Bulletin of Pure and Applied Sciences.
Vol.40 D (Physics), No.1,
January-June 2021 P.63-68
Original Research Article

Print version ISSN 0970 6569
Online version ISSN 2320 3218
DOI: 10.5958/2320-3218.2021.00009.9
Available online at www.bpasjournals.com

Single Electron Transistors as Position Detectors for Nano-Electomechanical Systems

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Received on 21.02.2021
Accepted on 21.05.2021

ABSTRACT We have studied the single electron transistors are extremely sensitive devices as position detectors for nanoelectromechanical systems. Suspended carbon nanotubes have observed a reduction of the mechanical resonance frequency of the fundamental bending at low bias voltages near the degenerate region. This effect is a precursor of the mechanical instability and thus of the current blockade. We have found that the consequence of a capacitive electromechanical coupling in a suspended single electron transistor when the supporting beam is brought close to the Euler buckling instability by a lateral compression force. The result is that the low bias current blockade originating from the coupling between the electronic degrees of freedom and the classical resonator enhanced by several orders of magnitude in the vicinity of the instability. These results are a direct consequence of the continuous nature of the Eular buckling instability and the associated critical slowing down of the fundamental bending mode of the beam at the instability. Our results frequently have close and instructive analogies with mean field theory of second order phase transitions. In order to increase energy, we can increase the electrostating coupling between the oscillator and the single electron transistor, since energy depends quadratically on electrostatic force and a large change in the gate voltage is found when electrons tunnel. In the way of reducing oscillator spring constant in a controlled manner is to operate a doubly clamped beam subject to a lateral compression force. Under the action of the lateral compression force the system exhibited a continuous transition from a flat to a blockade state, while the fundamental bending mode became softer as approached the mechanical instability. We have found that near the buckling instability the current blockade induced by the mechanical resonator is strongly enhanced, rendering this effect. The obtained results were found in good agreement with previously obtained results.

KEYWORDS Detector, resonator, electromechanical system, current blockade, mechanical instability, compression force, buckling, oscillator, phase transition.

1. INTRODUCTION

Single electron transistors suspended nano-beam exhibited effects originating from the coupling of electronic degrees of freedom to the mechanical oscillations of the suspended structure [1-3]. The reduced size of the mechanical resonator implies that the back action of the single electron transistor have significant effects on the mechanical degree of freedom such as generation of self oscillations [4-6]. Pistolesi and Labarthe [7] presented new effects displayed by device is the current blockade that appears at low bias voltage when the single electron transistor is coupled capacitively to a classical oscillator. The presence of an extra electron on the central island of the single electron transistor induces an additional electrostatic force (F_e) on the oscillator. Thus the equilibrium position of the oscillator is shifted by a distance $\frac{F_e}{k}$, where k is the oscillator spring constant. After such a displacement the gate voltage V_g by the single electron transistor changed by a quantity of the order of $F_e \times \frac{F_e}{ek} \equiv \frac{E_E}{e}$, where e is the electron charge, E_E is the energy. The dimension of the conducting windows in V_g is controlled by V, since at low temperatures, current flow through device of $|V_g| \leq V$ when measuring V_g from the degeneracy point. The current is blocked for $e_V < E_E$ and a mechanical bistability appears. This phenomenon is the classical counterpart of the Frank-condon blockade in molecular devices [8,9].

Leturcq et al. [10] observed this phenomenon in suspended carbon nanotubes for high energy vibrational modes. The classical case has been theoretically studied in the case of a single level quantum dots [11-12] as well as in the metallic case [13]. Kirton et al. [14] analysed the response of a nanomechanical resonator to an external device when it is also coupled to a single electron transistor. The interaction between the single electron transistor electrons and the mechanical resonator depends on the amplitude of the mechanical motion leading to a strongly non linear response to the derive which is similar to that of a Duffing oscillator. They showed that the average dynamics of the resonator is well described by a simple effective model which incorporated damping and frequency renormalization term which are amplitude dependent. They also found that for a certain range of parameters the system displays interesting bistable dynamics in which noise arising from charge fluctuations causes the resonator to switch slowly between different dynamical states. Nanoelectromechanical systems in which the transport electrons in a mesoscopic conductor are coupled to a nanomechanical resonator [15] have been studied extensively over the last few years. Prominent examples include resonator coupled to tunnel Junctions, single electron transistors or quantum dots [16]. The change in the charge transport also necessarily affects the way in which the charges act back on the mechanical system leading to a feedback process which can also generate strongly non linear mechanical dynamics. Such effects have recently studied theoretically and seen experimentally in suspended carbon nanotube systems [17-18].

Weick et al. [19] showed that as long as the displacement of the resonator compared to the distance between the resonator and the single electron transistor island when the two are uncoupled, they made a linear approximation for the dependence of the gate capacitance on the position of the resonator. The rates can be calculated within the orthodox model [20] and in the zero temperature limits. The noise in the current flowing through nanoelectromechanical system is known to provide important information about the dynamics of the system [21]. Assuming that the current is measured on a time scale slower than the derive the relevant quantity is the period averaged current [22]. Alam and Aparajita [23] studied mutual capacitive coupling and tunneling in the single electron transistor coupled to a dopant atom. They observed a spectacular enhancement of the conductance through the single electron transistor when transport occurred by resonant tunneling via the dopant atom. They found that in certain range of temperature of mesoscopic fluctuations of coulomb blockade peaks were suppressed. The coulomb blockades have equal height which was determined by the transparencies of the single electron transistor tuned barriers and was independent of the electron motion inside the single electron transistor island. The peak height remained constant over a large number of coulomb blockade oscillations and various weakly with gate voltage due to the way the gate coupled to the single electron transistor tunnel barriers. Giri et al. [24] studied about single electron transistor realized by a suspended carbon nanotube. Analysis of a single frequency dip has shown that a broadening was obtained increasing the source drain voltage. Some of the observed effects in terms of a model in which the gate voltage acquired assigned time dependence. This phenomenological model, the back action of the nanotube motion on detected current was observed.

2. METHOD

We have considered quantum dot embedded in a doubly clamped beam. The presence of the metallic gate near the dot is responsible for the coupling of the bending modes of the beam to the charge state of the dot. The Hamiltonian of the system can then be written as

$$H = H_{vib} + H_{SET} + Hc$$

where H_{vib} is the oscillating modes of the nanobeam, H_{SET} is the oscillating modes of the nanobeam, H_{SET} is the electronic degrees of freedom of the single electron transistor and Hc is the coupling between the SET and the resonator. The model describes transport through suspended carbon nanotube. Using standard methods of elasticity theory we have shown that, close to the bukling instability the frequency ω of the fundamental bending mode of the nanobeam vanishes, while those of the higher mode remain finite. This allowed us to retain only the fundamental mode parametrized by the displacement $\times$ of the centre of the beam. The Hamiltonian representing the oscillations of the nanobeam thus takes the Laudau-Ginzburg form

$$H_{vib} = \frac{P^2}{2m} + \frac{m\omega^2}{2} X^2 + \frac{\alpha}{4} X^4$$

where P is the momentum conjugate to X. For a doubly clamped uniform nanobeam of length L, linear mass density σ, and bending rigidity k, we can show that close to the instability the effective mass of the beam is $m = \frac{3\sigma L}{8}$. The fundamental bending mode frequency is written as

$$\omega = \omega_0 \sqrt{1 - \frac{F}{F_c}}$$

where F is the compression force, $F_c = \kappa \left(\frac{2\pi}{L} \right)^2$ is the critical force at which bukling occur and $\omega_0 = \sqrt{\frac{\kappa}{\sigma}} \left(\frac{2\pi}{L} \right)^2$.

The positive parameter $\alpha = F_c L \left(\frac{\pi}{2L} \right)^4$ ensures the stability of the system for $F > F_c$. For $F < F_c \left(\omega^2 > 0 \right)$, $X = 0$ is the only stable solution and the beam remains straight. For $F > F_c$, it buckles into one of the two metastable states at $X = \pm \sqrt{\frac{-m\omega^2}{\alpha}}$. We have assumed that the nanobeam can ot rotate around axis due to clamping at its two ends. Electronic transport is accounted for the single electron transistor Hamiltonian consisting of three parts is as follows

$$H_{SET} = H_{dot} + H_{leads} + H_{tun}$$

where H_{dot} is the quantum dot, H_{leads} is the left and right leads and H_{tun} is the tunneling between leads and dot, then

$$H_{dot} = \left(\in_d - eV_g \right) n_d + \frac{U}{2} n_d \left(n_d - 1 \right)$$

where $n_d = d^\dagger d$ and $d^\dagger (d)$ creates (annihates) and electron on the dot.

$$V_g = C_g \frac{V_g}{C_\Sigma},$$

where C_g and C_Σ as the gate and total capacitances of the single electron transistor. The intradot coulomb repulsion is denoted by U. putting $\in_d = 0$, V_g is measured from the degeneracy point. The left and right leads are assumed to be Fermiliquids at temperature T with chemical potentials μ_L and μ_R as measured from $\in_d$. A symmetric bias voltage V is applied to the junction such that $\mu_L = -\mu_R = \frac{eV}{2}$. The lead Hamiltonian is written as

$$H_{leads} = \sum_{ka} \left(\in_k - \mu_a \right) c_{ka}^\dagger c_{ka}$$

where c_{ka} is the annihilation operator for spinless electron of momentum k in lead $a = L.R.$ tunneling is accounted for by the Hamiltonian

$$H_{tun} = \sum_{ka}\left(t_a C_{ka}^{\dagger} d + H.c.\right)$$

where t_a is the tunneling amplitude between the quantum dot and the lead *a*.

Two different kinds of couplings exist between the electronic occupation of the dot n_d and the vibrational degrees of freedom. An intrinsic one that originates from the variation of the electronic energy due to elastic deformation of the beam and an electrostatic one induced by the capacitive coupling to the gate electrode of the single electron transistor. By symmetry the former is quadratic in the amplitude X and its effect on the Euler instability has been considered. The later is linear in X and we have taken the case where the second coupling dominates over the first one. Their relative intensity is controlled by the distance h between the gate electrode and the beam, since the intrinsic coupling does not depend on h, while the electrostatic force depends logarithmically on h. Assuming that the beam is sufficiently close to the gate electrode such that the capacitive coupling dominates, then we have

$$H_c = F_e X n_d$$

Where $-F_e$ is the force exerted on the tube when one excess electron occupies the quantum dot. The model assumed that the gate voltage is such that only charge states with $n_d = 0$ and 1 are accessible. For larger gate voltages overcoming the charging energy of quantum dot, the charges on the dot fluctuate between N and (N+1). This induces an additional constant force bending the tube further. The obtained results have been compared with previously obtained results.

3. RESULTS AND DISCUSSION

The obtained results were compared with numerical calculations of the gap as shown in Figure (1) (a) for $\tilde{\alpha} = 10^{-3}$ and $\tilde{\alpha} = 10^{-6}$. It is evident from the Figure 1 (1) (a) that there is a dramatic increase of the gap close to the instability. For smaller $\tilde{\alpha}$ i.e. the smaller the electromechanical coupling, the larger is the increase of the gap at the instability relative to its value for vanishing compression force as shown in Figure (1) (a). The maximal value of the gap is absolute terms increased with the strength of the electromechanical coupling as F_e. The effect of the compression force is to continuously displace the coulomb diamond in the $v - v_g$ plane toward negative gate voltages as shown in Figure (1) (b) and to open a gap, which is maximal close to the Euler instability at $\delta = 0$. The shift in gate voltage is strongly asymmetric about the Euler instability. While shifts are only small below the Euler instability as shown in Figure (1) (b), the shift in gate voltage are order of magnitudes larger on the buckled side of the Euler instability as shown in Figure (1) (b). These shifts are easily detected consequence of the Euler buckling instability in nanoelectromechanical system. The bias and gate voltages have been measured in units of the elastic energy E_E^o, which is of the order of a few μeV on suspended carbon nanotubes. The smallness of this energy scale explains scaled numerical values of the shifts become so large on the buckled side of the Euler instability. The shape of the coulmb blockade diamond which have shown for the case of intrinsic electron-phonon coupling and for a metallic quantum dot, that the Euler buckling instability leaded to non linear deformations of the coulomb diamonds, a phenomenon that our present results shown that for a capacitive electromechanical coupling and for a single level quantum dot, the shape of coulomb diamond remains unchanged. We have found that the difference is due to the fact that in the single level case, the average occupation of the dot abruptly jumps as function of the gate voltage while in metallic as this occupation gradually changes due to the continuous density of states of the dot. For intrinsic electromechanical coupling the coulomb diamond is not influenced by the latter for compression forces below the critical force $\delta < 0$. This is due to the fact that in the flat state, the quadratic electromechanical coupling merely represents a renormalization of the fundamental bending mode frequency and does not lead to current blockade. The analogy with Landau theory is restricted to the mean field level since contrary to critical phenomena where an infinite number of modes are present; the system we have taken is constituted by a single mode. Beyond mean field theory fluctuation in Landau theory are purely thermal while in our context non equilibrium fluctuation plays an essential role. It is the effects of these fluctuations. It is physically clear that these fluctuations leaded to a smoothening of the current blockade at low bias voltages as the system explore more conducting states in phase space. Within the transport model of a single resonant electronic level with infinite charging energy that we have used, the numbers of electrons on the dot vary between 0 and 1. The range of gate voltages exceeded the charging energy and the average number of excess electron on the dot much larger than one. Due to these excess electrons an additional force- F_N further bends the nanotube and hence increases its vibrational frequency. Thus the bias voltage below which the current is blocked decreased when N increased. The obtained results were compared with previous results and were found in good agreement.

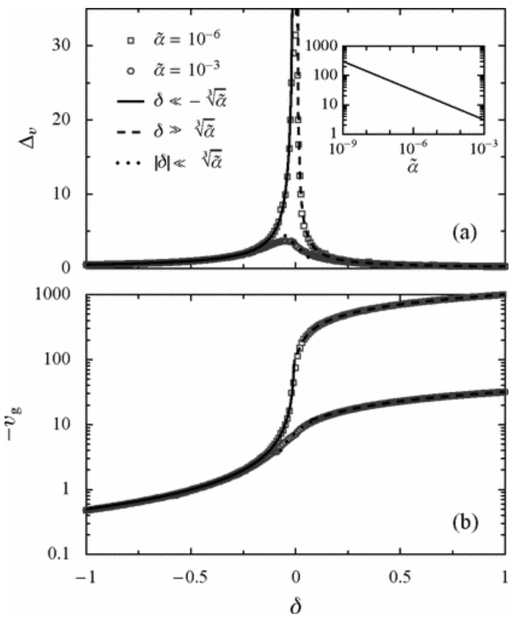

Figure 1: Gap Δ_v and gate voltage v_g

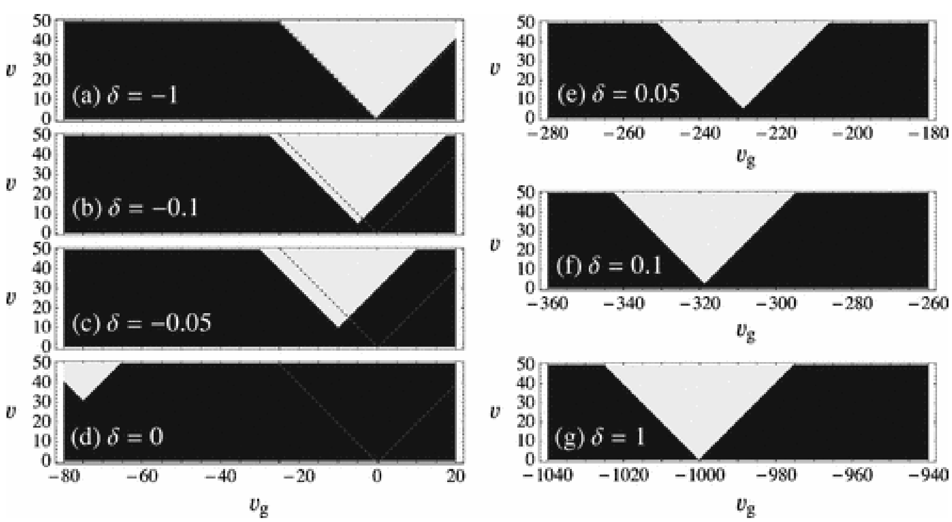

Figure 2: Mean-field current I at zero temperature and for symmetric coupling to the leads $\left(\Gamma_L = \Gamma_R = \frac{\Gamma}{2}\right)$ as a function of bias v and gate voltage v_g.

4. CONCLUSION

We have studied single electron transistors as position detectors for nanoelectro-mechanical systems. We have found that the low bias current blockade originating from the electromechanical coupling for the classical resonator is strongly enhanced near the Euler instability. The bias voltage below which transport is blocked increased by order of magnitude for typical parameters. By increasing the current blockade by exploiting the Euler instability modified anharmonic terms, the temperature and nonequilibrium fluctuations modified the previous results. We have found that near the buckling instability the current blockade induced by the mechanical resonator was strongly enhanced. The enhancement of energy was obtained at mean field level. The effects of thermal and charge fluctuations were found. The consequences of a finite average excess charge on the quantum dot were found. We have found that low bias current blockade due to coupling between the electronic degrees of freedom and the classical resonator was enhanced by several orders of magnitude in the vicinity of the instability. Our results were found close and instructive analogies with mean field theory of second order phase transitions. Single electron transistors embedded in a suspended nanobeam exhibited effects from the coupling of the lectronic degrees of freedom to the mechanical oscillations of the suspended structure. Consequences of a capacitive electromechanical interaction were found when the supporting beam brought close to the Euler buckling instability by a lateral compressive strain. The obtained results were compared with previously obtained results of theoretical and experimental works and were found in good agreement.

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How to cite this article: Amar, A. Bipin Kumar Singh, B.K. (2021). Single Electron Transistors as Position Detectors for Nano-Electomechanical Systems. *Bulletin of Pure and Applied Sciences- Physics*, 40D (1), 63-68.