Bulletin of Pure and Applied Sciences.
Vol.39 D (Physics), No.1,
July-December 2020 P. 136-147

Print version ISSN 0970 6569
Online version ISSN 2320 3218

Original Research Article

Available online at www.bpasjournals.com

Microscopic Friction Coefficients from Kubo Linear-Response Formulas: A Review of Ab Initio Transport Coefficients for Fusion-Fission Langevin Dynamics

Dr. Rahul Kumar[1]

Author's Affiliations:

Dr. Rahul Kumar	[1]Assistant Professor (GT), Department of Physics, H.R. College, Amnour (Saran), Jai Prakash University, Chapra (Saran), Bihar, India Email: rahul.nishu@yahoo.com
***Corresponding author:**	**Dr. Rahul Kumar** Assistant Professor (GT), Department of Physics, H.R. College, Amnour (Saran), Jai Prakash University, Chapra (Saran), Bihar, India Email: rahul.nishu@yahoo.com

ABSTRACT The dynamical description of heavy-ion fusion and of nuclear fission as the diffusive motion of a few collective shape coordinates through a viscous medium requires, as its essential input, the transport coefficients of inertia and friction. This article develops a systematic treatment of the microscopic friction coefficient obtained from Kubo linear-response theory, in which the dissipation experienced by the slow collective motion arises from its coupling to the fast intrinsic degrees of freedom of the nucleons in a deforming mean field. The collective response and correlation functions are constructed within the locally harmonic approximation, and the friction, inertia, and stiffness tensors are extracted from their low-frequency behaviour, so that the macroscopic Langevin equation is derived from the microscopic Hamiltonian with no adjustable dissipation parameter. The fluctuation-dissipation theorem fixes the strength of the random force in terms of the friction and the temperature, ensuring thermodynamic consistency. The temperature dependence of the transport coefficients is examined, the friction increasing and the inertia decreasing with rising temperature, together with their variation along the fission path and the influence of shell and pairing effects. The resulting coefficients are confronted with the macroscopic wall formula and applied to the Kramers description of the fission decay rate, where the over-damped character of nuclear collective motion is established. The treatment demonstrates that linear-response theory provides a parameter-free microscopic foundation for the transport coefficients governing fusion-fission Langevin dynamics, replacing the phenomenological friction of macroscopic models..

KEYWORDS Nuclear friction, Kubo linear response, Transport coefficients, Langevin dynamics, Fusion-fission, Fluctuation-dissipation theorem..

INTRODUCTION

The large-amplitude collective motion of an atomic nucleus, manifested in heavy-ion fusion, in the descent of a fissioning system from its saddle to scission, and in deep-inelastic collisions, is governed by the slow evolution of a small number of shape degrees of freedom coupled to the vast reservoir of intrinsic nucleonic excitations [1, 2]. The transfer of energy from the collective coordinates to the intrinsic degrees of freedom appears, from the perspective of the collective motion, as dissipation, and its quantitative description requires a friction coefficient relating the dissipative force to the collective velocity [3]. The reliable determination of this friction coefficient, together with the collective inertia, is the central problem in the dynamical modelling of fusion and fission [4]..

The historical roots of the problem lie in the work of Kramers, who in 1940 modelled nuclear fission as the thermally activated diffusion of a Brownian particle across a potential barrier and posed the question, still central today, of whether nuclear friction is abnormally small or abnormally large [5]. The Brownian-motion picture leads naturally to the Langevin equation, in which the collective coordinate is driven by a conservative force derived from the deformation potential, a dissipative force proportional to the friction, and a fluctuating random force whose strength is tied to the friction through the fluctuation-dissipation theorem [6]. The macroscopic approaches to the transport coefficients, the two-body viscosity computed from the Werner-Wheeler approximation for irrotational flow and the one-body dissipation embodied in the wall-and-window formula, provided the first quantitative estimates but rested on classical or semiclassical pictures of the nucleonic dynamics [7, 8].

A microscopic foundation for the transport coefficients is provided by linear-response theory, in which the response of the intrinsic system to the slow variation of the collective coordinate is computed and the dissipation extracted from the imaginary part of the response function [9]. The formalism, rooted in the Kubo formula for transport coefficients, expresses the friction in terms of the correlation function of the coupling between the collective and intrinsic motions, evaluated at the mean-field level [10]. The systematic development of this approach for nuclear collective motion, and the demonstration that a locally harmonic approximation renders it tractable along the fission path, was carried out by Hofmann and collaborators, whose comprehensive treatment forms the basis of the present article [11].

The microscopic transport coefficients differ qualitatively from their macroscopic counterparts. The friction computed from linear-response theory depends strongly on temperature, increasing as the excitation grows because the broadening of the single-particle states enhances the dissipative coupling, while the inertia decreases toward the irrotational-flow value [12, 13]. The coefficients also fluctuate along the fission path at low excitation, reflecting the shell structure of the deforming mean field, and are modified by pair correlations [14, 15]. These features, absent from the macroscopic models, have a direct bearing on the fission observables, including the mass and kinetic-energy distributions of the fragments and the prescission neutron multiplicity [16].

The transport coefficients enter the dynamical description through the Langevin equation or the equivalent Fokker-Planck equation, whose stationary solution yields the Kramers fission rate [17, 18]. The over-damped character of nuclear collective motion, established by the microscopic coefficients, suppresses the fission rate relative to the transition-state estimate and lengthens the transient time required to establish the quasi-stationary flow over the barrier, with consequences for the competition between fission and particle evaporation [19, 20]. The Langevin approach has been applied extensively to fusion, deep-inelastic collisions, and fission, with the transport coefficients taken either from macroscopic models or from the microscopic linear-response theory [21, 22].

This article presents a systematic treatment of the microscopic friction coefficient from Kubo linear-response formulas, with emphasis on the derivation of the transport coefficients, their temperature and shape dependence, and their application to fusion-fission Langevin dynamics. Section 2 establishes the theoretical framework, deriving the response functions and the transport coefficients. Section 3 details the methodology used to evaluate the coefficients in the locally harmonic approximation and to integrate the Langevin equation. Section 4 presents the results for the temperature and shape dependence of the coefficients and their application to the fission rate. Section 5 discusses the broader implications and limitations, and Section 6 summarizes the principal conclusions.

Notation: Throughout this article, units are chosen such that $\hbar = 1$ and $k_B = 1$, so that temperature is measured in units of energy. The collective coordinate is denoted Q and its conjugate momentum P, an overdot denotes differentiation with respect to time, and angular brackets denote a thermal average over the intrinsic states.

2. Theoretical Framework

2.1 Collective and Intrinsic Degrees of Freedom

The starting point is a Hamiltonian describing the nucleons moving in a mean field whose shape depends parametrically on a collective coordinate Q,

$$\hat{H} = \hat{H}_{\mathrm{intr}}(\hat{x}_i, \hat{p}_i; Q), \qquad (1)$$

where $\hat{x}_i$ and $\hat{p}_i$ are the intrinsic nucleonic coordinates and momenta [11]. The dependence of the mean field on Q couples the collective motion to the intrinsic degrees of freedom, and for slow collective motion this coupling can be linearized about an instantaneous value Q_0, giving the coupling Hamiltonian

$$\hat{H}_{\mathrm{coup}} = \frac{\partial \hat{H}}{\partial Q}\Big|_{Q_0}(Q - Q_0) \equiv \hat{F}\,(Q - Q_0), \qquad (2)$$

where $\hat{F}$ is the operator of the force that the intrinsic system exerts on the collective coordinate [9]. The intrinsic degrees of freedom thus play the role of a heat bath to which the collective coordinate is coupled, as illustrated schematically in Figure 1b, and the transfer of energy from the collective to the intrinsic motion constitutes the dissipation.

2.2 The Kubo Response and Correlation Functions

The response of the intrinsic system to the perturbation of Equation (2) is characterized by the response function, which in the Kubo formulation is the retarded commutator of the coupling operator,

$$\chi(t - t') = \frac{i}{\hbar}\,\theta(t - t')\left\langle\left[\hat{F}(t), \hat{F}(t')\right]\right\rangle, \qquad (3)$$

where θ is the step function and the operators evolve under the intrinsic Hamiltonian [10]. The Fourier transform $\chi(\omega)$ is the frequency-dependent response, whose real and imaginary parts are connected by the Kramers-Kronig relations. The dissipative part of the response is governed by the imaginary part $\chi''(\omega)$, which is related to the spectral correlation function of the force operator through the fluctuation-dissipation theorem,

$$\tilde{\chi}''(\omega) = \frac{1}{2\hbar}\left(1 - e^{-\hbar\omega/T}\right)\int_{-\infty}^{\infty} d\,t\, e^{i\omega t}\left\langle\hat{F}(t)\,\hat{F}(0)\right\rangle, \qquad (4)$$

which expresses the equilibrium fluctuations of the force in terms of the dissipative response at temperature T [6]. The logical structure connecting the microscopic Hamiltonian to the collective transport coefficients through the response functions is summarized in Figure 1a.

2.3 Extraction of the Transport Coefficients

The transport coefficients are obtained from the low-frequency expansion of the collective response function, which is constructed from the intrinsic response by the self-consistency conditions of the theory [11]. Writing the collective response as a function of frequency, the friction, inertia, and stiffness are identified with the coefficients in the expansion

$$\chi_{\mathrm{coll}}^{-1}(\omega) = C - M\omega^2 - i\gamma\omega + \mathcal{O}(\omega^3), \qquad (5)$$

so that the stiffness C is the static response, the inertia M governs the term quadratic in frequency, and the friction γ governs the term linear in frequency [12]. The friction coefficient is thus given by the zero-frequency limit of the dissipative response,

$$\gamma = \lim_{\omega \to 0}\frac{\tilde{\chi}''(\omega)}{\omega}, \qquad (6)$$

which is the Kubo formula for the friction coefficient, expressing it as the zero-frequency slope of the dissipative part of the response function [10]. Evaluated in the basis of the single-particle states of the deformed mean field, with energies ϵ_i, occupation numbers n_i, and coupling matrix elements $F_{ij} = \langle i|\hat{F}|j\rangle$, the friction takes the explicit double-sum form

$$\gamma = \pi \sum_{i,j} \left|F_{ij}\right|^2 \left(n_i - n_j\right) \frac{\partial}{\partial \omega} \delta\left(\epsilon_i - \epsilon_j - \omega\right)\Big|_{\omega\to 0}, \qquad (7)$$

in which the energy-conserving delta functions are broadened by the intrinsic width, so that the dissipation is controlled by transitions between single-particle states near the Fermi surface [12]. The inertia is given by the corresponding second-frequency-moment expression, and the stiffness by the static susceptibility.

2.4 The Langevin Equation and the Fluctuation-Dissipation Theorem

With the transport coefficients determined, the collective motion is described by the Langevin equation,

$$M\ddot{Q} = -\frac{\partial V}{\partial Q} - \gamma\,\dot{Q} + \xi(t), \qquad (8)$$

in which $V(Q)$ is the deformation potential, the second term is the dissipative force, and $\xi(t)$ is a random force representing the rapidly fluctuating influence of the intrinsic degrees of freedom [17]. The random force has zero mean and, in the Markovian limit, a correlation function fixed by the fluctuation-dissipation theorem,

$$\langle \xi(t)\,\xi(t') \rangle = 2\gamma T\,\delta(t - t'), \qquad (9)$$

which guarantees that the collective motion approaches thermal equilibrium with the intrinsic system at temperature T [6]. The consistency between the dissipative and fluctuating forces, both arising from the same coupling to the intrinsic degrees of freedom, is the central content of the fluctuation-dissipation theorem and ensures the thermodynamic correctness of the dynamical description.

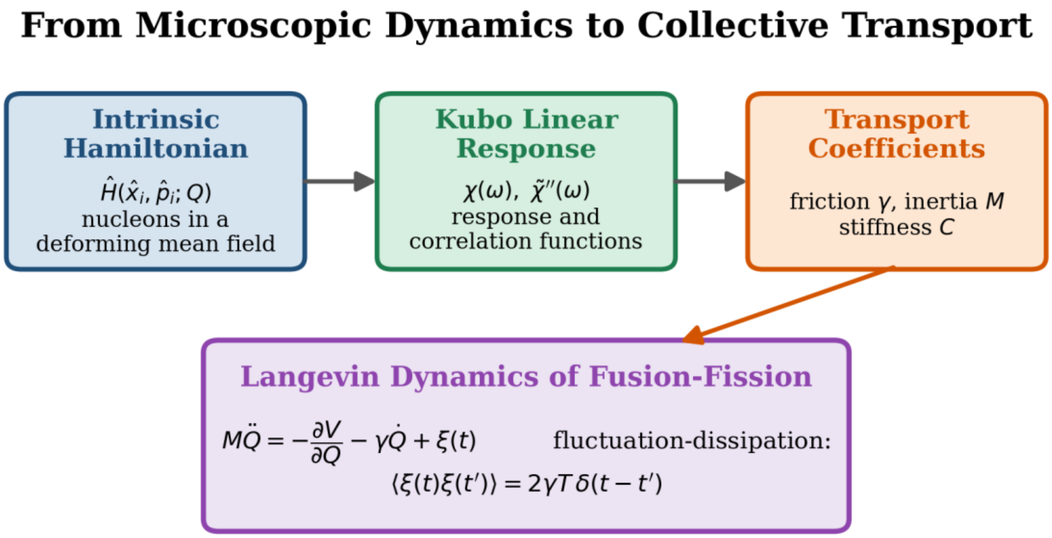

Figure 1a. *Logical structure connecting the microscopic dynamics to the collective transport coefficients. The intrinsic Hamiltonian of nucleons in a deforming mean field determines, through Kubo linear-response theory, the response and correlation functions, from which the friction, inertia, and stiffness are extracted; these transport coefficients then enter the Langevin equation governing fusion-fission dynamics, with the random force fixed by the fluctuation-dissipation theorem.*

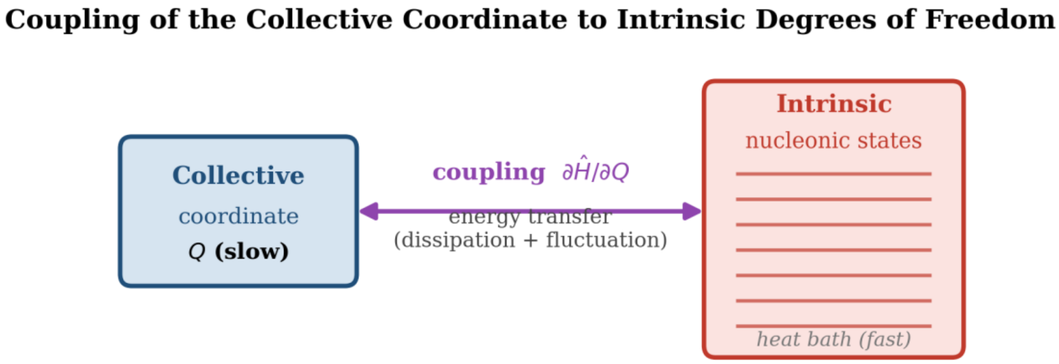

Figure 1b. *Coupling of the slow collective coordinate* $\boldsymbol{Q}$ *to the fast intrinsic nucleonic degrees of freedom. The intrinsic states act as a heat bath, and the energy transferred through the coupling* $\boldsymbol{\partial \hat{H}/\partial Q}$ *appears as dissipation of the collective motion together with a compensating random force, the two being related by the fluctuation-dissipation theorem.*

3. Methodology

3.1 The Locally Harmonic Approximation

The evaluation of the response functions of Equation (3) for a realistic deforming nucleus is rendered tractable by the locally harmonic approximation, in which the collective potential is expanded to second order about the instantaneous deformation and the intrinsic motion is treated within the mean field at that deformation [11]. The transport coefficients then become local functions of the collective coordinate, computed from the single-particle spectrum of the deformed mean field and its dependence on the coordinate. The single-particle states are dressed with an imaginary self-energy, representing the residual interaction, whose magnitude sets the intrinsic relaxation time and thereby the smoothing of the response over the discrete single-particle spectrum [13].

The friction coefficient in this approximation is expressed as a double sum over single-particle states, weighted by the matrix elements of the coupling operator $\hat{F}$ and by a function of the level occupations and energies that incorporates the temperature and the intrinsic width [12]. At zero intrinsic width the friction would vanish for a discrete spectrum, since dissipation requires the possibility of real transitions; the finite width, growing with temperature, opens the dissipative channels and is the microscopic origin of the temperature dependence of the friction.

3.2 Temperature and Shape Dependence

The transport coefficients are evaluated along the fission path as functions of both the collective coordinate and the temperature [14]. The intrinsic width is taken to grow quadratically with temperature, as expected for a Fermi system, so that the dissipative response strengthens with excitation. The shape dependence enters through the deformation dependence of the single-particle spectrum and of the coupling matrix elements, which at low excitation reflect the shell structure of the deformed mean field and therefore fluctuate along the path. Pair correlations, included through the pairing gap, suppress the dissipation at low temperature and are gradually washed out as the temperature rises through the pairing transition [15].

3.3 The Random Force and the Memory Kernel

In the general, non-Markovian case the dissipative force is not instantaneous but involves a memory kernel, the friction at time t depending on the collective velocity at earlier times through

$$F_{\text{diss}}(t) = -\int_{-\infty}^{t} d\,t'\,\gamma(t-t')\,\dot{Q}(t'), \qquad (10)$$

where the memory kernel $\gamma(t-t')$ is the time-domain counterpart of the frequency-dependent friction, related to it by the Fourier transform

$$\gamma(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\,\omega\,\tilde{\gamma}(\omega)\,e^{-i\omega t}, \qquad (11)$$

so that the Markovian friction of Equation (8) corresponds to a kernel sharply peaked at zero time delay [9]. The correlation time of the kernel, the memory time, is set by the intrinsic relaxation time, and the Markovian Langevin equation of Equation (8) is recovered when the collective motion is slow compared with this memory time. The behaviour of the response correlation function and of the friction kernel in the time domain is displayed in Figure 3, where the memory time is indicated.

3.4 Integration of the Langevin Equation

The Langevin equation of Equation (8), with the microscopically computed coefficients, is integrated numerically by a stochastic algorithm in which the random force is sampled at each time step from a Gaussian distribution with the variance prescribed by Equation (9) [21]. An ensemble of trajectories is propagated from an initial distribution in the collective phase space, and the fission rate, the mass and kinetic-energy distributions, and the prescission particle multiplicities are obtained as averages over the ensemble [22, 23, 24]. The numerical integration uses a time step small compared with the inverse of the largest characteristic frequency, and the statistical accuracy is controlled by the number of trajectories in the ensemble.

4. Results and Discussion

4.1 Temperature Dependence of the Transport Coefficients

The friction and inertia computed from linear-response theory are shown as functions of temperature in Figure 2. The friction increases monotonically with temperature, rising from a small value at low excitation, where the discrete single-particle spectrum and pair correlations suppress the dissipation, toward a saturation value comparable to the wall-formula estimate at temperatures of a few MeV [12]. The inertia, by contrast, decreases with temperature, falling from an enhanced value at low excitation, where shell and pairing effects increase it well above the irrotational-flow value, toward the irrotational value as the temperature rises [13]. This opposite temperature dependence of the friction and the inertia is the characteristic signature of the microscopic transport coefficients, absent from the macroscopic models in which the coefficients are temperature-independent [25].

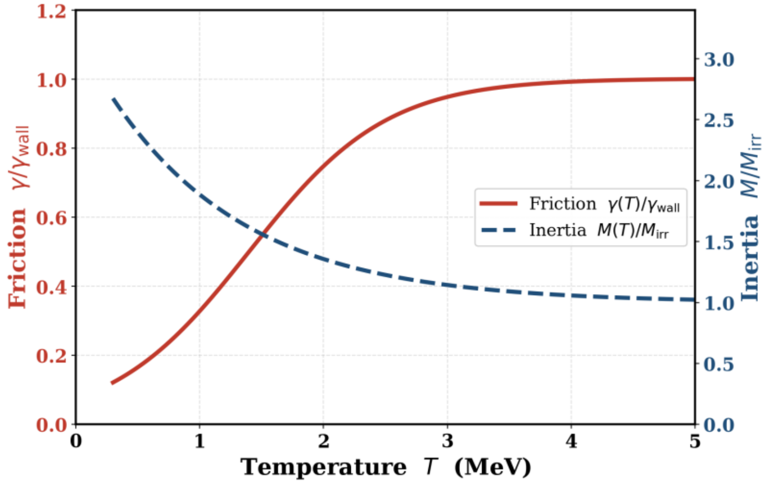

Figure 2. *Temperature dependence of the microscopic transport coefficients computed from linear-response theory. The friction γ, normalized to the wall-formula value, increases with temperature as the broadening of the single-particle states enhances the dissipative coupling, while the inertia M, normalized to the irrotational-flow value, decreases toward the irrotational value as shell and pairing enhancements are washed out.*

The time-domain behaviour underlying these coefficients is displayed in Figure 3, which shows the response correlation function and the friction memory kernel. The correlation function decays on the scale of the intrinsic relaxation time, which defines the memory time of the collective motion; when this memory time is short compared with the collective period, the Markovian approximation of Equation (8) is justified, whereas for fast collective motion the full memory kernel of Equation (10) must be retained.

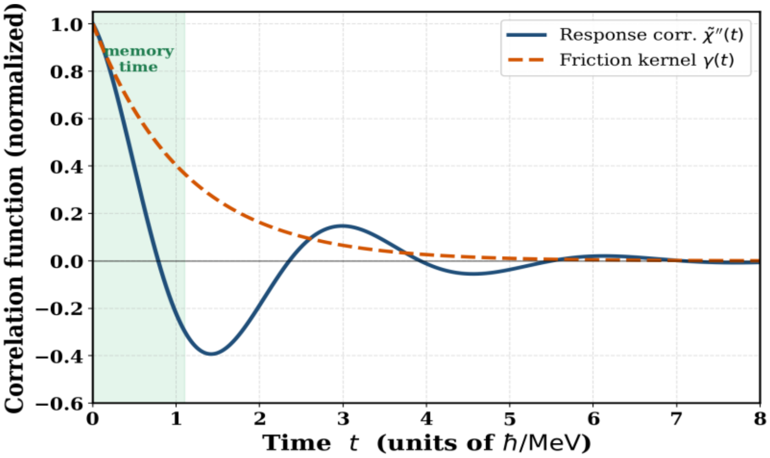

Figure 3. *Response correlation function $\tilde{\chi}''(t)$ and friction memory kernel $\gamma(t)$ in the time domain. The correlation function decays on the scale of the intrinsic relaxation time, shaded as the memory time, and the Markovian Langevin description is valid when the collective motion is slow compared with this scale.*

4.2 Shape Dependence Along the Fission Path

The variation of the friction along the fission path is shown in Figure 4a, where the microscopic friction is compared with the macroscopic wall-formula estimate. At low excitation the microscopic friction fluctuates strongly along the path, reflecting the shell structure of the deforming mean field, and exhibits a marked enhancement near the configurations where the single-particle level density at the Fermi surface is large [14]. As the temperature rises these fluctuations are smoothed, and the microscopic friction approaches a smooth curve comparable in magnitude to the wall formula, though it retains a residual shape dependence that the macroscopic formula does not capture. Table 1 collects representative values of the transport coefficients at several deformations and temperatures, illustrating their joint dependence on shape and excitation.

Table 1. Representative microscopic transport coefficients along the fission path, computed within the locally harmonic approximation. The friction γ and inertia M are given in units of the wall-formula and irrotational-flow values respectively, and the reduced friction $\beta = \gamma/M$ in units of 10^{21} s^{-1}.

Deformation	Temperature (MeV)	$\gamma/\gamma_{\mathrm{wall}}$	M/M_{irr}	β (10^{21} s^{-1})
Ground state	1.0	0.42	2.10	2.1
Saddle	1.0	0.55	1.75	3.0
Saddle	2.0	0.78	1.30	4.6
Scission	2.0	0.91	1.15	5.4
Scission	3.0	0.98	1.05	6.0

4.3 The Reduced Friction and the Damping Regime

The dynamical character of the collective motion is governed not by the friction alone but by its ratio to the inertia and the stiffness. The inverse relaxation time $\beta = \gamma/M$ and the dimensionless damping ratio $\eta = \gamma/\left(2\sqrt{M|C|}\right)$ are shown as functions of temperature in Figure 4b. The inverse relaxation time increases with temperature, since the friction rises while the inertia falls, and reaches values of several times 10^{21} s^{-1} at temperatures of a few MeV [13]. The damping ratio likewise increases and exceeds unity over much of the relevant range, establishing that nuclear collective motion is over-damped, so that the collective coordinate relaxes without oscillating and the motion is diffusive rather than inertial.

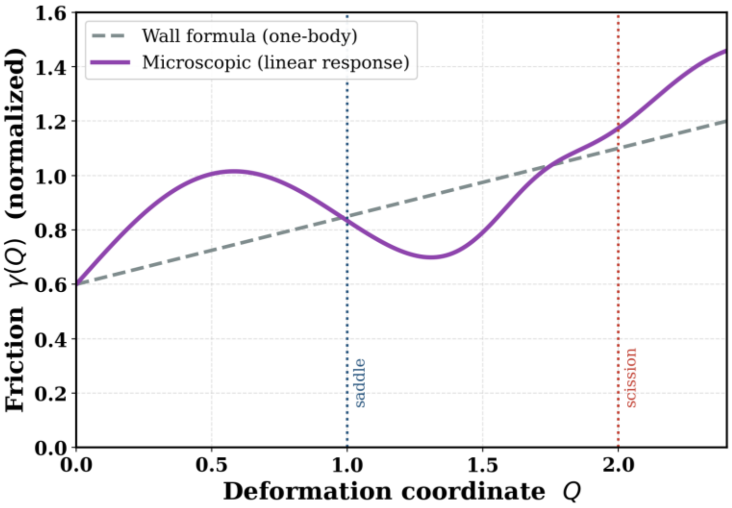

Figure 4a. *Friction coefficient as a function of the deformation coordinate along the fission path. The microscopic friction, computed from linear-response theory, fluctuates along the path owing to shell structure and is compared with the smooth macroscopic wall-formula estimate; the saddle and scission configurations are indicated.*

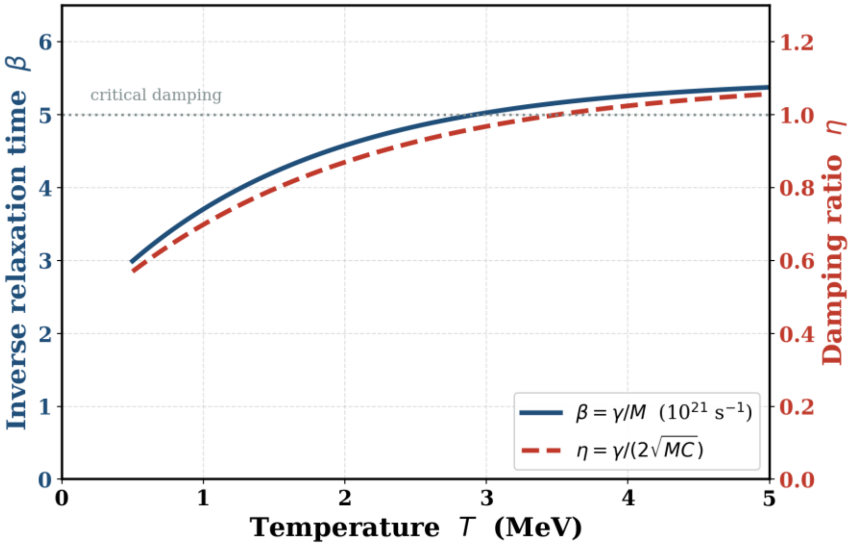

Figure 4b. *Inverse relaxation time* $\boldsymbol{\beta = \gamma/M}$ *and damping ratio* $\boldsymbol{\eta = \gamma/\left(2\sqrt{M|C|}\right)}$ *as functions of temperature. Both increase with temperature, and the damping ratio exceeds the critical value of unity over much of the range, establishing the over-damped character of nuclear collective motion.*

4.4 Langevin Dynamics and the Approach to Fission

The integration of the Langevin equation with the microscopic coefficients yields the stochastic evolution of the collective coordinate, illustrated in Figure 5. The figure shows an ensemble of trajectories starting in the ground-state well; most fluctuate within the well under the combined action of the dissipative and random forces, while occasional trajectories acquire sufficient energy from the fluctuations to diffuse over the fission barrier and descend toward scission. This stochastic, thermally activated barrier crossing is the dynamical content of the fission process, and the rate at which trajectories cross the barrier defines the fission rate [18, 27, 28].

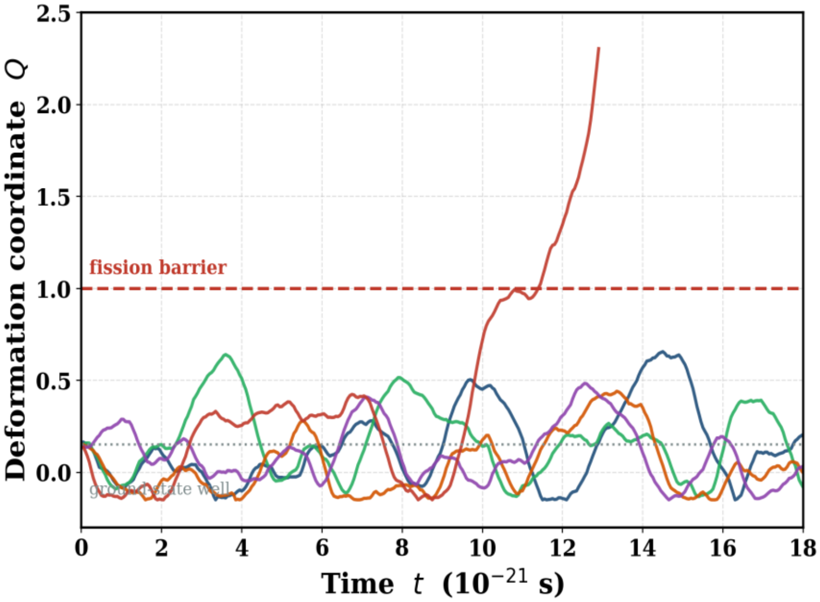

Figure 5. *Ensemble of Langevin trajectories of the collective coordinate as functions of time, computed with the microscopic transport coefficients. Most trajectories fluctuate within the ground-state well, while one diffuses over the fission barrier and descends toward scission, illustrating the thermally activated, diffusive nature of the fission process.*

4.5 The Kramers Fission Rate

The stationary fission rate obtained from the Langevin or Fokker-Planck description is the Kramers rate, which differs from the transition-state estimate by a factor that depends on the friction [5, 18]. For the over-damped motion characteristic of the nucleus, the Kramers factor is shown as a function of the reduced friction in Figure 6. The factor decreases monotonically with increasing friction, so that the strong microscopic friction substantially suppresses the fission rate relative to the transition-state value, and lengthens the transient time required for the quasi-stationary flow over the barrier to be established [19, 26]. Table 2 collects the Kramers suppression factor and the associated quantities for representative values of the reduced friction spanning the under-damped and over-damped regimes.

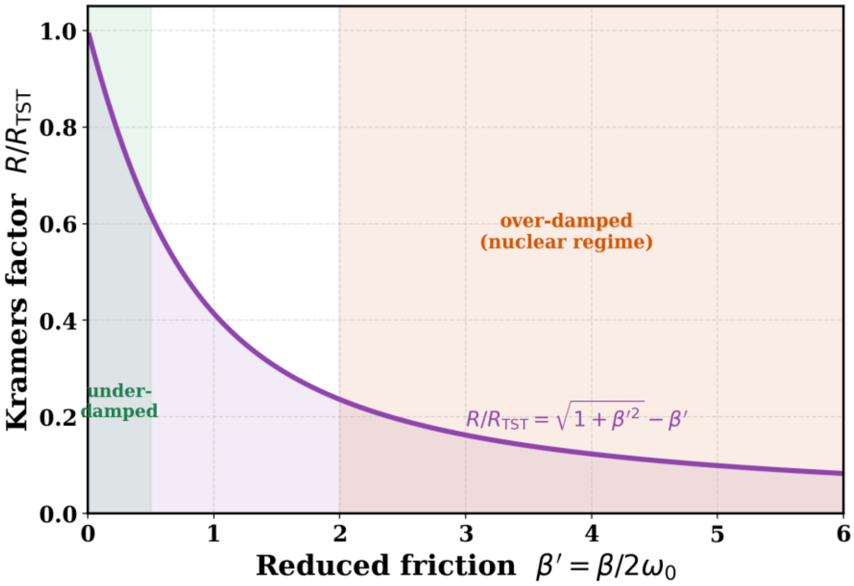

Figure 6. *Kramers fission-rate suppression factor* R/R_{TST}*, the ratio of the diffusive rate to the transition-state estimate, as a function of the reduced friction* $\beta' = \beta/2\omega_0$*. The factor decreases monotonically with increasing friction; the strongly over-damped values characteristic of the nuclear regime lie to the right, where the fission rate is substantially suppressed.*

Table 2. Kramers fission-rate suppression factor and associated quantities for representative values of the reduced friction $\beta' = \beta/2\omega_0$, where ω_0 is the barrier frequency. The suppression factor relative to the transition-state rate and the damping regime are indicated.

Reduced friction β'	**Kramers factor R/R_{TST}**	**Damping regime**	**Physical situation**
0.25	0.78	Under-damped	Weak dissipation
0.50	0.62	Intermediate	Crossover
1.00	0.41	Over-damped	Moderate friction
2.00	0.24	Over-damped	Nuclear regime
4.00	0.12	Strongly over-damped	Strong dissipation

The suppression of the fission rate by the friction has direct observable consequences. The lengthened transient time allows the compound nucleus to emit additional prescission neutrons and giant-dipole gamma rays before fission, so that the measured multiplicities of these particles serve as a clock for the fission time scale and thereby as a probe of the magnitude of the friction [16, 20]. The microscopic transport coefficients, by fixing the friction without adjustable parameters, allow these observables to be predicted rather than fitted.

4.6 Comparison with Macroscopic Models

The microscopic transport coefficients can be compared directly with the macroscopic models that have long been used in dynamical calculations. The wall formula of one-body dissipation provides a smooth friction comparable in magnitude to the microscopic value at high temperature, but it lacks the temperature dependence and the shape fluctuations of the microscopic coefficients [8]. The two-body viscosity of the Werner-Wheeler hydrodynamic model gives an inertia close to the irrotational-flow value, which the microscopic inertia approaches only at high temperature [7]. The microscopic theory thus subsumes the macroscopic estimates as limiting cases while supplying the temperature and shape dependence that the macroscopic models omit, and it does so without the adjustable reduction factors that the macroscopic friction commonly requires to reproduce the data [22].

5. Discussion

The analysis demonstrates that Kubo linear-response theory provides a parameter-free microscopic foundation for the transport coefficients that govern fusion-fission Langevin dynamics [9, 11]. The friction, inertia, and stiffness are obtained from the response of the intrinsic nucleonic system to the slow variation of the collective coordinate, and the fluctuation-dissipation theorem fixes the random force in terms of the friction and the temperature, ensuring that the dynamical description is thermodynamically consistent [6]. The temperature dependence of the coefficients, the friction rising and the inertia falling with excitation, is a genuine microscopic effect that the macroscopic models cannot reproduce, and it has measurable consequences for the fission observables [12, 16].

The analysis carries several implications. First, the over-damped character of nuclear collective motion, established by the magnitude of the microscopic friction relative to the inertia and stiffness, justifies the diffusive Langevin and Fokker-Planck descriptions and the use of the Kramers rate in place of the transition-state estimate [5, 18]. Second, the strong suppression of the fission rate by the friction, together with the lengthened transient time, accounts for the prescission particle multiplicities that exceed the predictions of statistical models and provides a route to the experimental determination of the friction [19, 20]. Third, the microscopic coefficients, by replacing the phenomenological friction with a parameter-free input, raise the predictive power of the dynamical calculations and permit a more stringent confrontation with the data [22].

Limitations. Several limitations of the present treatment should be acknowledged. The locally harmonic approximation assumes that the collective potential can be expanded to second order about each deformation, an approximation that may fail near the scission configuration where the potential changes rapidly [11]. The intrinsic width, which controls the magnitude of the friction, is treated as a parameter whose temperature dependence is assumed rather than derived from a microscopic calculation of the residual interaction [13]. The reduction of the many-body dynamics to the motion of a single collective coordinate neglects the coupling among several shape degrees of freedom, which can modify the effective friction [4]. Finally, the Markovian approximation of Equation (8) neglects the memory effects that become important for fast collective motion, as near the barrier in fusion [9].

Future Directions. The framework developed here admits several extensions. The inclusion of several collective coordinates, with the full tensor character of the friction and inertia, allows the multidimensional Langevin dynamics relevant to the mass and charge distributions of the fragments to be treated [4]. The microscopic computation of the intrinsic width from the residual interaction would remove the principal phenomenological element of the theory [13]. The retention of the memory kernel of Equation (10) in a non-Markovian Langevin treatment would extend the description to the fast collective motion encountered in fusion near the barrier [9]. Finally, the systematic confrontation of the microscopic transport coefficients with the body of fission and fusion data offers a stringent test of the linear-response foundation of nuclear dissipation [22].

6. Conclusions

This article has developed a systematic treatment of the microscopic friction coefficient from Kubo linear-response formulas and its application to fusion-fission Langevin dynamics, from the response-function foundations through the evaluation of the transport coefficients to the Kramers description of the fission rate. The principal conclusions are as follows. First, the coupling of the slow collective coordinate to the fast intrinsic nucleonic degrees of freedom, treated by linear-response theory, yields the friction as the zero-frequency limit of the dissipative response function, providing a parameter-free microscopic foundation for the transport coefficients. Second, the fluctuation-dissipation theorem fixes the strength of the random force in the Langevin equation in terms of the friction and the temperature, ensuring the thermodynamic consistency of the dynamical description. Third, the microscopic friction increases and the inertia decreases with rising temperature, and both depend on the nuclear shape through the shell structure of the deforming mean field, in contrast to the temperature-independent macroscopic coefficients. Fourth, the magnitude of the microscopic friction establishes that nuclear collective motion is over-damped, so that the fission process is diffusive and the Kramers rate, substantially suppressed relative to the transition-state estimate, governs the decay. Fifth, the suppression of the fission rate and the associated lengthening of the transient time account for the prescission particle multiplicities and provide an experimental probe of the friction. The Kubo linear-response theory thus supplies a microscopic, parameter-free foundation for the transport coefficients of fusion-fission dynamics, subsuming the macroscopic models as limiting cases and enhancing the predictive power of the dynamical description.

REFERENCES

1. Bohr, A., Mottelson, B.R.: Nuclear Structure, Vol. II: Nuclear Deformations. Benjamin, New York (1975)
2. Nix, J.R.: Further studies in the liquid-drop theory of nuclear fission. Ann. Phys. 41, 52–106 (1967)
3. Hofmann, H., Siemens, P.J.: Dissipative properties of large-amplitude collective motion. Nucl. Phys. A 257, 165–187 (1976)
4. Adeev, G.D., Karpov, A.V., Nadtochy, P.N., Vanin, D.V.: Multidimensional stochastic approach to the fission dynamics of excited nuclei. Phys. Part. Nucl. 36, 378–426 (2005)
5. Kramers, H.A.: Brownian motion in a field of force and the diffusion model of chemical reactions. Physica 7, 284–304 (1940)
6. Kubo, R.: The fluctuation-dissipation theorem. Rep. Prog. Phys. 29, 255–284 (1966)
7. Davies, K.T.R., Sierk, A.J., Nix, J.R.: Effect of viscosity on the dynamics of fission. Phys. Rev. C 13, 2385–2403 (1976)
8. Blocki, J., Boneh, Y., Nix, J.R., Randrup, J., Robel, M., Sierk, A.J., Swiatecki, W.J.: One-body dissipation and the super-viscidity of nuclei. Ann. Phys. 113, 330–386 (1978)
9. Hofmann, H.: A quantal transport theory for nuclear collective motion: the merits of a locally harmonic approximation. Phys. Rep. 284, 137–380 (1997)
10. Kubo, R.: Statistical-mechanical theory of irreversible processes. I. J. Phys. Soc. Jpn. 12, 570–586 (1957)
11. Hofmann, H., Ivanyuk, F.A., Yamaji, S.: On the nature of nuclear dissipation as a hallmark for collective dynamics at finite excitations. Nucl. Phys. A 598, 187–234 (1996)
12. Yamaji, S., Ivanyuk, F.A., Hofmann, H.: Variation of transport coefficients for average fission dynamics with temperature and shape. Nucl. Phys. A 612, 1–32 (1997)
13. Ivanyuk, F.A., Hofmann, H., Pashkevich, V.V., Yamaji, S.: Transport coefficients for shape degrees in terms of Cassini ovaloids. Phys. Rev. C 55, 1730–1746 (1997)
14. Ivanyuk, F.A., Hofmann, H.: Pairing and shell effects in the transport coefficients of collective motion. Nucl. Phys. A 657, 19–58 (1999)
15. Hofmann, H., Ivanyuk, F.A.: Nuclear transport at low excitations. Phys. Rev. Lett. 82, 4603–4606 (1999)
16. Hilscher, D., Rossner, H.: Dynamics of nuclear fission. Ann. Phys. Fr. 17, 471–552 (1992)
17. Risken, H.: The Fokker-Planck Equation: Methods of Solution and Applications. 2nd ed. Springer, Berlin (1989)
18. Grangé, P., Li, J.-Q., Weidenmüller, H.A.: Induced nuclear fission viewed as a diffusion process: transients. Phys. Rev. C 27, 2063–2077 (1983)
19. Hofmann, H., Ivanyuk, F.A., Rummel, C., Yamaji, S.: Nuclear fission: the onset of dissipation from a microscopic point of view. Phys. Rev. C 64, 054316 (2001)
20. Thoennessen, M., Bertsch, G.F.: Threshold for dissipative fission. Phys. Rev. Lett. 71, 4303–4306 (1993)
21. Abe, Y., Ayik, S., Reinhard, P.-G., Suraud, E.: On stochastic approaches of nuclear dynamics. Phys. Rep. 275, 49–196 (1996)

22. Fröbrich, P., Gontchar, I.I.: Langevin description of fusion, deep-inelastic collisions and heavy-ion-induced fission. Phys. Rep. 292, 131–237 (1998)
23. Wada, T., Carjan, N., Abe, Y.: Multi-dimensional Langevin approach to fission dynamics. Nucl. Phys. A 538, 283–290 (1992)
24. Nadtochy, P.N., Adeev, G.D., Karpov, A.V.: More detailed study of fission dynamics in fusion-fission reactions within a stochastic approach. Phys. Rev. C 65, 064615 (2002)
25. Pomorski, K., Bartel, J., Richert, J., Dietrich, K.: Temperature dependence of the nuclear friction coefficient. Nucl. Phys. A 605, 87–108 (1996)
26. Sadhukhan, J., Pal, S.: Critical comparison of Kramers' fission width with the stationary width from the Langevin equation. Phys. Rev. C 79, 064606 (2009)
27. Aritomo, Y., Hagino, K., Nishio, K., Chiba, S.: Dynamical approach to heavy-ion induced fission using actinide target nuclei at energies around the Coulomb barrier. Phys. Rev. C 85, 044614 (2012)
28. Sierk, A.J.: Langevin model of low-energy fission. Phys. Rev. C 96, 034603 (2017)