the change of temperature. A minimal device was made from Majorana bound states coupled to regular fermionic degree of freedom associated with quantum dot and was connected to normal leads. The presence of Majorana bound states was found in the conductance through the quantum dot. The study of transport through the noninteracting quantum dot coupled to metallic leads and connected to topological super conducting nanowires revealed that the presence of Majorana bound states influenced the linear conductance.

KEYWORDS Electron-Phonon Interaction, Transport, Quantum Dot, Majorana Bound State, Antiresonance, Linear Conductance, Tunneling.

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INTRODUCTION

Read et al. [1] and Stone et al. [2] presented that semiconducting nanowires was detected in transport measurements as a zero bias anomaly in the differential conductance. Kitaev [3] studied topological materials allowed the realization of localized edge states with zero energy excitations called Majorana bound states. Lutchyn et al. [4] and Oreg et al. [5] studied a realized Majorana bound states in nanodevice containing a semiconducting nanowires with strong spin-orbit coupling. Mourik et al. [6] showed Majorana bound states demonstrated the conductance measurement in hetrostructure. Liu and Baranger [7] studied the transport through a non interacting quantum dot coupled to metallic leads and connected to topological super conducting nanowires revealed that the presence of Majorana bound states dramatically affected the linear conductance. Quantum dot Majorana devices have been detected for the study of transport properties such as the conductance spectrum [8-12], Current noise [13-16], thermal conductance [17-19], optical schemes [20-21] and Fano resonance [22-27]. Zen et al. [28] analysied transport inside a ring system formed by an asymmetrically biased coupled to two Majorana bound states located at the ends of a topological super conducing nanowires and differential conductance was found. Calle et al. [29-30] have demonstrated that quantum transport in a conventional T-shaped double quantum dot system connected a topologically trivial super conducting lead corresponded to quantum transport in a quantum dot Majorana bound states ring system when fine tuning the system parameters. Hung and Zho [31] showed the case of other systems based on shot noise probing in mono-layer graphene quantum dots or in toroidal carbon nanotubes [32] have been studied to detect the presence of Majorana bound states. Sherman et al. [33] and Deng et al. [34] presented that quantum dot topological superconducting nanowire junction was detected Majorana bound states and explored their transport properties. Wang et al. [35] showed that the trans properties of quantum dot connected to one Majorana bound states in the presence of electron-phonon interaction study resulted in diverging conclusion about the zero temperature behavior of zero bias conductance. Kitaev [36] presented that Majorana bound states obey non Abelian statistics and provided non local electronic degrees of freedom to encode quantum information and served as potential platform for topological quantum computation. Some investigators presented that the topological qubits realized by Majorana bound states stored quantum information non locally and provided robust protection against decoherence [37-41]. Wang [42] studied that phonon assisted Andreev tunneling was observed experimentally for a carbon nanotube quantum dot coupled to nitrogen and $N_b Sc$ leads. Only a few theoretical works have showed the mechanisms of phonon assisted transport at Majorana bound states.

METHOD

We have studied quantum transport coupled quantum dot with Majorana bound states. We have studied the phonon assisted transport properties in a device when the central quantum dot interacted with a single ling wave optical phonon mode. We have considered a quantum dot coupled to two Majorana bound states located at the ends of topological superconducting nanowire loop threaded by external magnetic flux. Topological superconducting nanowires loop hosted Majorana bound states at its ends for magnetic field applied perpendicularly to the surface enclosed by loop. This allowed the magnetic field for the quantum dot system transport by modulating the magnetic flux. The current was measured through metallic electrodes. For the study of the influence of electron-phonon interaction on transport properties of the system, we have considered the localized electrons in the quantum dot interacting with a single optical phonon mode. Majorana bound states were coupled to the dot in the coulomb blockade regime when the quantum dot was modeled as a single fermion level. The applied magnetic field was large enough in order to drive the nanowire into topological superconducting phase. The Hamiltonian of the system was taken as

$$H = H_{leads} + H_{MBS} + H_{ph} + H_D + H_T$$

where H_{leads} is the Hamiltonian of the metallic electrodes which describes the free electrons in the lead

$$H_{lead} = \sum_{\alpha,k} \varepsilon_{\alpha k} c_{\alpha k}^{\dagger} c_{\alpha k}$$

$c_{\alpha k}^{\dagger}$ and $c_{\alpha k}$ are the creation and annihilation operators for non interacting electrons with momentum K in the lead

$$\alpha\left[\equiv Left(L), right(R)\right].$$

The $\varepsilon_{\alpha k} = \varepsilon_k - \mu_\alpha$ are the single particle energies μ_α is the chemical potential in the lead α and is assumed to be temperature-independent. We have considered the equal temperatures in the leads as $T_\alpha = T$. H_{MBS} is the Majorana bound states Hamiltonian and models the coupling between the two Majorana bound states.

$$H_{MBS} = i\varepsilon_M \gamma_1 \gamma_2$$

where ε_M is the overlap energy which exponentially depended on the length of the topological super conducting nanowire loop (*L*) as $\varepsilon_M \propto e^{-\frac{L}{\varepsilon}}$, with ε being the superconducting coherence length. The longitudinal optical phonon mode $H_{ph} = \hbar\omega_0 a^{\dagger} a$ where ω_0 is the frequency of the single level phonon mode. $a^{\dagger}(a)$ phonon creation or annihilation operator phonon distribution is given by Bose-Einstein function $N_{ph} = \frac{1}{\left(e\frac{\hbar\omega_0}{k_B T} - 1\right)}$.

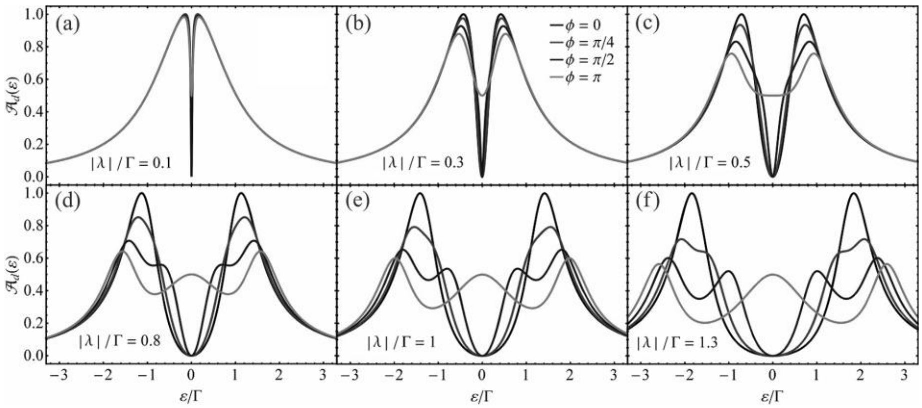
We have considered $\hbar$ and Boltzmann constant k_B to unity. H_D modeled the quantum dot and it is expressed as

$$H_D = \varepsilon_d d^{\dagger} d + \beta\left(a + a^{\dagger}\right) d^{\dagger} d .$$

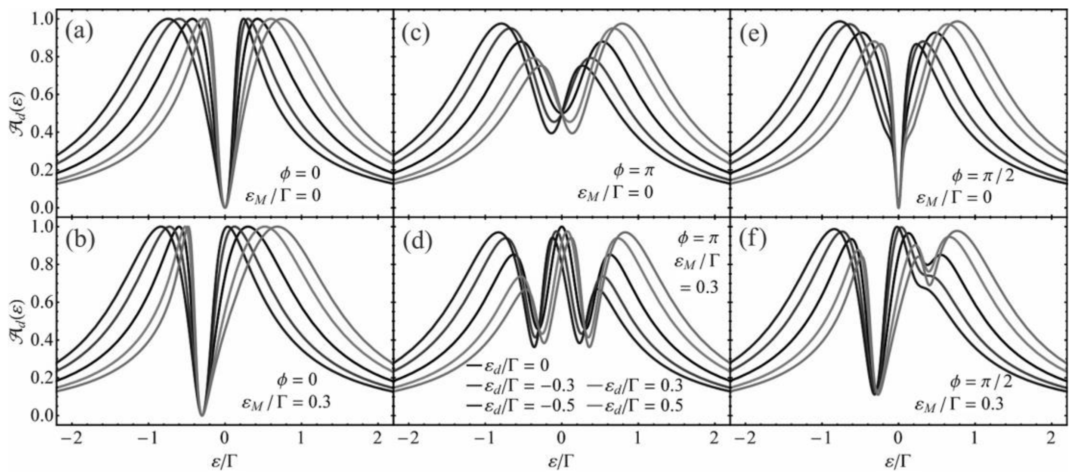
Where ε_d is the energy of the dot level, $d^{\dagger} d$ is the fermionic creation or annihilation operator for the localized electron in the quantum dot. The dot level ε_d was tuned by the gate voltage V_g through the gate electrode, β is the electron-phonon coupling strength. H_T is the tunneling Hamiltonian which represents the coupling of the quantum dot to the Majorana bound states to the metallic electrodes. We have considered the wide band limit by assuming electron-hole symmetry in the system. The transport properties of the quantum dot Majorana bound state system in the absence and presence of electron-phonon interaction. We have considered that super conductor was grounded i.e. $\mu_s = 0$ and there was an opposite bias applied between the metallic electrode $\mu_L = -\mu_R = \frac{eV}{2}$ using KirChoff's law the current from the superconductor was expressed as $I_S = -\left(I_L + I_R\right)$. The calculations of the corresponding differential and linear conductances were made. The current was obtained by performing a decoupling treatment for the electron phonon system on the spectral function.

RESULTS AND DISCUSSION

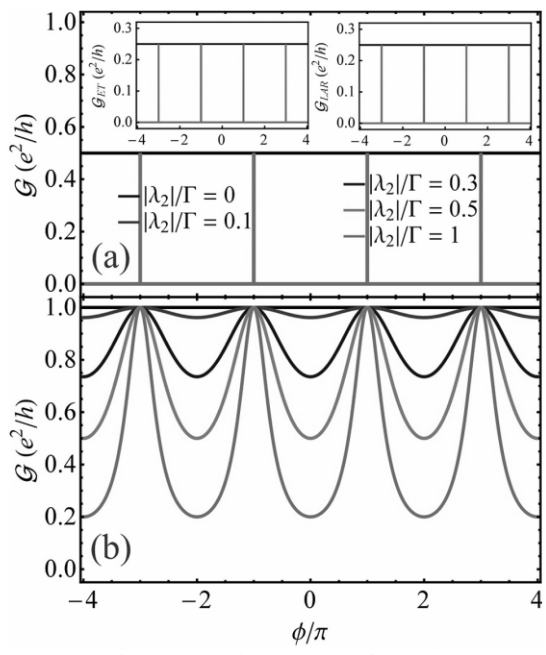
The probe in Majorana bound states in a topological superconducting nanowire required embedding the nanowire in a mesoscopic circuit. A minimal device was fabricated from Majorana bound states coupled to regular fermionic degrees of freedom associated with quantum dot and was connected to normal leads. The study of transport by a non interacting quantum dot, symmetrically coupled to metallic leads and was connected to a topological superconducting nanowire revealed the presence of Majorana bound states influenced the linear conductance. It was found that the linear conductance of quantum dot connected two Majorana bound states confined at the ends of a topological super conducing nanowire was a 2π periodic function of magnetic flux when the overlap energy between the Majorana bound states was zero. Graph (1) shows the plot of spectral function of the quantum dot as function of energy in the case of sufficiently long topological superconducting nanowire with vanishing overlap energy between Majorana bound states. It was seen that for small values of the symmetrical quantumdot-Majorana bound states coupling the spectral function presented two maxima at different positions. It was also found that with increasing quantum dot-Majorana bound state coupling, the maxima of shift to higher values of $|\varepsilon|$. The spectral function at $\varepsilon = 0$ vanished for any magnetic phase value except for $\phi = (2n+1)\pi$ with $n = 0, \pm 1, \pm 2$. Graph (2) shows the plot of the influence of dot level ε_d and overlap energy ε_H as a function of quantum dot and as function of energy of three different magnetic flux phase values and dot level energies with finite Majorana bound state overlap and weak symmetrical quantum dot Majorana bound state couplings. Graph (2)(a) and 2(b) show the antiresonance point of the spectrum decreased to zero and shifted to the point of $\varepsilon \approx -\varepsilon_H$ for any values of the dot level. The broadened peak at $\varepsilon \approx \varepsilon_d$ is due t resonant transmission through a quantum dot at the tuned dot level even in the absence of the Majorana bound states. The sharp peaks were mainly attributed to the regular fermionic states originating from the Majorana bound sates. Graph (2)(c) and (2)(d) show the two peak structure in the spectrum evolves with increasing ε_M into a three peak structure showing the presence of Majorana bound states in the system. Graph (3) shows the linear conductance as a function of magnetic flux phase for different electron-phonon coupling strengths for dot level at zero and finite temperature with and without electron-phonon interaction with either $\varepsilon_M = 0$ or $\varepsilon_M \neq 0$. At zero temperature when $\varepsilon_M = 0$, conductance became independent of the electron-phonon coupling strength β. This behavior was also found where a single Majorana bound state was connected to a normal lead through quantum dot. It was found that at finite temperatures the conductance peaks were broadened and suppressed with β. In the presence of electron phonon interaction at $\varepsilon_M \neq 0$, the linear conductance depended ε_d and β both at zero and finite temperatures. The electron phonon coupling strength β suppressed the magnitude of conductance at fixed ε_d. We have analysed the temperature effect on linear conductance in the presence of electron phonon interaction. The temperature strongly affected the linear conductance. We have also examined differential conductance $\frac{dI}{dV}$ as a function of bias voltage eV in the presence of electron-phonon interaction at zero and finite temperatures for different values of overlap energy between Majorana bound states ε_M, magnetic flux phase ϕ and electron-phonon coupling strength β, when the quantum dot weakly coupled to the Majorana bound states with symmetrical coupling and at $\varepsilon_d = 0$. The zero bias peak was found immused to the electron-phonon interaction at zero temperature. The phonon side bands in the spectrum were characterized by the main Majorana bound state induced peaks. The obtained results were compared with previously obtained results of theoretical and experimental research works were found in good agreement.



Graph 1: Plot of spectral function of quantum as function of energy.



Graph 2: Plot of spectral function with symmetrical quantum dot Majorana bound state coupling for different magnetic flux phases.



Graph 3: Plot of linear conductance as function of magnetic flux phase.

CONCLUSION

We have studied the influence of electron-phonon interaction on transport characteristics of bound states of nanowires. The quantum transport in quantum dot coupled to Majorana bound states of nanowires. A minimal device was fabricated from Majorana bound states coupled to regular fermionic degrees of freedom associated with quantum dot connected to normal leads. The presence of Majorana superconducting bound states was present in the conductance in quantum dot. The study of transport through a non-interacting quantum dot symmetrically coupled to metallic leads and connected to a topological superconducting nanowires which influenced the linear conductance. The study of phonon assisted transport properties in device was made when the central quantum dot interacted with a single long wave optical phonon modi. The electron-phonon interaction was incorporated in the model using the canonical transformation within the nonequilibrium Green's function formalism. The retarded Green's function was determined by using equation of motion. The influence of electron-phonon interaction on transport characteristics in the presence of Majorana bound states was analysed. The device electrodes were oppositely biased which resulted in a tunneling current by the dot with two components, one from the electron tunneling and the other from local Andreev reflection. The crossed Andreev reflection was present only when the bias was asymmetrically applied. It was found that the finite values of dot produced Majorana coupling. The linear conductance periodically charged to 4π due to a finite electron-phonon coupling strength. The differential conductance periodicity changes from 2π to 4π when the Majorana bound states hybridized and the electron-phonon coupling is finite. The obtained results were found in good agreement with previous obtained results.

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