Bulletin of Pure and Applied Sciences.
Vol.38 D (Physics), No.2,
July-December 2019 P.86-90

Print version ISSN 0970 6569
Online version ISSN 2320 3218
DOI: 10.5958/2320-3218.2019.00014.9

Original Research Article

Available online at www.bpasjournals.com

Change in Mass of Spinning Black Holes Due to Change in Angular Momentum

Anuradha Kumari, Alok Ranjan, Dipo Mahto*

Author's Affiliations:

Anuradha Kumari	Research Scholar Department of Physics, T. M. B. University, Bhagalpur, Bihar 812007, India E-mail: anuphy1987@gmail.com
Alok Ranjan	Research Scholar Department of Physics, T. M. B. University, Bhagalpur, Bihar 812007, India
Dipo Mahto	Associate Professor, Department of Physics, Marwari College, T. M. B. University, Bhagalpur, Bihar 812007, India.
***Corresponding author:**	**Dipo Mahto** Associate Professor, Department of Physics, Marwari College, T. M. B. University, Bhagalpur, Bihar 812007, India. E-mail: dipomahto@hotmail.com

Received on 22.07.2019
Accepted on 07.12.2019

ABSTRACT The present paper reviews the change in mass of spinning black holes w.r.t. the angular momentum in AGN as proposed by Mahto, D. & Kumari, A. to show that the change in mass of spinning black holes due to change in the angular momentum decreases with increasing their masses for both categories of spin a*=1/2 & a*=1 in the same manner provided that the change in mass of spinning black holes w.r.t. the angular momentum for a*=1/2 is lesser than to that of spinning black holes of the same mass for a*=1.

KEYWORDS Black holes, Angular momentum.

INTRODUCTION

Quantum mechanically, there is a possibility that one of a particle production pair in a black hole is able to tunnel the gravitational barrier and escapes the black hole's horizon and hence, it can radiate or evaporate particles [1]. This emission of radiation is known as Hawking radiation. The mass, area and surface gravity of the black hole mechanics are analogous quantities to the energy, entropy and temperature in the ordinary laws of thermodynamics respectively[2]. In 2006, Aschenbach has shown that the orbital velocity of a test particle is no longer a monotonic function of the orbit radius when the spin of the black hole is greater than 0.9953 , but displays a local minimum-maximum structure for radii smaller than 1.8 gravitational radii [3]. In 2009, Richard B Larson suggested that in all cases, gravitational interactions with other stars or mass concentrations in a forming system play an important role in redistributing angular momentum and thereby enabling the formation of a compact object [4]. In 2018, Mahto and Kumari gave the comparative study for the change in mass of the spinning black holes due to corresponding change in the angular momentum for half spin and maximum spin of the black holes [5]. In the present paper, we have reviewed the above work to show the change in mass of spinning black holes w.r.t. the angular momentum decreases with increasing their masses for both categories of spin a*=1/2 & a*=1 in the same manner provided that the change in mass of spinning black holes w.r.t. the angular momentum for a*=1/2 is lesser than to that of spinning black holes of the same mass for a*=1.

THEORETICAL MODEL

For spinning black holes having maximum spin, spin parameter $a*$=1 [5,6].

$$\frac{\delta M}{\delta J} = \frac{1}{2M} \quad (1)$$

For half spin parameter (a*=1/2), the equation (1) becomes [5,7]

$$\frac{\delta M}{\delta J} = \frac{1}{4M} \quad (2)$$

The equation (1) & (2) show the change in mass of the spinning black holes due to corresponding change in the angular momentum for maximum and half spinning rate of black holes respectively.

DATA IN THE SUPPORT FOR MASS OF BLACK HOLES AND SUN

There are two categories of black holes classified on the basis of their masses clearly very distinct from each other, with very different masses M ~ 5 - 20$M_\odot$ for stellar – mass Black holes in X-ray binaries and M ~10^6 – $10^{9.5}M_\odot$ for super massive black holes in Active Galactic Nuclei[8]. Mass of sun ($M_\odot$) = 1.99 x 10^{30} kg[8].

On the basis of the data mentioned in the section 3, the change in mass of different test spinning black holes due to corresponding change in angular momentum are calculated in AGN to plot the graphs with the help of equation(1) & (2) as shown in figure.1.

Table 1: Change in mass of spinning black holes due to corresponding change in angular momentum in AGN for a*=1.

S. No.	Mass of BH_s (M) in solar mass	Mass of BH_s (M) in solar mass	$\frac{\delta M}{\delta J}$	$\frac{\delta M}{\delta J}$
1	1 x 10^6 $M_\odot$	0.1x10^7	5.000 x10^{-37}	50.000 x10^{-38}
2	2 x 10^6 $M_\odot$	0.2 x10^7	2.500 x10^{-37}	25.000 x10^{-38}
3	3 x 10^6 $M_\odot$	0.3 x10^7	1.666x10^{-37}	16.660 x10^{-38}
4	4 x 10^6 $M_\odot$	0.4 x10^7	1.250 x10^{-37}	12.500 x10^{-38}
5	5 x 10^6 $M_\odot$	0.5 x10^7	1.000 x10^{-37}	10.000 x10^{-38}
6	6 x 10^6 $M_\odot$	0.6 x10^7	0.833 x10^{-37}	8.330 x10^{-38}
7	7 x 10^6 $M_\odot$	0.7 x10^7	0.714 x10^{-37}	7.140 x10^{-38}
8	8 x 10^6 $M_\odot$	0.8 x10^7	0.625 x10^{-37}	6.250 x10^{-38}
9	9 x 10^6 $M_\odot$	0.9 x10^7	0.555 x10^{-37}	5.555 x10^{-38}
10	1 x 10^7 $M_\odot$	1 x10^7	5.000 x10^{-38}	5.000 x10^{-38}
11	2 x 10^7 $M_\odot$	2 x10^7	2.500 x10^{-38}	2.500 x10^{-38}

12	3 x 10^7 M_Θ	3 x10^7	1.666x10^{-38}	1.666 x10^{-38}
13	4 x 10^7 M_Θ	4 x10^7	1.250 x10^{-38}	1.125 x10^{-38}
14	5 x 10^7 M_Θ	5 x10^7	1.000 x10^{-38}	1.000 x10^{-38}
15	6 x 10^7 M_Θ	6 x10^7	0.833 x10^{-38}	0.833 x10^{-38}
16	7 x 10^7 M_Θ	7 x10^7	0.714 x10^{-38}	0.714 x10^{-38}
17	8x 10^7 M_Θ	8 x10^7	0.625 x10^{-38}	0.625 x10^{-38}
18	9 x 10^7 M_Θ	9 x10^7	0.555 x10^{-38}	0.555 x10^{-38}
19	1 x 10^8 M_Θ	10 x10^7	5.000 x10^{-39}	0.500 x10^{-38}
20	2 x 10^8 M_Θ	20 x10^7	2.500 x10^{-39}	0.250 x10^{-38}
21	3 x 10^8 M_Θ	30 x10^7	1.666x10^{-39}	0.166 x10^{-38}
22	4 x 10^8 M_Θ	40 x10^7	1.250 x10^{-39}	0.125 x10^{-38}
23	5 x 10^8 M_Θ	50 x10^7	1.000 x10^{-39}	0.100 x10^{-38}
24	6 x 10^8 M_Θ	60 x10^7	0.833 x10^{-39}	0.083 x10^{-38}
25	7 x 10^8 M_Θ	70 x10^7	0.714 x10^{-39}	0.071 x10^{-38}
26	8 x 10^8 M_Θ	80 x10^7	0.625 x10^{-39}	0.062 x10^{-38}
27	9 x 10^8 M_Θ	90 x10^7	0.555 x10^{-39}	0.053 x10^{-38}
28	1 x 10^9 M_Θ	100 x10^7	5.000 x10^{-40}	0.050 x10^{-38}
29	2 x 10^9 M_Θ	200 x10^7	2.500 x10^{-40}	0.025 x10^{-38}
30	3 x 10^9 M_Θ	300 x10^7	1.666x10^{-40}	0.016 x10^{-38}

Table 2: Change in mass of spinning black holes due to corresponding change in angular momentum in AGN for a*=1/2.

S. No.	Mass of BH_s (M) in solar masses	Mass of BH_s (M) in solar mass	$\frac{\delta M}{\delta J}$	$\frac{\delta M}{\delta J}$
1	1 x 10^6 M_Θ	0.1x10^7	2.500 x10^{-37}	25.000 x10^{-38}
2	2 x 10^6 M_Θ	0.2 x10^7	1.250 x10^{-37}	12.500 x10^{-38}
3	3 x 10^6 M_Θ	0.3 x10^7	0.833x10^{-37}	8.330 x10^{-38}
4	4 x 10^6 M_Θ	0.4 x10^7	0.625 x10^{-37}	6.250 x10^{-38}
5	5 x 10^6 M_Θ	0.5 x10^7	0.500 x10^{-37}	5.000 x10^{-38}
6	6 x 10^6 M_Θ	0.6 x10^7	0.416 x10^{-37}	4.160 x10^{-38}
7	7 x 10^6 M_Θ	0.7 x10^7	0.357 x10^{-37}	3.570 x10^{-38}
8	8 x 10^6 M_Θ	0.8 x10^7	0.312 x10^{-37}	3.120 x10^{-38}
9	9 x 10^6 M_Θ	0.9 x10^7	0.277 x10^{-37}	2.777 x10^{-38}
10	1 x 10^7 M_Θ	1 x10^7	2.500 x10^{-38}	2.500 x10^{-38}
11	2 x 10^7 M_Θ	2 x10^7	1.250 x10^{-38}	1.250 x10^{-38}
12	3 x 10^7 M_Θ	3 x10^7	0.833x10^{-38}	0.833 x10^{-38}
13	4 x 10^7 M_Θ	4 x10^7	0.625 x10^{-38}	0.625 x10^{-38}
14	5 x 10^7 M_Θ	5 x10^7	0.500 x10^{-38}	0.500 x10^{-38}
15	6 x 10^7 M_Θ	6 x10^7	0.416 x10^{-38}	0.416 x10^{-38}
16	7 x 10^7 M_Θ	7 x10^7	0.357 x10^{-38}	0.357 x10^{-38}
17	8x 10^7 M_Θ	8 x10^7	0.312 x10^{-38}	0.312 x10^{-38}
18	9 x 10^7 M_Θ	9 x10^7	0.277 x10^{-38}	0.277 x10^{-38}
19	1 x 10^8 M_Θ	10 x10^7	2.500 x10^{-39}	0.250 x10^{-38}
20	2 x 10^8 M_Θ	20 x10^7	1.250 x10^{-39}	0.125 x10^{-38}
21	3 x 10^8 M_Θ	30 x10^7	0.833x10^{-39}	0.083 x10^{-38}
22	4 x 10^8 M_Θ	40 x10^7	0.625 x10^{-39}	0.062 x10^{-38}
23	5 x 10^8 M_Θ	50 x10^7	0.500 x10^{-39}	0.050 x10^{-38}
24	6 x 10^8 M_Θ	60 x10^7	0.416 x10^{-39}	0.041 x10^{-38}

25	7 x 10^8 M_Θ	70 x10^7	0.357 x10^{-39}	0.035 x10^{-38}
26	8 x 10^8 M_Θ	80 x10^7	0.312 x10^{-39}	0.031 x10^{-38}
27	9 x 10^8 M_Θ	90 x10^7	0.277 x10^{-39}	0.027 x10^{-38}
28	1 x 10^9 M_Θ	100 x10^7	2.500 x10^{-40}	0.025 x10^{-38}
29	2 x 10^9 M_Θ	200 x10^7	1.250 x10^{-40}	0.012 x10^{-38}
30	3 x 10^9 M_Θ	300 x10^7	0.833x10^{-40}	0.008 x10^{-38}

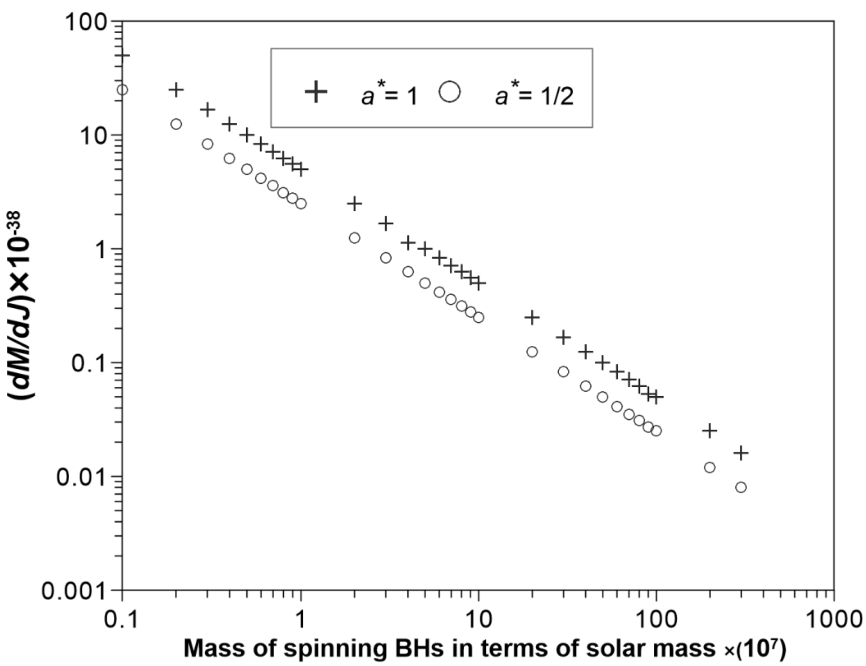

Figure 1: The graph plotted between the mass of spinning black holes and corresponding change in their angular momentum in AGN with spin parameter a*=1/2 and 1.

RESULTS AND DISCUSSION

The work is started with use of equations (1) and (2) to calculate the change in mass of the spinning black holes due to corresponding change in the angular momentum for maximum spinning rate of black holes (a*=1) given by the following equation

$$\frac{\delta M}{\delta J} = \frac{1}{2M}$$

and the change in mass of the spinning black holes due to corresponding change in the angular momentum for maximum spinning rate of black holes (a*=1/2) given by the following equation.

$$\frac{\delta M}{\delta J} = \frac{1}{4M}$$

We have plotted the graph between the change in mass of the spinning black holes w.r.t. the change in the angular momentum for maximum spinning rate of black holes (a*=1) and spinning parameter (a*=1/2) $(\delta M / \delta J)$ and corresponding value of the mass of black holes (M) and we have shown that the change in mass of the spinning black holes due to corresponding change in the angular momentum for maximum spinning rate of black holes (a*=1) is greater than to that of the spinning black holes having spinning parameter (a*=1/2). This fact is also clear from the comparative study of table (1) & (2) and nature of the graph plotted in the figure1 for both cases.

The figure 1 shows the joint graph plotted the graph between the change in mass of the spinning black holes w.r.t. the change in the angular momentum for maximum spinning rate of black holes $(\delta M / \delta J)$ and corresponding value of the mass of black holes (M) for (a*=1) and spinning parameter (a*=1/2). This shows that the change in mass of the spinning black holes w.r.t. the change in the angular momentum for maximum spinning rate of black holes (a*=1) is twice times to that of spinning of black holes with spinning parameter (a*=1/2) for the same mass.

From the graph in the figure 1, it is clear that the change in mass of the spinning black holes w.r.t. the change in the angular momentum for spinning rate of black holes (a*=1& a*=1/2) decreases with increasing the value of the mass of the different spinning black holes.

CONCLUSION

The change in mass of the spinning black holes w.r.t. the change in the angular momentum for maximum spinning rate of black holes (a*=1) is similar manner to that of the spinning black holes with spin parameter (a*=1/2), provided that the change in mass of the spinning black holes due to corresponding change in the angular momentum for maximum spinning rate of black holes (a*=1) is greater than to that of the spinning black holes having spinning parameter (a*=1/2).

ACKNOWLEDGEMENT

The authors are grateful to Prof. Ashutosh Prasad, HOD & Prof. Kamal Prasad, University Department of Physics, TMBU Bhagalpur for finding out the shortcomings in the original manuscript. The authors are also obliged to Prof. Jagdhar Mandal, Dr. S.K. Jaiswal and Dr. S. Prasad, Associate Professor to give good suggestions to make it better.

REFERENCES

[1]. Triyanta & Bowaire, A. N.: Hawking Temperature of the Reissner-Nordstrom-Vaidya Black Hole, J. Math. Fund. Sci., Vol. 45, No. 2, 114-123, 2013.

[2]. Bardeen, J.M., Carter,B. and Hawking, S.W.(1973): The four laws of black hole mechanics. Comm. Math. Phy. 31 (2): 161-170. doi: 10.1007/BFO1645742, 1973.

[3]. Aschenbach, B.: Mass and Angular Momentum of Black Holes: An Overlooked Effect of General Relativity Applied to the Galactic Center Black Hole Sgr A∗, Chinese Journal of Astronomy and Astrophysics, 2006.

[4]. Richard, B. L.:Angular momentum and the formation of stars and black holes, Reports on Progress in Physics, **73,** 014901 pp14, 2010.

[5]. Mahto D. & Kumari A.: Change in mass of spinning black holes w.r.t. the angular momentum, Journal of Modern Physics, p1037-1042, 2018.

[6]. Mahto, D., Kumara, A. and Singh, K.M.: Energy and Entropy Change of Spinning Black Holes, International Journal of Engineering and Innovative Technology, Vol. 3, 2013.

[7]. Mahto, D., Singh, A.K. & Kumari, N.: Change in Internal Energy and Enthalpy of Spinning Black Hole with Half Spin Parameter in XRBs, International Journal of Astronomy and Astrophysics, , 6, 328-333, 2016.

[8]. Narayan, R.: Black holes in Astrophysics, arXiv:gr-qc/0506078 V1, 2005.