Bulletin of Pure and Applied Sciences.
Vol.39 D (Physics), No.2,
July-December 2020 P.191-195
Original Research Article

Print version ISSN 0970 6569
Online version ISSN 2320 3218
DOI: 10.5958/2320-3218.2020.00029.9
Available online at www.bpasjournals.com

Coupling of Electron and Magnetic Ion Spin in Quantum Dots by Electron-Electron Interaction

Sanjiv Kumar*, Manish Kumar Bhatt, Sunil Kumar Mishra

Author's Affiliations:	
Sanjiv Kumar	Department of Physics, M.S. College, Motihari, East Champaran, Bihar 845401, India. E-mail: drsanjivkumar01@gmail.com
Manish Kumar Bhatt	Research Scholar, University Department of Physics, B. R. A. Bihar University, Muzaffarpur, Bihar 842001, India. E-mail: m.k.bhatt05@gmail.com
Sunil Kumar Mishra	Department of Physics, M.P.S. Science College, Muzaffarpur, Bihar 842001, India. E-mail: sunilmishra.phy@gmail.com
***Corresponding author:**	**Sanjiv Kumar** Department of Physics, M.S. College, Motihari, East Champaran, Bihar 845401, India. E-mail: drsanjivkumar01@gmail.com
Received on 11.06.2020 **Accepted on 14.10.2020**	

ABSTRACT We have presented a microscopic model of interacting electrons with a magnetic ion spin localized in the centre of a self assembled quantum dot. We have found that the electrons occupying finite angular momentum orbitals interact with the localized spin through an effective exchange interaction mediated by electron-electron interactions. With a localized spin placed in the centre of the dot, only the spins of electrons occupying the zero angular momentum states of the *s*, *d*, shells couple directly to the localized spin via a contact exchange interaction. The effective interaction for p-shell electrons is obtained using exact diagonalization of the microscopic Hamiltonian as a function of the number of electronic shells, shell spacing and anisotropy of the exchange interaction. The anisotropy of exchange interpolates between the interaction types characteristics for conduction band electrons and valence band holes. The obtained results were found in good agreement with previously obtained results.

KEYWORDS Microscopic, Interacting, Magnetic, Localization, Quantum Dot, Spin, Hamiltonian, Diagonalization.

INTRODUCTION

Fauzi et al.[1] and Akiba[2] presented that nuclear spin coupled to fractional quantum hall states. The coupling of a localized spin, either magnetic impurity or nuclear spin, with spins of interacting electrons were studied by Hawrylak[3] and Koenraad et al.[4]. This included the Kondo effect in metals were studied by Li et al.[5] and Jacob et al.[6]. The quantum dots and the impurity spin in diamond were presented by Latta et al.[7] and Mamin et al.[8]. Qu et al.[9] and Nguyen et al.[10] presented the interplay between electron-electron coulomb interactions and the electron-Magnese exchange interaction has been studied using exact diagonalization techniques and it was also studied by Govorov[11] and Kalameitsev et al.[12] using mean field approach. Mendes et al.[13], Trojnar et al.[14] and Korkusinki etal[15]

studied on electron-electron interactions in excitonic complexes coupled with localized spins. Roy et al.[16] studied the influence of the nearest antiferromagnetic interaction with $t-t'-J-J'$ model in an exact technique. An 8 site square cluster has been chosen as the representative of the system. The effect of the next-nearest neighbor antiferromagnetic interaction on various ground state properties such as electron-electron correlation, spin-spin correlation, effective hopping amplitude etc has been studied. Kumari and Kumar[17] studied the Kondo box effect by varying the coupling between the dots and the chain. A Kondo box can be realized in systems of two impurities coupled between them by a finite number of non interacting sites. They have shown that finite size effects can take place together with a magnetic interaction between the impurities of the Ruderman-Kittel-Kasaya-Yosida type. They have found that when Kondo effect is present, the fourth order Ruderman-Kittel-Kasaya-Yosida interaction between the impurities is mediated by the electron of the non-interacting sites, which are participating simultaneously in the Kondo screeing of each impurity. Roy and Ghosh[18] studied the effect of next nearest neighbor hopping and exchange interactions on pairing mode (s or d-wave) and hole binding the $t-t'-J-J'$ model, had been considered within a eight-site square cluster i.e. tilted. Using the Lanczos exact diagonalization method, the dominance of s-wave was identified. The formation of two hole pair was observed. Leet et al.[19], Morello et al.[20] and Saeedi et al.[21] showed the case of interaction between electrons and single nuclear spins and presented that it is possible to manipulate a few nuclear spins in diamond, silicon and carbon nanotubes. The obtained results were compared with the previous obtained results.

METHOD

We have considered a model system of N electrons confined in a two dimensional parabolic quantum dot with a single magnetic impurity in the centre. We have taken an iso-electronic impurity, a magnese ion with a total spin $M=\frac{5}{2}$ in a quantum dot. In the effective mass and envelope function approximations, the single particle states of a two dimensional harmonic oscillator with the characteristic frequency ω_o. The two orbital quantum numbers are represented by $i=\{n,m\}$ and the electron spin $\sigma=\pm\frac{1}{2}$. The single particle states are characterized by energy $E_{n,m}=\omega_o\left(n+m+1\right)$ and angular momentum $L_e=n-m$. We have expressed all energies in units of the effective Rydberg, $Ry^*=\frac{m^*e^4}{2\in^2\hbar^2}$ and all distances in units of the effective Bohr radius $a_B^*=\frac{\in\hbar^2}{m^*e^4}$, where m^*, e, $\in$ and $\hbar$ are the electron effective mass and charges, the dielectric constant and the reduced plank constant. We have taken $m^*=0.1m_0$ and $\in=10.6$, where m_0 is the free electron mass and $Ry^*=12.11$ meV and $a_B^*=5.61$ nm. We have also taken the harmonic oscillator frequency $\omega_0=1.98Ry^*$.

The Hamiltonian of N electrons confined in quantum dot and interacting with a magnese spins is written as

$$H=\sum_{i,\sigma}E_{i,\sigma}c_{i,\sigma}^{\dagger}c_{i,\sigma}+\frac{\gamma}{2}\sum_{\substack{i,j,k,l\\ \sigma,\sigma'}}\left\langle i,j\left|V_{ee}\right|k,l\right\rangle c_{i,\sigma}^{\dagger}c_{j,\sigma'}^{\dagger}c_{k,\sigma'}c_{l,\sigma}$$

$$-\sum_{i,j}\frac{J_{i,j}(R)}{2}\Big[\left(c_{i,\uparrow}^{\dagger}c_{j,\uparrow}-c_{i,\downarrow}^{\dagger}c_{j,\downarrow}\right)M_z+\varepsilon\left(c_{i,\downarrow}^{\dagger}c_{j,\uparrow}M^{+}\right.$$

$$\left.+c_{i,\uparrow}^{\dagger}c_{j,\downarrow}M^{-}\right)\Big]$$

where $c_{i,\sigma}^{\dagger}\left(c_{i,\sigma}\right)$ creates (annihilates) an electron on the orbital $i=\{m,n\}$ with σ.

The anisotropy of the exchange interaction is tuned by the factor ε. By setting $\varepsilon=0$ we have obtained the anisotropic Ising *e*-magnese exchange Hamiltonian and setting $\varepsilon=1$, we have obtained the isotropic Heisenberg exchange Hamiltonian. The spin projections S_z and M_z are separately good

quantum numbers. The total spin projection of the electron depends on the number and polarization of the particles. For the case $\varepsilon = 1$ we have established the total spin quantum number J of the given manifold of states by considering its degeneracy $g(J) = 2J + 1$.

The eigenenergies and eigen states of the Hamiltonian have been obtained in the configuration interaction approach. In this approach we have constructed the Hamiltonian matrix in the basis of configuration of N-electron and one magnese spin and $N = N_{\uparrow} + N_{\downarrow}$ is the number of electrons in which $N_{\uparrow}$ and $N_{\downarrow}$ are the number of electrons with spin up and spin down. The total number of configurations depends on the number of electrons and on the number of the harmonic oscillation shells available in the quantum dot. With magnese impurity in the centre the total orbital angular momentum of electrons $L = \sum_{i=1}^{N} L_e^i$ is conserved by the Hamiltonian. Moreover depending on the anisotropy of e-magnese interactions, the Hamiltonian also conserved the total projections S_z and M_z of the electron and magnese spin separately or the projection $J_z = s_z + M_z$ of the total spin. Based on these conservation rules, we have divided the basis of configurations into sub-spaces labeled by the numbers *L*, s_z and M_z or *L* and J_z and diagonalized the Hamiltonian in each sub space separately. Our model is also suitable for electrons interacting with a single nuclear spin. The obtained results have been compared with previously obtained results.

RESULTS AND DISCUSSION

Figure (1)(a) shows the effect of the electron-electron coulomb interaction on the low energy spectrum of the four electron and magnese complex. The appearance to triplet and singlet energy shells, separated by the electron-electron exchange interaction. The splitting of the triplet shell is governed by the electron-electron and electron-magnese exchange interactions. Figure (1) (b) shows the energy difference Δ, i.e. the effective exchange coupling as a function of ω_0 for four interacting election confined in the magnese doped quantum dot with three confined shells. We have also found a ferromagnetic to antiferromagnetic crossing as a function of the quantum dots with four shells the ferromagnetic to antiferromagnetic crossing occurred at $\omega_0 \approx 0.04\ Ry^*$. We have presented the effect of the symmetry of the electron-magnese coupling on the four electron ground state.

Figure (1)(c) shows comparative effects of the anistropic for $\varepsilon = 0$ and isotropic for $\varepsilon = 1$ coupling for a magnese doped quantum dot with three confined shells and in the presence of a full electron-electron coulomb interaction for $\gamma = 1$. For the anistropic coupling the triplet state is split into nine doubly degenerate levels. In this case S_z and M_z are good quantum numbers and therefore $s_z = 1$ and $s_z = -1$ breaks the magnese spin degeneracy into six. As the energy of the state with $s_z = 1$ and M_z is equal to the energy of the state $s_z = -1$ and $M_z = -1$, these six states are double degenerate. The $s_z = 0$ configurations split into three, where the degeneracy is given by M_z, i.e. the $s_z = 0$ configurations are degenerate and labeled by $|M_z|$, as for the two electrons interacting with the magnese via an anistropic interaction. The singlet state is also split into three doubly degenerate levels. The obtained results were compared with previous obtained results and were found in good agreement.

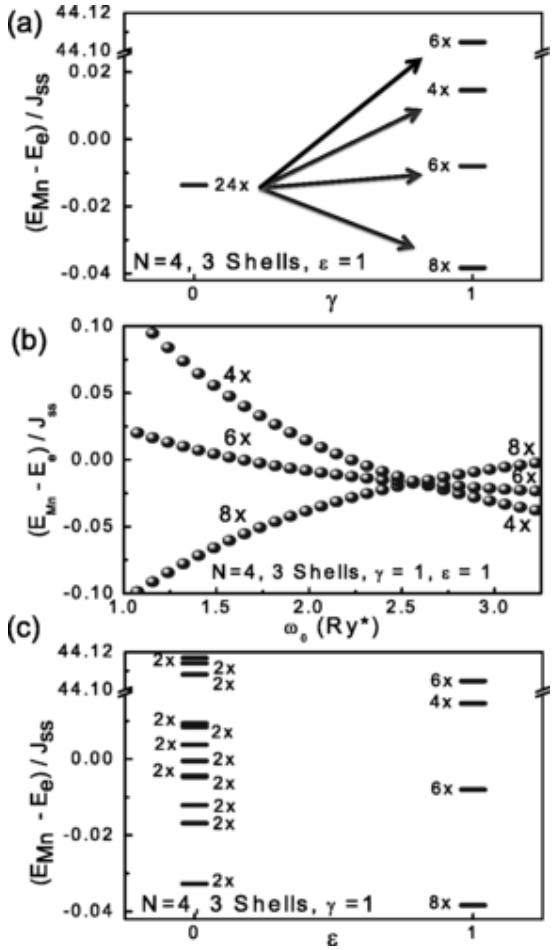

Figure 1: (a) Energy difference Δ for noninteracting $(\gamma = 0)$ and ineracting $(\gamma = 1)$ electrons in the four electron magnetic dot. (b) Ground state energy difference as a function of the Quantum dot shell spacing ω_0 for three confined in the quantum dot. (c) Ground state energy difference for the anisotropic $(\varepsilon = 0)$ and isotropic $(\varepsilon = 1)$ e-Mn coupling.

CONCLUSION

We have studied indirect coupling of electron and magnetic ion spin in quantum dots by electron-electron interaction. We have shown that the effective electron-electron mediated exchange interaction be ferromagnetic by varying quantum dot parameters. We have found that when the electron-electron interactions are included in the study, the electrons occupying fininte angular momentum orbitals interact with the localized spin. This effective interaction is obtained using exact diagonlization of the microscopic Hamiltonian as a function of the number of electronic shells, shell spacing and anisotropy of the electron-magnese exchange interaction. We have used the model of a self assembled quantum dot with a single magnese impurity in its centre and a controlled number of electrons. We have the results of exact diagonalization of the model Hamiltonian for quantum dots confining from two to six electrons and the emergence of the indirect electron magnese coupling for quantum dots with a partially filled *p*-shell. The obtained results were compared with previously obtained results of theoretical and experimental works and were found in good agreement.

REFERENCES

1. Fauzi. M. H, Watanabe. S and Hirayama. Y, (2012), Appl. Phys. Lett. 101, 162105.
2. Akiba. K, Yuge. T, Kahasugi. S, nagase. K and Hirayama. Y, (2013), Phys. Rev. B, 87, 235309.
3. Hawrylak. P, (2011), in the Physics of Diluted Magnetic semiconductors edited by Gaj. J and Kossut. J, Springer series in Materials Science, (Springer, Berlin)
4. Koenraad. P. M and Flatte. M. E, (2011), Nat. Mater. 10, 91.
5. Li. J, Schneider. W. D, Berndt. R and Delley. B, (1998), Phys. Rev. Lett. 80, 2893.
6. Jacob. D, Soriano. M and Polacios. J. J, (2013), Phys. Rev. B, 88, 134417.
7. Latta. C, Haupt. F etal, (2011), Nature (London), 474, 627.
8. Mamin. H. J, Kim. M, Sherwood, M. H, Rettner. C. T, Ohno. K, Awschalom. D. D and Rugar. D, (2013), Science, 339, 557.
9. Qu. F and Hawrylak. P, (2005), Phys. Rev. Lett. 95, 217206.
10. Nguyen. N. T. T and Peeters. F. M, (2008), Phys. Rev. B, 78, 245311.
11. Govrov. A. O, (2004), Phys. Rev. B, 70, 035321.
12. Kalameitsev. A. V and Govorov. A. O, (2005), Phys. Rev. B, 71, 035338.
13. Mendes. U. C, Korkusinski. M, Trojnar. A. H and Hawrylak. P, (2013), Phys. Rev. B, 88, 115306.
14. Trojnar. A. H, etal, (2011), Phys. Rev. Lett. 107, 207403.
15. Trojnar. A. H, Korkusinski. K, Goryca. M etal, (2013), Phys. Rev. B, 87, 2053011.
16. Roy. K and Ghosh. N. K (2018), J. BPAS, Vol – 37, Phys, No -1, p-27.
17. Kumari Rambha and Kumar Ashok, (2019), J. BPAS, Vol -38D, Phys., No-2, p- 55.
18. Roy. K and Ghosh. N. K, (2020), J. BPAS, Vol – 39D, Phys., No -1, p-66.
19. Lee. S. Y etal (2013), Nat. Nanotechnol, 8, 487.
20. Morello. A, etal (2010), Nature (London), 467, 687.
21. Saeedi. K, etal, (2013), Science, 342, 830.